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# Clustering Wireless Channel Impulse Responses in Angular-Delay Domain

Dmitriy Shutin
Institute of Signal Processing and
Speech Communications
Graz University of Technology
Inffeldgasse 12
A-8010, Graz,
Austria
Email: dshutin@inw.tugraz.at

*Abstract* — **This paper introduces a novel wireless MIMO channel clustering technique implemented in angular and delay domains. The clustering is preceded by extraction of angular and delay information from the sampled impulse response. Akaike Information Criterion (AIC) has been used to find the number of multipath components arriving at the same delay bin but having different directions of incidence. The latter have been resolved with ESPRIT algorithm. A modified Saleh-Valenzuela model is used as a basis for the derivation of the unsupervised clustering algorithm. To avoid estimation of the joint angle-delay density function required for the clustering algorithm, it is 'factorized' algorithmically by first identifying clusters in the delay domain and then finding angular clusters 'conditioned' on the corresponding delay cluster. The algorithm has been applied to measured MIMO channel impulse responses and is able to find visually identifiable clusters as well as their width.**

## I. Introduction

Wireless systems are subject to fading - time variations of the receiving conditions caused by multipath propagation and transceiver movements. A wireless channel contains all the information about the propagation environment and, in general, the receiving side should be aware of the channel. Due to the fading nature of the wireless channel, it is imperative for the system to follow the variations of the receiving conditions along time to adapt itself and sustain reasonable communication quality.

Generally, the wireless channel consists of a series of attenuated, time-delayed, phase shifted replicas of the transmitted signal. In the baseband this is represented as follows:

$$h_t(\tau) = \sum_{i=0}^{N-1} \beta_i(t,\tau)\exp\left(j\theta_i(t,\tau)\right)\delta(\tau - \tau_i(t)) \qquad (1)$$

where $\beta_i(t,\tau)$ and $\tau_i(t)$ are the real amplitudes and excess delays of the $i$th multipath component at the time $t$, respectively (see, for example, [1, ch.4]). The phase term $\theta_i(t,\tau)$ lumps together all the mechanisms for phase shifts of a single multipath component within the $i$th excess delay bin. Equation (1) is a starting point for further analysis. The straight-forward procedure to follow the channel is to follow each of the contributing reflections. Obviously, it is difficult to do, since the number of contributions could be significant, but if successful, this approach will provide quite a precise instantaneous description of the channel dynamics. Although it is computationally impractical to follow all of the contributing paths, it is still possible to follow at least several strong ones [2, 3].

Alternatively, a number of statistical models provide probability density functions (PDFs) for the parameters of interest [4, 5], thus giving a probabilistic description of the channel behavior. It would seem beneficial to join both ideologies and devise an algorithm to extract some information from the instantaneous impulse response based on the probabilistic model of the channel parameters. This could be accomplished if channel impulse responses (or channel parameters) are regarded as sample realizations of the corresponding statistical models.

Recently, directional systems have attracted much of attention [6]. There are numerous advantages of using antenna arrays to communicate and among them is a much richer structure of the impulse response that can be exploited by the transceiver. Numerous measurement campaigns have shown the presence of clusters both in delay as well as in angular domains [7, 8]. The idea of clusters is quite appealing for it provides a very promising framework for tracking, analyzing and simulating wireless channels: instead of following every reflector or scatterer we can concentrate on more tractable dynamics of the clusters. Clusters inherently impose structure on the impulse response that provides a basis for a hierarchical analysis (i.e., from the impulse response to clusters and then to the rays within each cluster), as well as a straightforward physical interpretation of the clusters and model parameters.

We propose a clustering algorithm that relies on the statistical structure of the channel parameters, i.e., distribution of angles and delays within the cluster: the delays and angles are estimated from instantaneous MIMO channel impulse responses and then grouped into clusters.

The rest of the paper is organized as follows: Section II describes the sampled channel tap model. Section III explains how the angular and delay information has been estimated from the sampled channels. The details of the clustering algorithm are summarized in section IV, and finally, Section V presents some of the clustering results for the measured channel impulse responses.

