

# THE DESIGN OF POLYPHASE-BASED IIR MULTIBAND FILTERS

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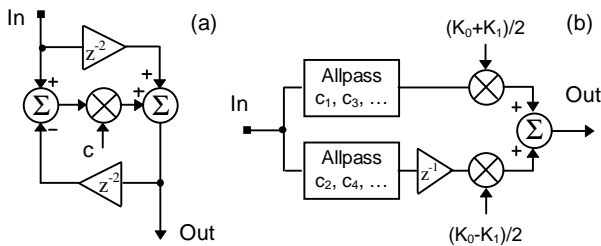
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## ABSTRACT

This paper addresses a new approach for the design of multiband filters with step like magnitude responses and extremely flat, weighted passbands down to  $\mu$ dBs. Our new technique can also be used for the multiband step-wise approximation of arbitrary filter magnitude responses with precise transition band control. One marked advantage of the technique is that our basic building blocks are the modified polyphase (IIR) filters as reported in [1]-[3]. The points of transition from one flat-top to another, namely the multiple transition bands of our filter are completely free of cross-over oscillations. One other advantage of this technique is that we are not confined to employing IIRs only. To this end perfect reconstruction FIR filters as in [4] can also be used. In both the FIR and IIR cases our technique is general purpose and works for both real and complex valued filter coefficient cases.

## THE MODIFIED POLYPHASE HALFBAND BUILDING BLOCK

Our polyphase structures suggested in [1]-[2] are very attractive for the design and implementation of halfband Lowpass and Highpass filters achieving very small passband ripples for a very small coefficient budget. Lowpass and highpass filter functions can be effected using the same coefficients, with exception of a sign change at one of the summation block. By combining lowpass and highpass filters as in Figure 1(a) we can get a very efficient two-band filter.



**Figure 1:** Two-path Polyphase Basic Building Block: (a) The Structure, (b) The Allpass Filter Block

It has controllable band  $K_0$  and  $K_1$ , where  $K_0$  is the gain of the lower half band and  $K_1$  is the gain of the upper half band. Figure 1(a) shows the form of the computationally as well as hardware efficient single coefficient second-order allpass building block employed in the parallel bands of the structure of Figure 1(b). Lowpass and highpass prototype filters combined in

the basic building block in Figure 1(a) are complementary filters. When they are added with same gain factors  $K_0$  and  $K_1$ , they result in the perfect reconstruction:

$$H(z) = \left[ \frac{K_0 + K_1}{2} \prod_{i=0}^{K_{out}-1} A_{2i+1}(z) + \frac{K_0 - K_1}{2} z^{-1} \prod_{i=0}^{K_{out}-1} A_{2i}(z) \right]_{K_0=K_1} = K_0 \prod_{i=0}^{K_{out}-1} A_{2i+1}(z) \quad (1a)$$

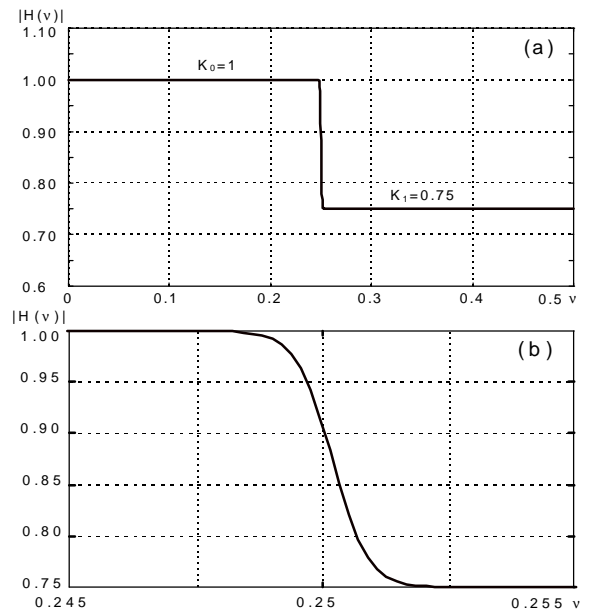
where  $A_i(z)$  is the 2<sup>nd</sup>-order allpass filter as in Figure 1(a):

