Noncoherent Detection and Differential Detection

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***Abstract*— From the communication channel and implementation aspects of communications, the transmission environement may be sufficiently degraded, e.g. a multipath fading channel, that acquiring and tracking a coherent demodulation reference signal is practically difficult. Coherent receivers require exact knowledge of the channel phase for optimum performance. Due to the difficult task of estimation channel phase, differential detection is an attractive alternative to coherent detection.**

A conventional differential detector uses the signal received in the previous symbol interval as a phase reference for the received signal in the current interval. As long as the phase distortion introduced by channel varies slowly relative to the symbol rate, conventional differential detection will work quite well.

This paper shows a short overview of some algorithms for noncoherent detection and differential detection, particularly with PSK modulation technique.

I. Introduction

The use of differential detection are widely acknowleged and understood. It provides an inherent robustness to phase and frequency sets meanwhile at the other aspect it offers a low complexity solution.

There are several algorithms for differential detection that are commonly found in modulation technique. A differential detection technique for M-ary Phase Shift Keying (MPSK) for detection data transmitted, which uses multiple-symbol observation interval is one of differential detection. Due to their good energy as well as spectral efficiency, MPSK as a class of linear digital modulation techniques which is quite often used and suitable for mobile communications.

A multiple-symbol differential detector (MSDD) is a detector that makes a decision about a block of N consecutive PSK symbols based on N+1 received samples. The other differential detection that minimizes the quadratic erros of the certain number of symbol detectors and uses decision feedback, this detection refers to differential feedback detection [3]. Furthermore, since we consider the initial phase of the modulator to be unknown at the receiver, this detection is refered to noncoherent detection [4].

We note that differential detection eliminates the need for carrier acquisition and tracking in the receiver, it suffers from a performance penalty (additional required SNR at a given bit error rate) when compared to ideal (with perfect carrier phase reference) coherent detection. As long as the parameters of the detectors (differential detection and noncoherent detection) are chosen properly, they can provide an error performance sufficiently close to that of ideal coherent detection.

II. Differential Detection

In the following we consider several differential detection algorithms that are based on maximum likelihood sequence estimation and differential feedback detection.

A. Multiple Symbol Differential Detection (MSDD) for Uncoded MPSK

Divsalar and Simon [1] who first proposed the idea of multiple symbol differential detection (MSDD) of differentially multiple phase-shift keying (M-PSK) signals transmitted over an additive white Gaussian noise (AWGN) channel. The method was presented to bridge the gap between the performances of coherent M-PSK and non-coherent differential M-PSK (M-DPSK).

They assumed that the channel phase was unknown to the receiver, but was constant over multiple symbol intervals. The detection was made based on an observation interval consisting of more than two received symbols. They showed that for a long observation interval, the performance of MSDD (in terms of the required signal-to-noise ratio (SNR) for a given bit-error-probability (BEP)) approached that of coherent detection. The main advantage of MSDD is that it does not require a coherent phase reference at the receiver wince we consider noncoherent detection.

In this case, the received signal of the transmission of MPSK signals over an AWGN channel is given as

$$r_k = s_k e^{j\theta_k} + n_k = \sqrt{2P} e^{j\phi_k} e^{j\theta_k} + n_k; kT \leq t \leq (k+1)T \quad (1)$$

where P denotes the constant signal power, T denotes the MPSK symbol interval, ϕ_k the transmitted phase (which takes on one of M uniformly distributed values $\beta_m = 2\pi m/M; m = 0, 1, \cdots, M-1$ around the unit circle), n_k is a sample of zero-mean complex Gaussian noise and θ_k is an arbitrary phase introduced by the channel.

Furthermore, we note $\theta_k = \theta$ due to a received signal of length N with assumption that θ_k is independent of k over the length of this sequence, thus the received sequence r is then expressed as

$$r = se^{j\theta} + n \quad (2)$$

The posteriori probability of r given s and θ is then defined as

$$p(r|s,\theta) = \frac{1}{(2\pi\sigma_n^2)^N}\ exp -\frac{\|r - se^{j\theta}\|^2}{2\sigma_n^2}. \quad (3)$$

Generally they assumed θ to be uniformly distributed, then the a posteriori probability of r given s is simply

$$\begin{aligned} p(r|s) &= \int_{-\pi}^{\pi} p(r|s,\theta)p(\theta)\ d\theta \\ &= \frac{1}{(2\pi\sigma_n^2)^N}\ exp\ -\frac{1}{(2\pi\sigma_n^2)} \sum_{i=0}^{N-1} [|r_{k-i}|^2 - |s_k - i|^2] \\ &\times I_o(-\frac{1}{\sigma_n^2}| \sum_{i=0}^{N-1} r_{k-i}s_k^* - i|) \end{aligned} \tag{4}$$

where $I_o(x)$ is the zeroth order modified Bessel function of the first kind. We note $|s_k|^2$ is constant for all phases. Thus $I_o(x)$ is a monotonically increasing function of its argument.

