Cluster Analysis Of Wireless Channel Impulse Responses

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***Abstract*— This paper introduces a novel wireless channel clustering technique based on the Saleh-Valenzuela channel model. The impulse response is regarded as a single realization of the statistical channel model based on which the prior density functions of cluster arrival times are derived. Cluster analysis is done by means of extending the original Saleh-Valenzuela model by allowing different cluster arrival rates. It is shown that the estimation of cluster arrival rates is essentially equivalent to clustering the impulse response and estimating the expected cluster centers. Having defined the clusters, other parameters of the Saleh-Valenzuela model can be estimated from a single realization. The proposed method has been applied to simulated as well as measured channel impulse responses.**

I. Introduction

Wireless systems are subject to fading - time variations of the receiving conditions caused by multipath propagation and transceiver movements. A wireless channel contains all the information about the propagation environment, and, in general, the receiving side should be aware of the channel. Due to the fading nature of the wireless channel, it is imperative for the system to follow the variations of the receiving conditions along time to adapt itself and sustain reasonable communication quality.

Generally, the wireless channel consists of a series of attenuated, time-delayed, phase shifted replicas of the transmitted signal. In the baseband this will be represented as follows:

$$h_t(\tau) = \sum_{i=0}^{N-1} \beta_i(t,\tau) \exp\left(j\theta_i(t,\tau)\right)\delta(\tau - \tau_i(t)) \qquad (1)$$

where $\beta_i(t,\tau)$ and $\tau_i(t)$ are the real amplitudes and excess delays of the ith multipath component at the time t, respectively (see, for example, [1], ch. 4). The phase term $\theta_i(t,\tau)$ lumps together all the mechanisms for phase shifts of a single multipath component within the ith excess delay bin. Equation (1) is a starting point for further analysis. The straight-forward procedure to follow the channel is to follow each of the contributing reflections. Obviously, it is a hard task to accomplish, since the number of contributions could be significant, but if successful, it will provide quite a precise instantaneous description of the channel dynamics. Although it is computationally impractical to follow all of the contributing paths, it is still possible to follow at least several strong ones [2], [3]. Alternatively, a number of statistical models provide probability density functions (PDFs) for the parameters of interest [4], [5], thus giving a probabilistic description of the channel behavior. However, for on-line operation the communication system has to know the instantaneous parameter values. It would seem beneficial to join both ideologies and devise a method to extract some information from the instantaneous impulse response based on the probabilistic model of the channel. This could be accomplished if the channel impulse response is regarded as a sample realization of the corresponding probabilistic model. We can think of the instantaneous channel parameters as of samples from the corresponding distributions. Among a number of different statistical models, the Saleh-Valenzuela (S.-V.) channel model[6] has attracted our attention because it provides a very promising framework for implementing these ideas. The basic idea behind the S.-V. model is based on the assumption that rays arrive in clusters. The clusters arrive with a certain rate at random time instances, as well as the individual rays within each cluster. Such a structure imposed on the impulse response provides, first of all, a basis for hierarchical analysis (i.e., from the impulse response to clusters and rays), as well as physical interpretation of the clusters and model parameters. Although the S.-V. model was initially developed for indoor channels, the same ideas could still be valid for outdoor communications with wideband and directional systems [7]. We show how the cluster arrival rate could be learned from the instantaneous impulse response, as well as how the clusters can be identified.

The rest of the paper is organized as follows: Section II describes the parameter estimation algorithm and clustering procedure, along with some practical considerations; Section III shows the application of the proposed clustering algorithm to simulated channels; and Section IV provides some results of algorithm application to measured channel impulse responses.