$$A_i(z) = \frac{\alpha_i + z^{-2}}{1 + \alpha_i z^{-2}} \quad (1b)$$

When  $K_0=K_1$ , the filter becomes an allpass function having the phase response  $\phi(v)$ :

$$\phi(v) = -2 \sum_{i=0}^{K_{out}-1} \tan^{-1} \left( \frac{1 - \alpha_{2i+1}}{1 + \alpha_{2i+1}} \tan(2\pi v) \right) \quad (2)$$

where  $v$  is the normalized frequency.



**Figure 2** Magnitude Response of the Basic Building Block with Gains  $K_0=1$  and  $K_1=0.75$ . (a) Full-band, (b) Zoomed in the Transition Band

If gain factors  $K_0 \neq K_1$  then in the result the two-band filter will have band gains  $K_0$  in the first band and  $K_1$  in the second. Figure 2 shows the magnitude response of a sample four coefficient (ninth order) two-band building block filter as in Figure 2(a) having gain  $K_0=1$  in band 1 and  $K_1=0.75$  in band 2,

displaying smooth transition band characteristics as in Figure 2(b). Additionally there are no oscillations in the transition band which follows the transition band of the original highpass filter (Figure 2).

It can be proved that the characteristics of the building block filter is equiripple in both bands and displays a monotonic cosine like shape in its transition region from one band to another. Their symmetric monotonicity in these transition regions ensure oscillation free behavior. We observe the detailed behavior of the building block filter in its bands as well as in its transition region through the calculation of the filter frequency response. This is obtained by evaluating the transfer function (1) on the unit circle, i.e.

$$H(v) = \frac{K_0 + K_1}{2} e^{j\phi_0(v)} + \frac{K_0 - K_1}{2} e^{j(\phi_1(v) - 2\pi v)} \quad (3a)$$

where

$$\begin{aligned} \phi_0(v) &= -2 \sum_{i=0}^{K_{\text{odd}}-1} \tan^{-1} \left( \frac{1 - \alpha_{2i+1}}{1 + \alpha_{2i+1}} \tan(2\pi v) \right) \\ \phi_1(v) &= -2 \sum_{i=0}^{K_{\text{even}}-1} \tan^{-1} \left( \frac{1 - \alpha_{2i}}{1 + \alpha_{2i}} \tan(2\pi v) \right) \end{aligned} \quad (3b)$$

which gives

$$\begin{cases} H(v) = \sqrt{\frac{K_0^2 + K_1^2}{2} + \frac{K_0^2 - K_1^2}{2} \cos(\phi_0(v) - \phi_1(v) + 2\pi v)} \cdot e^{j\theta(v)} \\ \phi(v) = \tan^{-1} \left( \frac{(K_0 + K_1) \sin \phi_0(v) + (K_0 - K_1) \sin(\phi_1(v) - 2\pi v)}{(K_0 + K_1) \cos \phi_0(v) + (K_0 - K_1) \cos(\phi_1(v) - 2\pi v)} \right) \end{cases} \quad (3c)$$

As the building block filter is a linear combination of two complementary equiripple polyphase filters, [1]-[2], the building block filter will also have an equiripple behavior in both its bands. For the class of allpass filters (1b), which we use here, their phase response changes monotonically between zero and  $-2\pi$ . Therefore the overall filter (3a) has magnitude response at frequencies near DC (small  $v$ ),  $|H(v)| \approx K_0$ , and for frequencies near Nyquist frequency ( $v$  close to 0.5),  $|H(v)| \approx K_1$ . The magnitude response for frequencies between DC and Nyquist is determined by the dynamics of  $\phi_0(v)$  and  $\phi_1(v)$ . With reference to (3c) functions  $\phi_0(v)$  and  $\phi_1(v)$  have to be custom designed to force the argument  $\{\phi_0(v) - \phi_1(v) + 2\pi v\}$  to be approximately zero in the first band and equal to  $\pi$  in the second one.