The main idea of this detection is to maximize $p(r|s)$ over s is equivalent to finding

$$\underset{i}{max}| \sum_{i=0}^{N-1} r_{k-i}\ s_{k-i}^*|^2 \tag{5}$$

and the decision statistics is given

$$\eta\ =\ |r_{k-N+1} + \sum_{i=0}^{N-2} r_{k-i}e^{-j\sum_{m=0}^{N-i-2}\Delta\phi_{k-i-m}}|^2 \tag{6}$$

where $r_{k-N+1}, r_{k-N+2}, \cdots, r_k$ is the received symbol sequence.

The detection tries to observe the received signal over N symbol time intervals. For the length of sequence N = 2, in Fig. 1, the decision statistics (6) becomes

$$\begin{aligned} \eta\ &=\ |r_{k-1} + r_k e^{-j\Delta\phi_k}|^2 \\ &= |r_{k-1}|^2 + |r_k|^2 + 2\text{Re}\{r_k r_{k-1}^* e^{-j\Delta\phi_k}\} \end{aligned} \tag{7}$$

therefore, the decision rule becomes

$$\text{choose } \Delta\hat{\phi}_k \text{ if } \text{Re}\{r_k r_{k-1}^* e^{-j\Delta\hat{\phi}_k}\} \text{ is maximum} \tag{8}$$

where $\hat{\phi}$ is a particular sequence of the β_m's. For N = 3, then the decision statistics becomes

$$\begin{aligned} \eta &= |r_{k-2} + r_k e^{-j(\Delta\phi_k + \Delta\phi_{k-1})} + r_{k-1}e^{-j\Delta\phi_k}|^2 \\ &= |r_{k-2}|^2 + |r_{k-1}|^2 + |r_k|^2 + 2\text{Re}\{r_k r_{k-2}^* e^{-j(\Delta\phi_k + \Delta\phi_{k-1})}\} \\ &+ 2\text{Re}\{r_{k-1}r_{k-2}^* e^{-j(\Delta\phi_{k-1})}\} \\ &+ 2\text{Re}\{r_k r_{k-1}^* e^{-j\Delta\phi_k}\} \end{aligned} \tag{9}$$

Thus, the decision rule becomes: choose $\Delta\hat{\phi_k}$ and $\Delta\hat{\phi}_{k-1}$ if the sum of real part of (9) is maximum. We note that the terms of the metric used in the decision rule are identical to those used to make successive, independent and an optimum joint decision on $\Delta\phi_k$ and $\Delta\phi_{k-1}$, respectively, in conventional MDPSK.

We can conclude MSDD algorithm as following: the first received sample is used to provide a phase reference for the entire block while the last sample is used to provide a reference for the next block. When N is equal to 1, we have

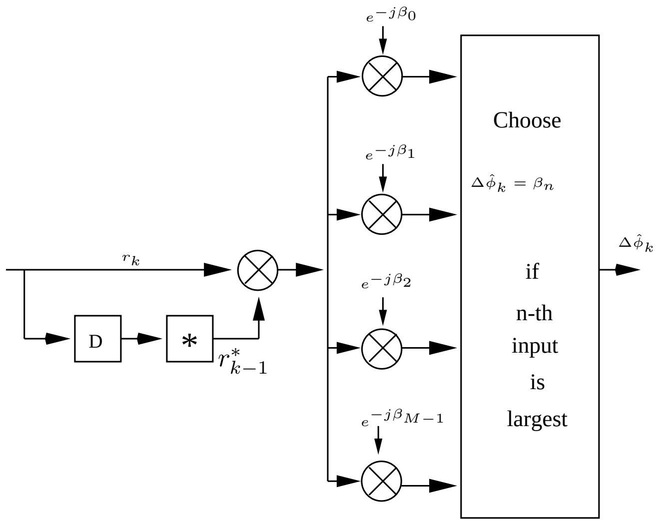

Fig. 1: Conventional differential detection of MPSK; N=2

a conventional differential detector. The larger the value of N is, the better the error performance. In the limiting case where N approached infinity, the performance of a multiple-symbol differential detector in AWGN channel approaches that of a coherent detector with differential encoding ot resolve phase ambiquity.