II. Cluster analysis algorithm

A. General description of the Saleh-Valenzuela model

The basic idea of the S.-V. model is easily understood from considering the impulse response in Fig. 1. The model assumes rays arriving in clusters. The time between cluster arrivals is random and distributed exponentially with a parameter Λ. Likewise, within the cluster the time between successive ray arrivals is also exponentially distributed with a parameter λ, $\lambda >> \Lambda$:

$$\begin{aligned} p(T_l|T_{l-1}) &= \Lambda \exp\left[-\Lambda(T_l - T_{l-1})\right] \\ p(\tau_{k,l}|\tau_{k-1,l}) &= \lambda \exp\left[-\lambda(\tau_{k,l} - \tau_{k-1,l})\right] \end{aligned} \quad (2)$$

Here, T_l is the arrival time of the lth cluster, given the previous arrival at T_{l-1}, $T_l \geq T_{l-1}$. Likewise, $\tau_{k,l}$ is the arrival time of the kth ray within the lth cluster, given the preceding ray at $\tau_{k-1,l}$, $\tau_{k,l} \geq \tau_{k-1,l}$.

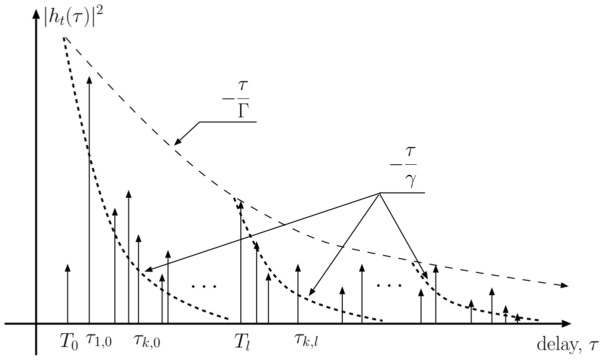

Fig. 1. A channel impulse response with clusters.

The power gain $\beta^2_{k,l}$ is assumed by the model to be independent of the associated delays and distributed exponentially:

$$p(\beta^2_{k,l}) = \left(\overline{\beta^2_{k,l}}\right)^{-1} \exp\left(-\beta^2_{k,l}/\overline{\beta^2_{k,l}}\right)$$

where the expected power gain $\overline{\beta^2_{k,l}}$ is a function of the delay τ:

$$\overline{\beta^2_{k,l}} = \overline{\beta^2_{0,0}} \exp\left(-\frac{T_l}{\Gamma}\right) \exp\left(-\frac{\tau_{k,l} - \tau_{0,l}}{\gamma}\right)$$

Here, $\overline{\beta^2_{0,0}}$ is the expected power gain of the first ray in the first cluster. Γ and γ are power-delay time constants for the clusters and rays, respectively (see Fig. 1).

Next, we formulate the clustering algorithm.

B. PDFs for cluster arrival times

Let us consider a snapshot of the time-varying channel impulse response $h_t(\tau)$, for example, the one shown in Fig. 1. We will drop explicitly the time dependency for notation simplicity and write $h(\tau)$ only. We can think of this impulse response as a single realization of the S.-V. model.

Let us assume, that this particular realization has L clusters, arriving at unknown times T_l, $l = 0, 1, \ldots L-1$. Without any loss of generality, let us also assume that $T_0 \equiv 0$, i.e., the first cluster arrives at the moment $\tau = 0$. Defining the time between consecutive cluster arrivals as $\Delta T_l = T_l - T_{l-1}$, the positions of the clusters are given as:

$$T_l = \sum_{m=1}^{l} \Delta T_m \quad (3)$$

From the original model it follows that the interarrival times ΔT_m's are statistically independent random variables. This leads to the derivation of the PDFs $p_l(T)$ of the corresponding cluster arrival times. Indeed, from (3) it follows that $p_l(T)$ is a convolution of l exponential distributions. It can be shown that $p_l(T)$ is a chi-square distribution with $2l$ degrees of freedom.

$$p_l(T) = \frac{\Lambda^l}{(l-1)!} T^{l-1} \exp\left(-\Lambda T\right) \quad (4)$$

Expression (4) could be used differently. On the one hand, let us say when $l = 1$, $p_1(T)$ is the PDF of the second cluster arrival time. However, the same function could also be used as a likelihood function of an arbitrary time instance τ belonging to the first cluster. Indeed, $p_1(\tau)$ decays exponentially with its maximum at zero, since by definition the first cluster starts at $\tau = 0$.