## II. Channel model

The detailed analysis of the sampled baseband representation of the MIMO channel impulse response reveals structure that enables estimation of the directional information about the objects interacting with the wavefront induced by a transmitter. It has been shown that a time-varying channel tap $h_k(t)$ of a sampled Multiple-Inputs-Single-Output (MISO) or Single-Input-Multiple-Outputs (SIMO) channel contains information about the angle of incoming wavefronts (MISO channel), or angles of departure in the SIMO case[9]:

$$h_{k,p}(t) = \sum_{n=1}^{N} \alpha_{k,n}(t) \mathrm{e}^{jw_{D,n}(t)t} \mathrm{e}^{j\frac{2\pi}{\lambda} pd \sin(\phi_n(t))} \tag{2}$$

where $h_{k,p}(t)$ is a time-varying channel tap at the $p$th sensor and delay bin $k$, $N$ is the number of the wavefronts arriving withing this delay bin and having complex gains $\alpha_{k,n}(t)$. The exponent term $w_{D,n}(t)$ denotes the time-varying Doppler shift induced by the $n$th wave source, and $\phi_n(t)$ is the corresponding Direction-of-Arrival (DoA) in case of MISO channel or Direction-of-Departure (DoD) in SIMO case, with $d$ being the distance between antenna sensors and $\lambda$ being the wavelength. It is important to note that this expression is the linearized representation of the tap dynamics resulted from the plane wave assumption and uniform linear array (ULA) antenna structure. This will result in the same Doppler shift induced on each of the antenna sensors. In case of MIMO channels, it is straightforward to show that the dynamics of the tap will include both DoD and DoA contributions:

$$h_{k,p,m}(t) = \sum_{n=1}^{N} \alpha_{k,n}(t) \mathrm{e}^{jw_{D,n}(t)t} \times$$
$$\times \mathrm{e}^{j\frac{2\pi}{\lambda} pd_{RX} \sin(\phi_n(t))} \mathrm{e}^{j\frac{2\pi}{\lambda} md_{TX} \sin(\psi_n(t))}$$

Here, $p$ and $m$ are receiving and transmitting antenna indices, $d_{RX}$ and $d_{TX}$ denote the distances between receiving and transmitting sensors, respectively, and $\phi_n(t)$ and $\psi_n(t)$ are the corresponding DoA and DoD of the wavefronts. Based on this representation the angular information can be estimated from the channel impulse response and then used as an input data to the clustering algorithm. These steps are summarized in the following section.

## III. Extraction of the delay and angular information

In the rest of the paper we concentrate on MISO channels and ULA on the receiving side to illustrate the idea of clustering. The study of SIMO channels is completely equivalent. The MIMO case is a bit more complicated since it requires simultaneous estimation of both DoA and DoD from the measurements. This could be accomplished subsequently, for example, by first estimating the DoA from the measurements and then 'canceling' the corresponding induced phase shift to estimate DoD. More details on this procedure can be found in [10]. Let us assume the receiving antenna is an ULA consisting of $P$ sensors spaced $0.5\lambda$ apart, i.e., $d_{RX} = 0.5\lambda$. Let us also assume that the sampled equivalent baseband MISO channel impulse response has been identified by some means and is available as a $P \times K$ complex matrix $\mathbf{H} \in \mathbb{C}^{P\times K}$, where $K$ is the number of channel taps:

$$\mathbf{H} = \begin{bmatrix} h_{0,1} & h_{1,1} & \dots & h_{K-1,1} \\ h_{0,2} & h_{1,2} & \dots & h_{K-1,2} \\ \vdots & & & \\ h_{0,P} & h_{1,P} & \dots & h_{K-1,P} \end{bmatrix} = \begin{bmatrix} \boldsymbol{h}_1 \\ \boldsymbol{h}_2 \\ \vdots \\ \boldsymbol{h}_P \end{bmatrix} \tag{3}$$

Each element of the matrix $\mathbf{H}$ varies with time according to (2) (although the time dependency is not explicit in the formulation (3)) and corresponds to the sum of multipath components received within the delay bin $\tau_k = kT_s$, where $T_s$ is the channel baseband sampling period. Finally, we will assume the channel $\mathbf{H}$ is estimated with a certain amount of noise,

$$\mathbf{H} = \tilde{\mathbf{H}} + \mathbf{E}$$

where $\tilde{\mathbf{H}}$ is the 'noise free' channel and $\mathbf{E} \in \mathbb{C}^{P\times K}$ is a zero mean additive complex Gaussian white noise with the spectral density $\sigma_E^2$.