The building block order, though the scaled sum of two complementary functions, will not increase in complexity and will retain the order of the prototype highpass/lowpass filter. This may seem wrong at the first glance as addition of two transfer functions usually leads to the filter order equaling to the sum of orders of the added filters. However, the key to the pegged order of complexity lies in the fact that we are using the same structure to generate both lowpass and highpass filters.

### THE MULTIBAND STRUCTURE

At the heart of the multiband polyphase filter lies the two-path halfband building block. As its name suggests, the halfband filter is restricted to having cut-off at half-Nyquist. However, this is not a problem as the cut-off can easily be changed through the use of frequency transformations, as outlined in [5]-[7]. If a number of such two-band filters are

cascaded, the resulting filter can be engineered to exhibit the desired multiband transfer function. If the simplest frequency transformation, that of real lowpass-to-lowpass as in (4a), or the lowpass-to-highpass as in (4b) is used, then the mapping can be performed with first-order allpass filters.

$$z_{\text{old}}^{-1} = \frac{z_{\text{new}}^{-1} - \alpha}{1 - \alpha z_{\text{new}}^{-1}} \quad \text{and} \quad \alpha = \frac{\sin \pi(v_{\text{old}} - v_{\text{new}})}{\sin \pi(v_{\text{old}} + v_{\text{new}})} \quad (4a)$$

$$z_{\text{old}}^{-1} = -\frac{z_{\text{new}}^{-1} + \alpha}{1 + \alpha z_{\text{new}}^{-1}} \quad \text{and} \quad \alpha = -\frac{\cos \pi(v_{\text{old}} + v_{\text{new}})}{\cos \pi(v_{\text{old}} - v_{\text{new}})} \quad (4b)$$

where  $v_{\text{old}}$  and  $v_{\text{new}}$  are the cut-off frequencies of the original and target filters respectively.

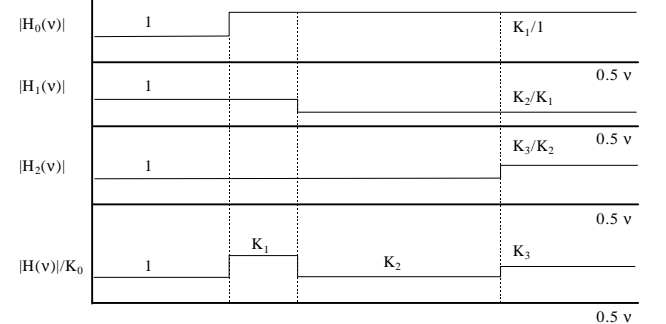
During transformation each delayor of the original filter is substituted with the function  $z_{\text{old}}(z_{\text{new}})$ . As a consequence, the order of the resultant filter is kept the same as the starting-point prototype order, and hence no increase in implementation complexity.

On the other hand the lowpass-to-lowpass transformation squeezes or stretches the rest of the filter frequency response to ensure that the target filter is real. It should be made clear that the amplitude of the ripples is unchanged as a result of the stretching and squeezing process. However, the location of the peaks and troughs of the ripples, as well as the cut-off frequency and transition-band of the filter are altered. In our application the prototype two-band filter is prewarped so that the resulting target filter has the required transition bandwidth, and it is centered around the new cut-off frequency. In order to create a multiband filter a set of  $M$  two-band filters are cascaded to form:

$$H(z) = \frac{K_0}{2} \prod_{j=1}^{M-1} \left\{ \frac{K_{j-1} + K_j}{K_{j-1}} \prod_{i=0}^{K_{\text{odd},j}-1} A_{j,2i+1}(z) + \frac{K_{j-1} - K_j}{K_{j-1}} z^{-1} \prod_{i=0}^{K_{\text{even},j}-1} A_{j,2i}(z) \right\} \Bigg|_{z = \frac{z - \alpha_j}{1 - \alpha_j z}} \quad (5)$$

where  $K_0 \dots K_{M-1}$  specify magnitude response gains of the overall filter in all their passbands.