The performance improvement over conventional differential detection is obtained by exploiting the correlation between the phase distortion experienced by the different transmitted PSK symbols. The computational complexity of an MSDD in AWGN channel is $O(NlogN)$ (where $O(.)$ means of the order of).

B. Fast Algorithm for MSDD

As described above, the multiple symbol detection of MPSK squences makes use of maximum likelihood sequence estimation (MLSE), rather than symbol-by symbol detection as in conventional differential detection, which eliminates the exact carrier phase recovery but still retain good E_bN_o performance.Furthermore, the complexity of MSDD grows exponentially with the sequence length. Yingqun et. al in 1999 [2], proposed new algorithm to make a fast computation for multiple symbol detection of uncoded MPSK sequence over an AWGN channel. The central idea is to define a new decision statistic for each symbol sequence.

Based on the result from [1], the decision statistic (6) for optimal multiple symbol detection, then the decision rule is equivalent to finding $\Delta\hat{\phi}_k = (\Delta\hat{\phi}_{k-N+2}, \Delta\hat{\phi}_{k-N+3}, \cdots, \Delta\hat{\phi}_k)$ that maximes η. They proposed to make such assumption that the carrier phase is constant over the observation of this sequence.

For the fast algorithm of MSDD, the decision statistic (6) is then multiplied at the right side by

$$|e^{j\sum_{m=1}^{N-2}\Delta\phi_{k-m}}|^2 \text{ (equal to 1)} \tag{10}$$

and we obtain new decision statistic

$$\eta = |\sum_{i=2}^{N-1} r_{k-1} e^{-j\sum_{m=1}^{i-1}\Delta\phi_{k-m}} r_{k-1} + r_k e^{-j\Delta\phi_k}|^2 \quad (11)$$

if the $N-1$ symbols before the kth symbol have been determined to be $\Delta\hat{\phi}_{k-i}$ $(i = 1, 2, \cdots, N-1)$, it must be established how this prior information can be used to determine the kth symbol. A decision statistic for the kth symbol can be defined as

$$\eta_k(\Delta\phi_k) = |\sum_{i=2}^{N-1} r_{k-1} e^{-j\sum_{m=1}^{i-1}\Delta\phi_{k-m}} r_{k-1} + r_k e^{-j\Delta\phi_k}|^2 \quad (12)$$

The decison rule for the kth symbol therefore involves choosing a value for $\Delta\hat{\phi}_k$ which maximise $\eta(\Delta\hat{\phi}_k)$. Generally, they devided (12) into two main part metric

$$\lambda_{k,i} = \begin{cases} r_{k-i} e^{j\sum_{m-1}^{i-1}\Delta\hat{\phi}_{k-m}} & \text{for } 2 \le i \le N-1, \\ r_{k-1} & \text{for } i = 1. \end{cases} \quad (13)$$

then new decision rule is defined as

$$\eta_k(\Delta\phi_k) = |\sum_{i=1}^{N-1} \lambda_{k-i} + r_k e^{-j\Delta\phi_k}|^2 \quad (14)$$

For $(k+1)th$ symbol, from equation (13), we can obtain

$$\lambda_{k+1,i} = \begin{cases} \lambda_{k,i-1} e^{j\Delta\hat{\phi}_k} & \text{for } 2 \le i \le N-1, \\ r_k & \text{for } i = 1. \end{cases} \quad (15)$$

For each symbol, a similar decision statistic and decision rule can be calculated while taking into account that a simple relation exists between successive symbol decision statistics.

The above algorithm dramatically reduces the computational complexity. The advantage of this detection that it only adds $N-2$ multiplications, one addition and some additional memory to conventional differential detection. The computational complexity of this symbol-by-symbol algorithm dramatically reduces the computational complexity with only grows linearly to the length of the observed symbols yet still gives good $E_b N_o$ performance [2].

C. Decision Feedback Differential Detection (DFDD)

This detection was proposed by Franz Edbauer (1992) [3]. The main idea of this algorithm is to use L symbol detectors with delays of $1, 2, \cdots, L$ symbol periods and to feed back detected PSK symbols. An improvement of BER performance can be achieved if symbol detectors with delay larger than a symbol period are used and if detected symbols are fed back to the detection unit.