The clustering could only make sense if the sequence of the channel parameters, i.e., delays and corresponding angles, is not uniformly distributed, which can be a valid assumption for the specular channel model (1). Methods utilizing (1) to estimate channel parameters usually give non-uniformly distributed values, of course if the nature of the channel supports it. SAGE algorithm[11], for instance, effectively approximate the Maximum Likelihood solution for the parameter estimates. It require a good initial estimate of the channel parameters as well as the number of the arriving multipaths $N$, but having those could prove to be an effective way to obtain estimates. Since our goal is to illustrate the idea of channel clustering, we propose a simple heuristic approach to estimate channel parameters, that, if necessary, can be treated as initial estimates and refined with iterative methods later on.

The estimated impulse response $\mathbf{H}$ is the input data for the algorithm. The estimation of the delays is done by means of sub-sampling the channel matrix $\mathbf{H}$, i.e. selecting the columns with power exceeding a certain threshold. The latter is based on the knowledge of the power of additive noise $\sigma_E^2$. For the taps that exceed the threshold the angular information is estimated by means of unitary ESPRIT algorithm [12]. These steps are summarized in the the following algorithm:

1. Accumulate $L$ MISO channel snapshots $\mathbf{H}[l]$, $l = 1, \dots, L$. This is necessary for the estimation of the sensor covariance matrices, required for the ESPRIT algorithm. The number $L$ should be chosen so that the structure of the channel has not been compromised, i.e., so that the channel remained stationary.
2. Compute the variance $\sigma_H^2$ of channel taps for the whole block of snapshots $\mathbf{H}[l]$, $l = 1, \dots, L$.
3. Compute the subsampling threshold as
$$A = \frac{\sigma_E^2 \sigma_H^2}{\sigma_H^2 - \sigma_E^2} \log(\frac{\sigma_H^2}{\sigma_E^2})$$
where, $\sigma_E^2$ is the additive noise variance, which is equivalent to the second moment of the zero mean additive noise $\mathbf{E}$. The threshold $A$ is obtained by a very simple reasoning: the distribution of the tap power could be easily found to be exponential with the mean equal to $\sigma_H^2$. Likewise, the distribution of the noise power will also be exponential with the parameter $\sigma_E^2$. The intersection point of the two distributions gives the value of the threshold $A$, which is an optimum decision boundary when we ask if the tap is a multipath or just the noise sample.
4. Select a channel tap $k$, $k$=0, $\dots, K-1$
5. Construct a matrix $\mathbf{G}_k = [\boldsymbol{g}_k[1], \boldsymbol{g}_k[2], \dots, \boldsymbol{g}_k[L]]$, where $\boldsymbol{g}_k[l]$ is a $k$th column from the channel matrix $\mathbf{H}[l]$. Here, the $k$th column corresponds to the multipath delay $\tau_k$.
6. Compute the variance $\sigma_G^2$ of the entries in $\mathbf{G}_k$.

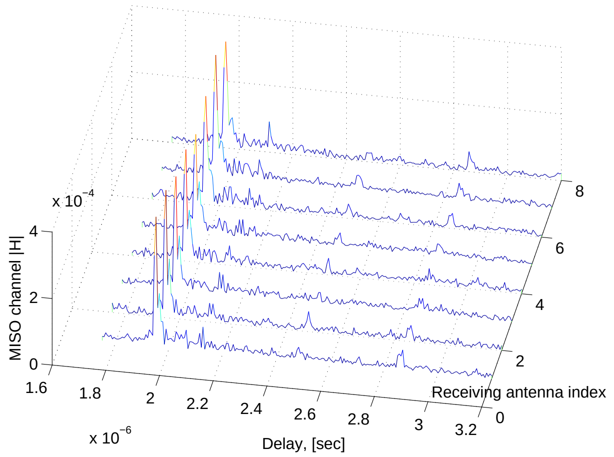


Fig. 1: A sample MISO channel impulse response.

7. If $\sigma_G^2 < A$, the corresponding taps are unlikely to contain any multipaths, and thus they are discarded.
8. Otherwise, the taps have sufficient power and we try to extract the angular information: the $P \times L$ matrix $\mathbf{G}_k$ is used to compute the sensor covariance matrix $\mathbf{R}_k$, based on which the number $N$ of multipath components present within the delay $\tau_k$ is identified using Akaike information theoretic criterion (AIC)[13].
9. Having estimated the number of component, ESPRIT algorithm is used to obtain the estimates of the angles $\phi_n$, $n = 1 \ldots N$, along with the power of the arriving components $|\alpha_{k,n}|^2$.
10. The triplets $(\tau_k, \phi_n, |\alpha_{k,n}|^2)$, $n = 1 \ldots N$, are appended to the set of the resulting angle-delay data points.
11. The tap index is iterated $k = k + 1$ and we go back to 5 unless all the taps have been scanned.