Cut-off frequencies are changed through the lowpass-to-lowpass frequency transformation as in (4a). The variable  $M$  is the number of bands of the overall target filter. Each of the building block filters must have magnitude response equal to unity in their first band, ensuring the magnitude response shape created by the preceding cascaded sections are unaltered. This idea is clearly exposed in Figure 3, for the case of  $M=4$ . The next band of the target filter is created through careful choice of the scaling factor  $K$ .



**Figure 3** Composing Multiband Magnitude Response from Frequency Transformed Basic Building Blocks

Every building block filter is designed with requirements for ripples and transition bands so as to match the specifications for the overall target filter. Furthermore each building block filter undergoes a lowpass-to-lowpass transformation. As a result of the stretching and squeezing on the magnitude response caused by the transformation, it is not possible to calculate coefficients  $\alpha_j$  in a straightforward manner as in (4). This is because the transition band after the transformation is not centered around the target cut-off frequency. The transformation process also complicates the calculation of the required transition band of the lowpass prototype filter. Therefore an iterative approach has been adopted for calculation of the frequency transformation coefficient for each basic two-band subfilter, and is summarized as follows:

1. Specify the target cut-off frequency  $v_{\text{cut}2}$  and target transition band  $\Delta v_2$  ( $v_{\text{cut}1}=0.25$  is the original filter cut-off frequency).
2. Calculate coefficient  $\alpha$  from (4a)
3. Inverse transform the upper and lower edges ( $T_{\pm}$ ) of the target filter transition band  $v_{2,T_{\pm}}=v_{\text{cut}2}\pm\Delta v_2$  into  $v_{1,T_{\pm}}$  using

$$v_{1,T_{\pm}} = t \operatorname{an}^{-1} \left( \frac{\alpha - 1}{\alpha + 1} \tan \pi v_{2,T_{\pm}} \right) / 2\pi \quad (6)$$

4. Modify the target cutoff frequency using:

$$v_{\text{cut}2} = v_{\text{cut}1} + \frac{\Delta v_2}{2} \frac{v_{1,T_+} + v_{1,T_-}}{v_{1,T_+} - v_{1,T_-}} \quad (7)$$

5. If the modification done is greater than the allowed frequency error, then go to step 2, else:
6. Calculate the required transition band of the prototype lowpass filter using:

$$\Delta v_1 = v_{1,T_+} - v_{1,T_-} \quad (8)$$

Another problem is the specification of the attenuation for prototype filters so that the ripples of the overall filter in each of its individual passbands have the desired value. The passband and stopband ripples,  $\epsilon_p$ , and  $\epsilon_s$ , respectively of the polyphase filter are related to each other through,  $(1-\epsilon_p)^2 + \epsilon_s^2 = 1$  [2] and the overall multiband filter is designed as a cascade of such filters. The implication of this is that one can not get magnitude response ripples to exactly match the specification. However, one can design the basic polyphase filters so that the resulting ripples are smaller than the specification calls for. The minimum values of the magnitude response in both passbands,  $|H|_{\min 0}$  and  $|H|_{\min 1}$  (passband ripples  $\epsilon_0$  and  $\epsilon_1$ ) of the basic two-band building block filter are:

