Equalizer Design for Multi-Path Channels

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Outline

- Paper title:
  "On the Distribution of Zeros of Mobile Channels with Application to GSM/EDGE"
- Presentation Outline:
  1. Multi-path channels (MPCs): intuitive characterization
  2. Equalizers (EQs): Application, types of interest
  3. Multi-path channels: statistical model
  4. Power Delay Profiles (PDPs): introduction, GSM/EDGE application examples

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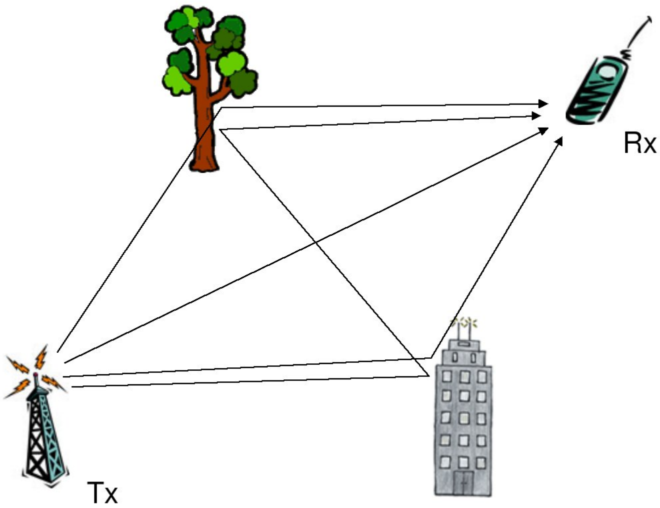
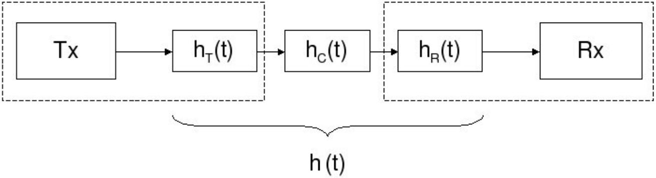
MPCs - Intuitive Characterization



MPCs - Intuitive Characterization

MPCs through rose-colored glasses

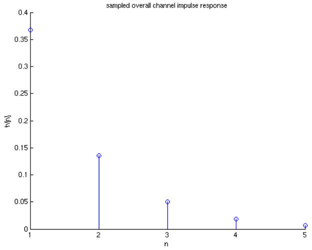
- Mobile channel can be seen as combination of several propagation paths with individual delay.

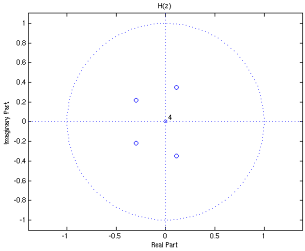


- Line of sight (LOS): one bin in channel impulse response h[n]
- LOS + one delayed path: two bins in channel impulse response
- LOS + ...

MPCs - Intuitive Characterization

- Consider $\{h[n]\} = e^{-n}; n = 1:5$





- Two effects / points of view when considering transmitted symbols:
  1. ISI - overlapping of impulse responses
  2. Zeros in complex plane

From Multipath Channels to Equalizers

- Equalizer removes ISI / places poles at zeros of h[n]
- Result: "clean" h[n] - only one bin / no zeros
- $\exists$ various approaches
- Paper focuses on some selected ones ...

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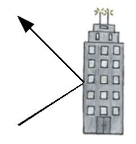
Which EQ to choose?

Equalizer Part...

Propagation Phenomena

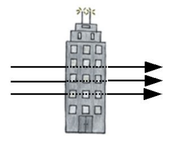
Reflection

On smooth surfaces



Transmission

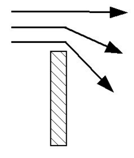
Through buildings, walls, etc.



Propagation Phenomena

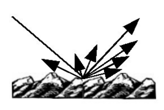
Dispersion

Plane wave = superposition of spherical waves



Scattering

On rough surfaces

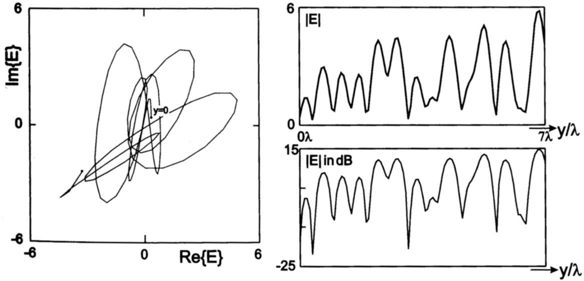


Fading

- Reflection, transmission, dispersion, scattering $\Rightarrow$ waves with different phase shifts arrive at reciever $\Rightarrow$
- Destructive interference which leads to *"Small-scale fading"* (up to 10λ), as opposed to
- "Large-scale fading" (e.g. shadowing by hills)

Fading (cont'd)

- Phasor and Amplitude of recieved field when Rx moves:



More realistic interpretation of $h[n]$

Relation between $h[n]$ and $h(t)$

- Overall time-variant impulse response $h_m[n]$ is sampled version of $h_\tau(t)$
- ∞ multipath components in $h_\tau(t_i)$ contribute to one tap in $h_m[n]$