The detection considers the received low pass equivalent signal in the form

$$r(t) = s(t)\exp[j(\omega t + \psi)] + n(t) \quad (16)$$

where ω is the radian frequency offset between transmitter and receiver and ψ is a phase offset in the time interval $(n-L-1)T < t \le nT$. Maximum signal delay in the receiver is denoted as $(L+1)T$.

The received signal is then passed through the matched filter and sampled at time nT yielding $y_n = \sqrt{2S}c_n\exp(j\psi) + n_n$, where S is the signal power, c_n is the differentially encoded M-DPSK symbol and n_k is zero-mean complex Gaussian noise.

In Fig. III-A, the approach for differential detection is based on minimizing the quadratic errors of the output of L symbol detectors of orders $j = 1, 2, \cdots, L$ and on the decision feedback.

The symbol detector of order j uses the jth previous symbol y_{n-j} as phase reference and performs the operation $y_n y_{n-j}^*$. Its output is

$$z_n^{(j)} = y_n y_{n-j}^* = 2S a_n a_{n-1} \cdots a_{n-j+1} + n_n^{(j)} \quad (17)$$

where a_n represents the complex baseband PSK signal. The metric, represents the quadratic error sum of the detector outpus under the hypothesis that at time n symbol $\tilde{a}_n$ has been sent, is given by

$$\begin{aligned} \eta = |z_n^{(1)} - 2S\tilde{a}_n|^2 + |z_n^{(2)} - 2S\tilde{a}_n\tilde{a}_{n-1}|^2 + \cdots \\ + |z_n^{(L)} - 2S\tilde{a}_n\hat{a}_{n-1}\cdots\hat{a}_{n-L+1}|^2, \; L > 1 \end{aligned} \quad (18)$$

The decision rule for $\tilde{a}_n$ is $\tilde{a}_n = \underset{\tilde{a}_n}{min}\{\eta\} = \underset{\tilde{a}_n}{max}\sum_{j=1}^{L}\{\tilde{a}_n M_j\}$.

$$M_j = \begin{cases} z_n^{(1)*}, & j = 1, \\ \tilde{a}_{n-1}, \tilde{a}_{n-2}, \cdots, \tilde{a}_{n-j+1} z_n^{(j)*}, & j = 2, 3, \cdots, L. \end{cases} \quad (19)$$

However, we distinguish this differential detection is completly different with MSDD [1] that is based on maximum likelihood (ML) sequence estimation, furthermore this detection proposed a simpler solution

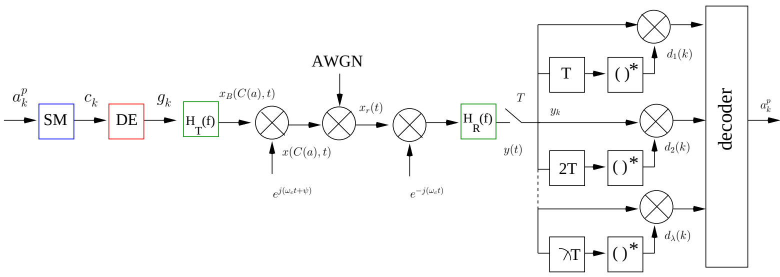

Fig. 2: Block diagram of noncoherent detection of PSK

III. Noncoherent Detection

A. Noncoherent Detection PSK

Since the initial phase of the modulator is considered unknown to the receiver, we refer this detection as noncoherent detection [4]. The main idea of this detection is to maximize a posteriory probability of the received signal and to estimate a certain amount of symbols known to the receiver as the initial reference at decoder.

In Fig. 2, block diagram of noncoherent detection for 8-PSK (2-bit words) and $\pi/4-QPSK$ (3-bit words) in AWGN channel is shown.