$$\begin{aligned} |H|_{\min 0}^2 &= K_0^2 + 10^{-A/10} (K_1^2 - K_0^2) & \text{and} & & \epsilon_0 &= 1 - |H|_{\min 0} & (9) \\ |H|_{\min 1}^2 &= K_1^2 + 10^{-A/10} (K_0^2 - K_1^2) & \text{and} & & \epsilon_1 &= 1 - |H|_{\min 1} \end{aligned}$$

where  $A$  is the stopband attenuation in dB of the polyphase lowpass filter employed in constructing the two-band building block. If a number of such basic two-band building blocks are cascaded to form a multiband filter, then the ripples in each passband become a function of all the prototype polyphase lowpass filter attenuations and the minimum gain  $|H|_{\min}$  in each passband (passband ripples,  $\epsilon$ ) can be calculated from (10).

$$|H|_{\min j}^2 = K_0^2 \left\{ \prod_{\substack{m=2 \\ \text{for } j \in M-1}}^{M-2} 1 + 10^{-\frac{A}{10}} \left( \frac{K_m^2}{K_{m-1}^2} - 1 \right) \cdot \prod_{\substack{n=M-j+1 \\ \text{for } j \neq 0}}^{M-2} \frac{K_n^2}{K_{n-1}^2} + 10^{-\frac{A}{10}} \left( 1 - \frac{K_n^2}{K_{n-1}^2} \right) \right\} \quad (10)$$

$$\epsilon_j = 1 - |H|_{\min j} \quad \text{for bands } j = 0, \dots, M-1.$$

The calculation of the required attenuation (passband ripples,  $\epsilon_s$ ) of the prototype polyphase filters requires solving a set of linear equations  $\mathbf{A}_{M \times M-1} \times \mathbf{E}_{M-1 \times 1} = \mathbf{B}_{M \times 1}$ . This is a standard linear programming problem which was solved in MATLAB with 'lp' function.

$$A = \begin{bmatrix} K_1 & K_2 & \cdots & K_M \\ K_0 & K_2 & \cdots & K_M \\ K_0 & K_1 & \cdots & K_M \\ \vdots & \vdots & \ddots & \vdots \\ K_0 & K_1 & \cdots & K_{M-1} \end{bmatrix} \quad E = \begin{bmatrix} \epsilon_{S,1} \\ \epsilon_{S,2} / K_1 \\ \vdots \\ \epsilon_{S,M} / K_M \end{bmatrix} \quad b = \begin{bmatrix} \epsilon_{R,0} \\ \vdots \\ \epsilon_{R,M} \end{bmatrix} \quad (11)$$

where  $\epsilon_{R,1 \dots M}$  are the required ripples.

## MULTIBAND COMPLEX STRUCTURES

The idea outlined for the design of multiband real IIR filters has been extended to the design of complex multiband IIR filters. This is done by combining the lowpass-to-lowpass frequency transformation with a complex rotation in the frequency domain:

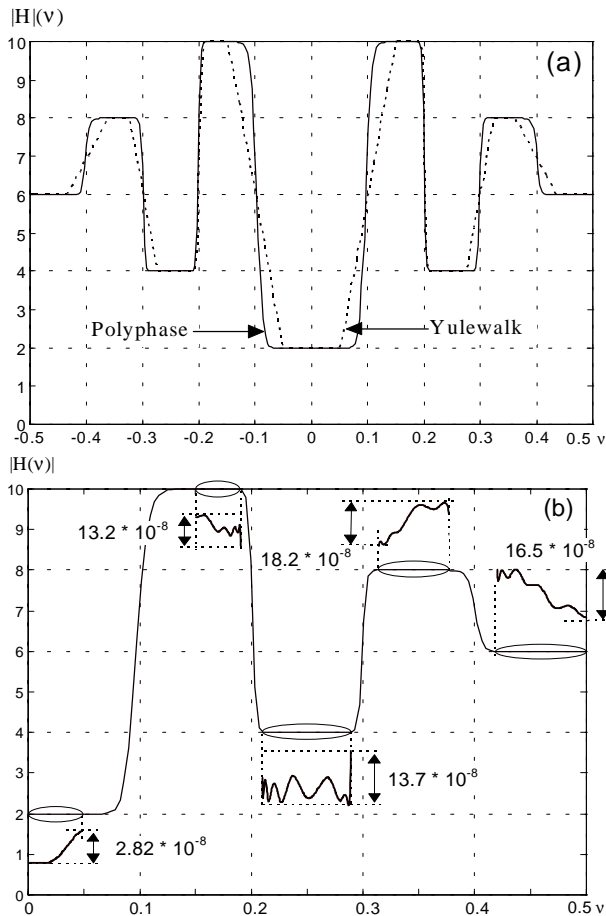
$$z = cz \quad \text{where} \quad c = e^{j2\pi(v_{\text{new}} - v_{\text{old}})} \quad (12)$$