The described algorithm has been applied to the measured data channel impulse responses obtained from the field-trial Multiple-Input-Multiple-Output (MIMO) channel measurements performed by Forschungszentrum Telekommunikation Wien, FTW, Vienna, Austria, under the supervision of Helmut Hofstetter[1][10]. The measurements were done with the MIMO-capable wideband vector channel sounder RUSK-ATM manufactured by MEDAV [14]. The sounder was specifically adapted to operate at the center frequency of 2 GHz. The transmitted signal is generated in the frequency domain to ensure a pre-defined spectrum over 120 MHz bandwidth, and an approximately constant envelope over time. Two simultaneously multiplexed antenna arrays have been used at the transmitter and receiver. The transmitter was a uniform circular array, with 15 sensors spaced at $\approx 6.45$ cm. The receiver was a fixed uniform linear array, with 8 antenna elements spaced at $\approx 7.5$ cm. For our purposes we select only a MISO subset by selecting a single transmitting antenna from the array. A sample waterfall plot of the MISO channel matrix $\mathbf{H}$ is shown in Fig.1.

In order to estimate the DoA, 25 consecutive channel snapshots were used. During the measurements, the channel snapshot was recorded each 20msec, while the transmitter was moved

[1]The author wish to thank Forschungszentrum Telekommunikation Wien for providing MIMO channel measurements data.

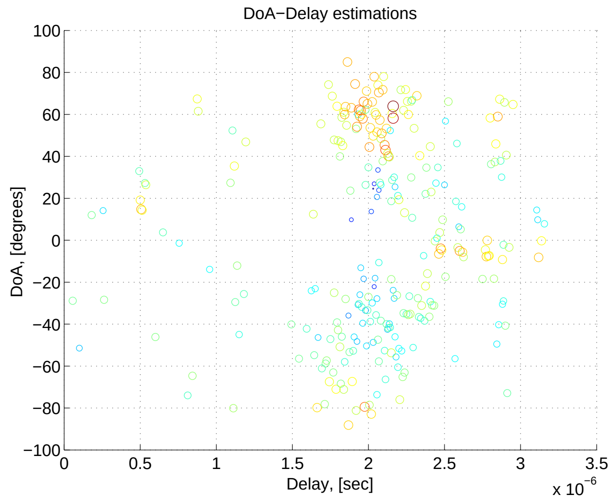


Fig. 2: Angle-Delay information extracted using the proposed algorithm.

with a velocity of $\approx$ 1m/s. The resulting 25 snapshots will correspond to the walked distance of $\approx$ 50cm. It is reasonable to assume that for the outdoor scenarios there will be no significant changes in the cluster position or their number for such a short change in the distance.

Estimation results are shown in Fig.2. The diameter of the markers is proportional to the estimated power $|\alpha_{k,n}|^2$ of the multipath components with respect to the highest averaged sensor power for each of the $L$ snapshots, i.e., $\max_l(\mathrm{mean}_p(\boldsymbol{g}_k[l]))$. This particular example shows a Line-of-Sight case, with most of the energy coming after 1.8$\mu$sec. The components appearing before the LOS are not used in the clustering. This finalizes the formulation of the angle-delay extraction procedure and brings us to the point of clustering.

## IV. Clustering algorithm

As it has been already mentioned, there are several statistical channel models that incorporate the clustering idea. This can be used to devise the unsupervised clustering algorithm that relies on this structure. In particular, the clusters in delay domain could be described by the Saleh-Valenzuela (S.-V.) model [7], based on which the densities of cluster arrival times could be inferred[15]. Similarly, in angular-domain the distribution of angles within a cluster has been found to conform well with a Laplacian distribution positioned at the cluster center[16].