Unlike the original formulation (2), expression (4) is not conditioned on any of the previous arrival instances and, in this sense, it is a prior distribution of cluster arrival times. This property makes it useful in the proposed clustering algorithm.

C. Parameter estimation and clustering

Now, when we have the prior distribution functions for cluster arrival times, we can begin with the clustering algorithm. We will make the following assumptions:

1) The samples come from a known number of L clusters.
2) The channel impulse response $h(\tau)$ is defined on a finite ordered set $\mathcal{D} = \{\tau_0, \tau_1, \ldots, \tau_{N-1}\}$ of time instances where the pulses were registered in the impulse response. Thus, $T_l \in \mathcal{D}$ and $\tau_{k,l} \in \mathcal{D}, \forall k, l$
3) We extend the original model by assuming different Λ_l for each cluster. In our approach each Λ_l uniquely specifies the position of the corresponding cluster in a nonstationary environment.

For a given impulse response, the time instance τ_j could possibly belong to any of the L clusters. Mathematically speaking, it belongs to a mixture of L density functions:

$$\begin{aligned} p(\tau_j|\mathbf{\Lambda}) &= \sum_{l=1}^{L} p(\tau_j|\Lambda_l, \omega_l) p(\omega_l) \\ p(\tau_j|\Lambda_l, \omega_l) &= \frac{\Lambda_l^l}{(l-1)!} \tau_j^{l-1} \exp\left(-\Lambda_l \tau_j\right) \end{aligned} \quad (5)$$

Here, $p(\omega_l)$'s are the prior probabilities for each of the clusters, $\omega_l \equiv l$ is a cluster index, and $p(\tau_j|\Lambda_l, \omega_l)$ is a likelihood of the τ_j belonging to the lth cluster.

Our basic goal is to estimate the unknown parameters Λ_l that determine the density functions. Once they are known, we can decompose the mixture into components and use a maximum *a posteriori* classifier [8] on the derived densities to classify all the τ_j's.

The class of Expectation-Maximization (EM) algorithms allows one to effectively learn mixture parameters, given the statistical structure of the problem, data realizations, and good initialization values. The description and properties of these algorithms could be found in a number of sources (see, for instance, [9], [8]). There is a slight modification to the parameter re-estimation formulas that accounts for the non-normal form of the observation distribution $p(\tau_j|\Lambda_l,\omega_l)$.

Once the parameters are learned, the $p_l(\tau)$ can be used to optimally (in the maximum likelihood sense) cluster the taps of the impulse response. The derived PDFs also allow one to estimate the *expected cluster arrival time*, i.e.,

$$\overline{\tau_{\cdot,l}} = E\{\tau|l\} = \int_0^\infty \tau p(\tau|\Lambda_l,\omega_l)d\tau = l/\Lambda_l \qquad (6)$$

D. Practical considerations

The practical implementation of the algorithm is quite straight-forward except for some particularities.

The distributions in the form (4) are very wide, meaning there is a significant uncertainty in classifying the points, especially belonging to the late clusters. To amend this, we regularize the classification algorithm by introducing an additional weighting function $w(\tau|\mu_l,\alpha_l)$ used as a window to taper the original density $p(\tau|\Lambda_l,\omega_l)$:

$$w(\tau|\mu_l,\alpha_l) = \frac{1}{\alpha_l\sqrt{2\pi}}\exp(-0.5(\tau-\mu_l)^2/\alpha_l^2)$$

Assuming functional independence of the window parameters μ_l and α_l from Λ_l, the former can be easily estimated from the data within the same EM framework by setting the derivatives of the likelihood function with respect to μ_l and α_l to zero.

The sparseness of the impulse response is crucial, since a uniform sequence of instances τ_j does not form clusters. Sparseness can be enforced by thresholding channel coefficients with power gains smaller than the noise level and then selecting local maxima.

Due to the iterative nature of the algorithm, the choice of the initial values is an important step. In our implementation the initial values were set as follows:

$$p(\omega_l) = 1/L, \quad \tilde{\Lambda}_l^{(0)} = l \Bigg/ \frac{\max_j(\tau_j)}{L+1}, l = 1,2,\ldots,L$$

i.e., uniformly in probability and time.