The idea is very similar to the real case and only requires small modifications to the equations used previously to incorporate an additional frequency transformation (12) on the prototype polyphase filter prior to changing its cut-off frequency through (4). The structure of Figure 1 is modified by the addition of a phase shift by  $-\pi/2$  to each delay operator in the transfer function. This rotates the magnitude response of the prototype filter by  $\Delta v=0.25$  ( $\Delta\phi=\pi/2$ ) in the frequency domain. A Hilbert transformer may be used to make the  $\pi/2$  phase shift.

## EXAMPLES

Here we show two examples, one for the real and one for the complex case. We set similar specifications to both designs in order to make comparisons between the real and complex design methods. The specifications for both filters are given in Table 1 and the results are presented in Figure 4 for both cases. In both cases the resulting overall IIR filter order is 50 (81 multiplications). This is a very good result considering the ripples that were achieved in each passband. These were better than what was required. The cut-off frequencies and transition bands were achieved with accuracy bordering on the floating-point precision of the computational platform. This level of accuracy can be apportioned to the prototype polyphase filters having only four to six coefficients as well as the simple first order frequency transformations employed. As a result of the small number of computations are involved, 81. For comparison we compared our algorithm with YULEWALK one (real filters) designed for identical specifications [8] (Figure 4a). Although the specifications for level values were achieved, bandedges

were all shifted approximately by  $\pm 0.0015$  and ripple values were a lot higher than for the polyphase approach (Table 1).



**Figure 4** Magnitude Responses of the Five-Band Example Filter, (a) Compared to Yulewalk, (b) Zoomed into Passbands

### SOME REMARKS

In this paper we presented a novel technique for the design of multiband IIR filters, employing two-path polyphase building blocks achieving solutions to the very stringently specified

magnitude response requirements for both real and complex cases. The structure suggested in Figure 1 is the most efficient, if not the only way, to implement this class of filters, as computing the equivalent IIR transfer function would necessitate too many convolutions and hence suffer from numerical error accumulation.

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Band Number	0	↔	1	↔	2	↔	3	↔	4
<b>Real Filter Case</b>									
Gain	2	-	10	-	4	-	8	-	6
Ripples required/achieved [ $10^{-8}$ ]	100/1.41	-	100/6.6	-	100/6.84	-	100/9.1	-	100/8.23
Ripples for "Yulewalk" [ $10^{-3}$ ]	0.42		1.5		1.15		0.4		0.1
Cutoff freq./Transition band	-	0.1/0.1	-	0.2/0.01	-	0.3/0.05	-	0.4/0.075	-
<b>Complex Filter Case</b>									
Gain	2	-	10	-	4	-	8	-	6
Ripples required/achieved [ $10^{-8}$ ]	100/0.307	-	100/6.6	-	100/6.84	-	100/9.1	-	100/8.23
Cutoff freq./Transition band	-	-0.3/0.2	-	-0.1/0.02	-	0.1/0.1	-	0.3/0.15	-

**Table 1** Specifications and Results for the, Real and Complex Multiband Filter Design Example.