The unsupervised clustering algorithm relies on the knowledge of the joint distribution $p(\tau_k, \phi_n)$ of angle and delay samples. The difficulty in estimating or approximating this distribution lies in the strong dependency of its form on a particular geometrical structure of scatterers/reflectors in the measurement location. To overcome this the following approximation could be used: the joint density is factorized as $p(\tau_k, \phi_n) = p(\tau_k)p(\phi_n|\tau_k)$ and then the clustering is performed in two stages: at the first stage, the clusters in the delay domain are identified, i.e., the sequence of estimated delays $\tau_k$ is clustered according to the modified Saleh-Valenzuela model[15]; then, we search for the clusters in the angular domain among those multipaths that arrive within the same cluster in the delay domain, i.e., implementing $p(\phi_n|\tau_k)$ factor.

Clustering in the delay domain We assume that the sequence of delays $\tau_k$ come from a known number of $Q$ clusters. It has been shown[15] that under certain assumptions the probability density function $p_q(\tau)$ of the $q$th cluster arrival time is a chi-square distribution with $2q$ degrees of freedom and distribution parameter $\Lambda_q$, where $q$ is the cluster index:

$$p_q(\tau) = \frac{\Lambda_q^q}{(q-1)!}\tau^{q-1}\exp\left(-\Lambda_q\tau\right) \tag{4}$$

For a given impulse response a time instance $\tau_k$ could possibly belong to any of the $Q$ clusters. Mathematically speaking, it belongs to a mixture of $Q$ density functions:

$$\begin{aligned} p(\tau_k|\mathbf{\Lambda}) &= \sum_{q=1}^{Q} p(\tau_k|\Lambda_q,\omega_q)p(\omega_q) \\ p(\tau_k|\Lambda_q,\omega_q) &= \frac{\Lambda_q^q}{(q-1)!}\tau_k^{q-1}\exp\left(-\Lambda_q\tau_k\right) \end{aligned} \tag{5}$$

Here, $p(\omega_q)$'s are the prior probabilities for each cluster, $\omega_q \equiv q$ is a cluster index, and $p(\tau_k|\Lambda_q,\omega_q)$ is a likelihood of the delay $\tau_k$ belonging to the $q$th cluster. The class of Expectation-Maximization (EM) algorithms can be used to learn the mixture parameters (see, for instance, [17, 18]). Once the parameters are learned, the $p(\tau_k|\Lambda_q,\omega_q)$ density can be used to optimally (in the maximum likelihood sense) assign the delays $\tau_k$'s to corresponding clusters.

Clustering in the angular domain Once we have identified clusters in the delay domain, we can select those triplets $(\tau_k, \phi_n, |\alpha_{k,n}|^2)$ that correspond to a certain cluster $q$ in the delay domain. For these data points a similar approach is used to find the clusters in the angular domain. As before, we assume the angles $\phi_n$ coming from a known number of $V$ clusters. The distribution of DoA within the $v$th cluster $p_v(\phi_n)$ is assumed to be Laplacian. This leads to the following mixture formulation of the clustering problem:

$$\begin{aligned} p(\phi_n|\mathbf{\Phi},\mathbf{\Psi}) &= \sum_{v=1}^{V} p(\phi_n|\Phi_v,\Psi_v)p(\omega_v) \\ p(\phi_n|\Phi_v,\Psi_v) &= \frac{1}{2\Psi_v}\exp\left(-\frac{|\phi_n-\Phi_v|}{\Psi_v}\right) \end{aligned} \tag{6}$$

Similarly, $p(\omega_v)$'s are the prior probabilities for each cluster in the angular domain, $\omega_v \equiv v$ is a cluster index, and $p(\phi_n|\Phi_v,\Psi_v)$ is a likelihood of the angle $\phi_n$ belonging to the $v$th cluster, centered around the mean $\Phi_v$ and having the cluster width $\Psi_v$. Again, these parameters could be identified with the EM algorithm.