III. Simulation Results

To test the algorithm we generate an impulse response according to the original S.-V. model (Fig. 2). The parameters used in the simulation are summarized in Table I.

$L=4$; $\Lambda = 5.7\cdot 10^6$, [sec^{-1}]	$\lambda = 1.0\cdot 10^8$, [sec^{-1}]
$1/\Gamma = 3.0\cdot 10^6$, [sec^{-1}]	$1/\gamma = 1.0\cdot 10^7$, [sec^{-1}]
$\beta_{0,0}^2 = 5.0\cdot 10^{-8}$, [W]	noise floor $= 1.0\cdot 10^{-9}$, [W]

TABLE I

Parameters used in the simulation.

This example shows the case when the 1st and 2nd clusters are strongly overlapping, the 3rd cluster only partially overlaps with the two previous ones, and cluster 4 is clearly distant from the preceding neighbors. By comparing the expected cluster arrival times $\overline{\tau_{\cdot,l}}$, computed as in (6), with the true values, we can judge the performance of the clustering algorithm. It should be noted that the comparison based on time of arrivals T_l is equivalent to comparing $\overline{\tau_{\cdot,l}}$'s.

Cluster index, l	1	2	3	4
True $\overline{\tau_{\cdot,l}}\times 10^{-6}$,[sec]	0.15	0.23	0.45	0.81
Estimated $\overline{\tau_{\cdot,l}}\times 10^{-6}$,[sec]	0.07	0.20	0.44	0.81
Estimated $\tilde{\Lambda}_l\times 10^6$, [sec^{-1}]	17.7	10.0	6.8	4.9

TABLE II

Comparison of true and estimated expectations of cluster arrival times for the channel shown in Fig.2.

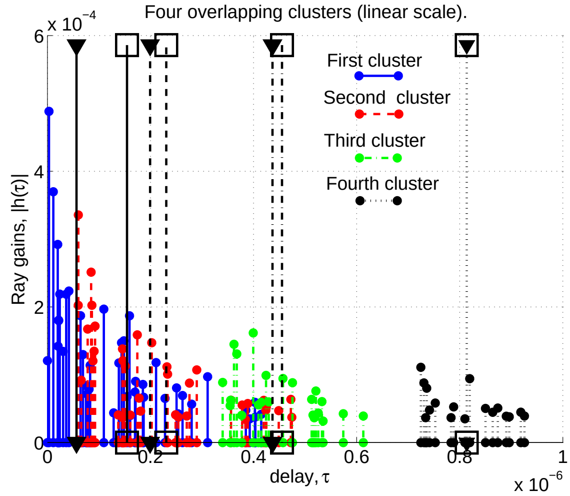

Fig. 2. Impulse response with overlapping clusters.

The cluster positions are indicated by vertical lines. Triangle-headed lines show estimated cluster arrival times; the ones with squares show the true positions. The corresponding values are summarized in Table II. We can see that the algorithm fails to properly distinguish strongly overlapping clusters, however the estimated arrival time of the 4th cluster is very close to the true one. We can also compute an empirical cluster arrival rate Λ^{emp} as an average of the $\tilde{\Lambda}_l$ (excluding the $\tilde{\Lambda}_1$ since the arrival rate is measured with respect to the first cluster). In this simulation $\Lambda^{emp} = 7.2\cdot 10^6$ sec^{-1}, i.e. it is overestimated.

IV. Application of the Clustering Algorithm to the Measured Channel Impulse Responses

This section presents some results of the clustering algorithm application to channel impulse responses obtained from the field-trial Multiple-Input-Multiple-Output (MIMO) channel measurements, performed by Forschungszentrum Telekommunikation Wien, FTW, Vienna, Austria, under the supervision of Helmut Hofstetter[1][10]. For our purposes we select only a Single-Input-Single-Output (SISO) subset by taking one transmitting and one receiving antenna from the array.