For the received signal $x_r(t) = x[C(a), t] + n(t)$ with $n(t)$ denotes the white Gaussian noise component with one-sided spectral density N_0, the decoder makes its decision on the maximation of

$$\begin{aligned} f[x_r(t)|C(a)] &= \int_{-\infty}^{\infty} f[x_r(t)|C(a), \psi]p(\psi)\ d\psi \\ &= \Gamma\ \exp\ \{-[\frac{1}{2N_0}\int_{-\infty}^{\infty}|x_r(t)|^2]\}\ dt \\ &\times\ \exp\{-(\frac{1}{2N_0}\int_{-\infty}^{\infty}|x_B(C(a),t)|^2)\ dt\} \\ &\times\ I_o\{\frac{|\int_{-\infty}^{\infty}[x_r(t)e^{-j(\omega_c t)}]x_B^*[C(a),t]\ dt|}{N_0}\}. \end{aligned} \tag{20}$$

where ψ is the initial phase of the modulator, I_o is zeroth order Bessel function and Γ is a constant. From the Bessel function tends to monotically increase, it means that the optimal noncoherent receiver is equivalent to the maximation of the nominator of the Bessel function argument.

Maximisation of its squared function means

$$\max\ \mathcal{R}_1[y, C(a)] \Rightarrow (\mathcal{R}_1[y, C(a)])^2 = (|\sum_{k=0}^{L} y_k c_k^*|)^2 \tag{21}$$

with $y(t) = \int_{-\infty}^{\infty}[x_r(\tau)e^{-j(\omega_c\tau)}]h_T^*(t-\tau)d\tau$ leads to the multiple differential detection structures. $d_l(k) = y_k y_{k-l}^*$ represents differential detector having delay element equal to lT second where $d_l^I(k) = \text{Re}\ \{d_l(k)\}$ and $d_l^Q(k) = \text{Im}\ \{d_l(k)\}$.

Due to range value of l between 1 and L (length of the information sequence), a total of L differential detectors is required and it leads to complexity structures. One way to reduce complexity is to choose a certain value of the truncated number (λ) rather than L. The task of decoder is then to initialize reference, by assuming the transmission of λ symbols $(c_0, c_{-1}, \cdots, c_{-(\lambda-1)})$ are known to the receiver, before the transmission of the current information. The complexity of this suboptimal structure is low compared with the optimal structure, meanwhile the performance of the suboptimal structure closes to the optimal receiver [4].

IV. Conclusions

We have seen that noncoherent detection is a detection technique that can be implemented without using carrier frequency which the performance asymptotically approached that of the coherent detection. However, differential detection is an attractive alternative to coherent detection. MSDD does not require, however the ability to measure relative phase differences which eliminates the exact carrier phase recovery but the computational complexity grows exponentially with the sequence length. Decision feedback differential detector is less complex than the MSDD based on maximum-likelihood detector.

References

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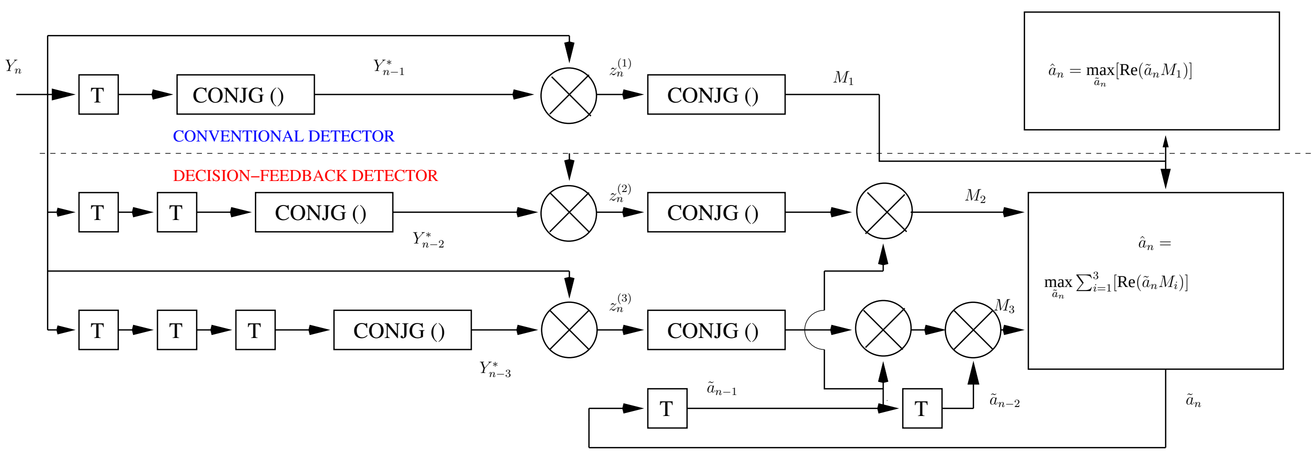

Fig. 3: Conventional and decision-feedback detector for DPSK signals with three symbol detectors (L = 3)