## V. Clustering results

In Fig.3 and Fig.4 one can see clustering results of the data shown in Fig.2. It has been assumed there are 2 clusters in the delay domain with 2 angular clusters for the first and 3 for the second cluster in the delay domain. In Fig.3 one can see the discovered clusters together with their centers and estimated width. The cluster centers are given as expectations of the corresponding distribution, which for the delay is given as:

$$\overline{\tau_{\cdot,q}} = E\{\tau|q\} = \int_0^\infty \tau p(\tau|\Lambda_q,\omega_q)d\tau = q/\Lambda_q \tag{7}$$

The centers in angular domain $\Phi_v$'s are computed directly from the EM-learning procedure. Thus, in this simulation the discovered clusters are represented by centers $(\overline{\tau_{\cdot,1}}, \Phi_1)$ and $(\overline{\tau_{\cdot,1}}, \Phi_2)$ ( depicted in Fig.3 with the black stars). The right-hand side of

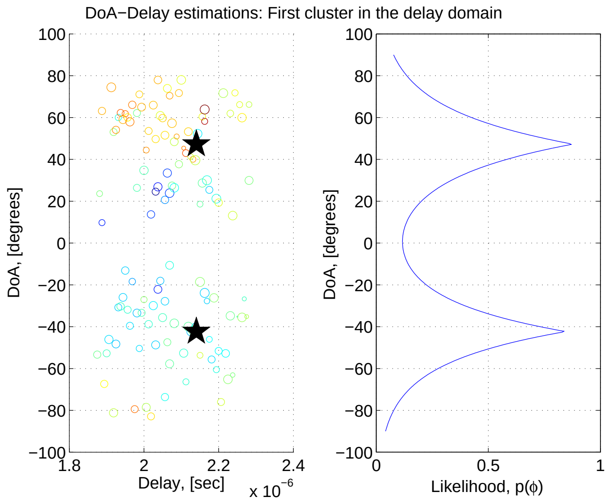


Fig. 3: Clustering results - first cluster in the delay and two clusters in the angular domain.

the plot depicts the estimated mixture density $p(\phi_n|\mathbf{\Phi},\mathbf{\Psi})$ for two angular cluster.

Clustering results for the second cluster are shown in Fig.4. It is expected it will be more difficult to identify late clusters, since taps belonging to those contain less energy, which prohibits effective multipath detection and parameter estimation.

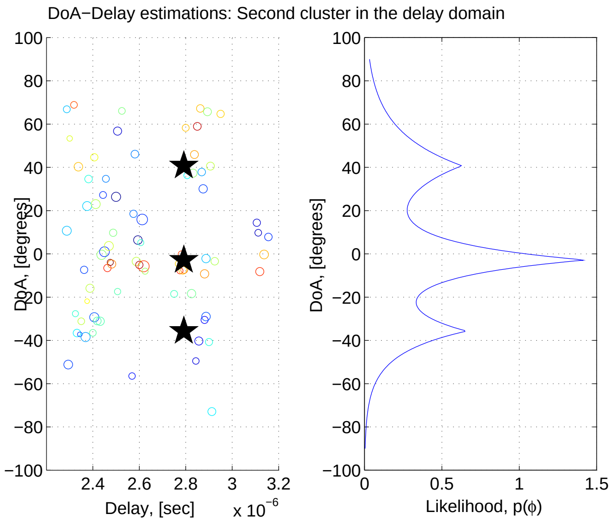


Fig. 4: Clustering results - second cluster in the delay and three clusters in the angular domain.

## VI. Conclusion

Clustering the channel impulse responses could be an interesting approach to extract important channel parameters. However as a pre-step to clustering, channel parameters must be estimated by some means. As a staring point, a simple heuristic algorithm has been proposed to extract delay and angular information from the channel impulse responses. Although quite simple, this approach is too optimistic, since it utilizes incoherent parameter estimation scheme (the effect of the Doppler shift has been completely neglected) and thus is inferior in comparison to joint estimation. Alternative methods that perform joint channel parameter estimation like SAGE could be a very good substitution for the method used in this paper. In fact our recent research have proved that. Nonetheless, in this case the clustering algorithm itself is not influenced by the parameter estimation scheme.

Based on the angular-delay information the unsupervised clustering algorithm has been devised. The latter is based on the developed statistical structure of the clusters, mainly inspired by the Saleh-Valenzuela channel model. The parameters of this structure could be effectively learned with a class of Expectation-Maximization algorithms.

The absence of the joint density $p(\tau_k, \phi_l)$ of the channel parameters hinders the straight-forward clustering of the data. A two-stage approximation has been proposed to overcome this problem by first performing clustering in the delay domain, and only then clustering the corresponding angular information for a certain delay cluster.

The algorithm has been applied to the measured data and has been found to perform quite good in finding the clusters, that could be identified by visual inspection. Estimation of the true number of clusters is still an open question that must be considered, since the performance strongly depends on the correct number of cluster.

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