Fig.3 shows consecutive measurements of the channel impulse response as the transmitter moves. The channel snapshot

[1]The authors wish to thank Forschungszentrum Telekommunikation Wien for providing MIMO channel measurements data.

was taken every 20msec while the transmitter was moving at $\approx$ 1m/s. This particular example shows a subset from 2000 consecutive channel snapshots which is equivalent to 40sec of measurements. From this picture one can clearly see several

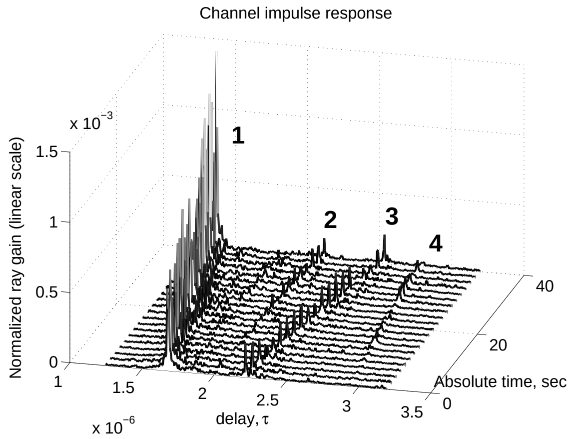

Fig. 3. Measured channel impulse response (non-stationary behavior).

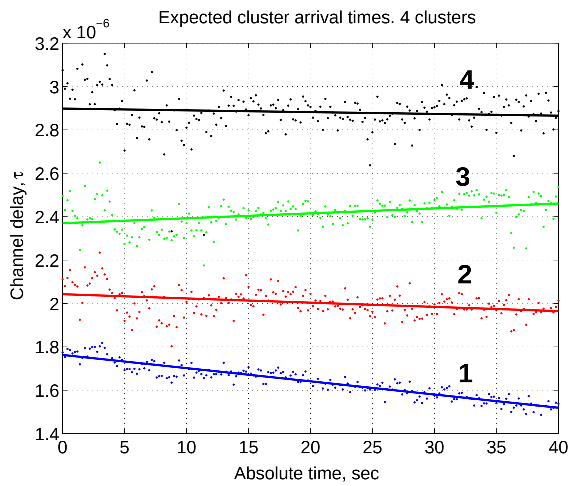

Fig. 4. Expected arrival time tracks of four clusters with superimposed linear regression lines.

prominent stripes. It is reasonable to assume that the clusters are formed in the neighborhood of these peaks. Under the assumption of four clusters we run the clustering algorithm on every 5th channel snapshot from the measurement data. On each step, the clustering algorithm was initialized with parameters estimated on the preceding snapshot. The corresponding results are shown in Fig.4.

The scatter plots show instantaneous values of the expected cluster arrival times. From considering Fig.3, it is clear that we are dealing with a non-stationary channel, since clusters change their relative positions. From visual inspection of the impulse response evolution, we conclude that the clusters move on straight lines. This allows us to fit a linear regression to these estimates in order to have a clearer picture of the cluster dynamics.

In general, the obtained estimates are unbiased due to the property of Maximum Likelihood estimators. However, the variance of the estimates strongly depends on the number of available impulses used to compute the parameters. Due to the thresholding and peak-peaking, required to obtain a sparse representation, this number is not very high, thus leading to the increased variance. Therefore, it might be advantageous to filter the estimates by a narrow low-pass filter to decrease the variance.

V. Conclusion

Clustering the channel impulse responses could be an interesting approach to extract important channel parameters. Based initially on the S.-V. channel model, the expressions for the PDFs of the cluster arrival times are derived. The latter turn out to be chi-square distribution with $2l$ degrees of freedom, with l being the cluster index starting from 1. These could be thought of as prior distributions over cluster arrival times. The classical Maximum Likelihood approach is used to learn the parameters of the distributions and classify the taps in the impulse response. By exploiting the property of the chi-square distributions the expected time of cluster arrivals can be effectively computed. This value can then used as the cluster center.

To fully exploit this approach, the stationarity restriction has been relaxed, i.e., an independent parameter for each distribution has been used. This allows a better adaptation of the clustering algorithm to the non-stationary scenarios.

The algorithm has been applied to simulated, as well as to real data and, in both cases, it has shown reasonable performance.

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