Small-Scale Fading Without Dominant Component

$h_\tau(t)$, $h_m[n]$ as Random Processes

- $h_\tau(t)$, $h_m[n]$ modeled as random process
- $h_\tau(t)$ are assumed to be identically, independently distributed $\Rightarrow$
- via central limit theorem:
  $h_m[n]$ - distribution is circular symmetric zero-mean Gaussian distribution
- Amplitude $|h_m[n]|$ is Rayleigh-distributed
- Phase $arg(h_m[n])$ is uniformly distributed

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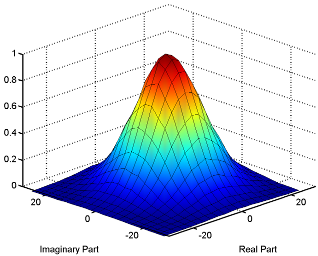
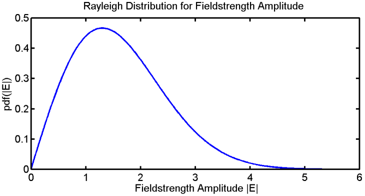
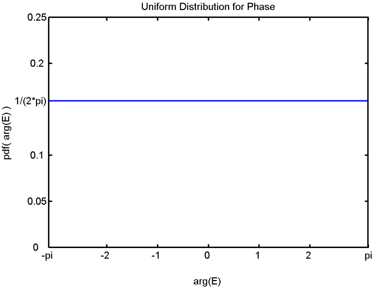
Amplitude- and Phase-Distribution of E resp. $h_m[n]$

$f_{xy}(Re(E), Im(E))$, $pdf(|E|)$, $pdf(arg(E))$



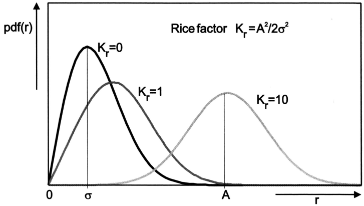




Small-Scale Fading With a Dominant Component

What if one component gets dominant?

- Rayleigh distribution $\rightarrow$ Gaussian distribution with $\mu > 0$
- Rice Distribution



A...amplitude of dominant component

Underlying Mathematics

- From [PA]: marginal density of zeros

$$f_r(r) \triangleq r \cdot \int_0^{2\pi} f_z(r \cdot cos(\varphi) + j \cdot r \cdot sin(\varphi)) \cdot d\varphi \qquad (1)$$

- Expected number of zeros in the disc $|z| = r \leq R$

$$n(R) = \int_0^R f_r(r) \cdot dr \qquad (2)$$

- Expected number of zeros inside $\rho \leq |z| \leq 1/\rho, 0 < \rho < 1$

$$d(\rho) = n(1/\rho) - n(\rho) \qquad (3)$$

  - Of special interest for EQ design: $\rho = 0.9$ - region $0.9 \leq |z| \leq 1.11$

Uncorrelated Impulse Response coefficients

- Marginal density

$$f_r(r) = \frac{2}{r}\left(\frac{\sum_{n=0}^{L-1} n^2\sigma_h^2[L-1-n]r^{2n}}{\sum_{n=0}^{L-1} \sigma_h^2[L-1-n]r^{2n}} - \left(\frac{\sum_{n=0}^{L-1} n\sigma_h^2[L-1-n]r^{2n}}{\sum_{n=0}^{L-1} \sigma_h^2[L-1-n]r^{2n}}\right)^2\right) \tag{4}$$

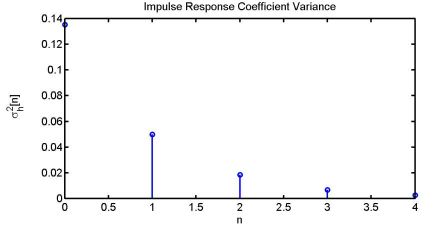
- $\Rightarrow$ Expected number of zeros inside the disc $|z| \leq R$

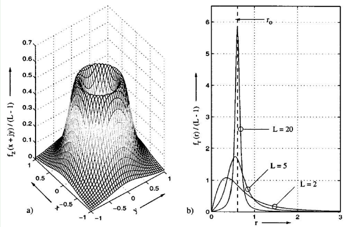
$$n(R) = \frac{\sum_{n=0}^{L-1} n \cdot \sigma_h^2[L-1-n] \cdot R^{2\cdot n}}{\sum_{n=0}^{L-1} \sigma_h^2[L-1-n] \cdot R^{2\cdot n}} \tag{5}$$

Distribution Based on Mathematical Model

Example

Channel impulse response with exponential decay of tap variances.





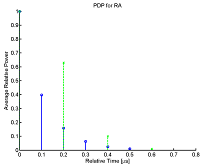
PDP Introduction

- Also known as "Delay Power Spectral Density".
- Simple measure to characterize channel impulse responses.
- European Telecommunications Standards Institute (ETSI) recommends PDPs for selected environments in GSM/EDGE (Global System for Mobile Communications / Enhanced Data rates for GSM Evolution [1])

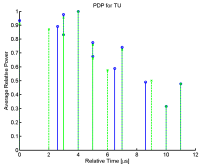
[1]GMSK - Gaussian Minimum (Phase) Shift Keying vs. 8-PSK

Recommended Propagation Models

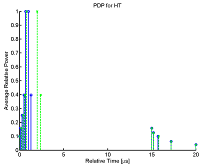
- Rural Area - RA



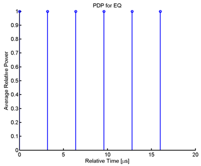
- Typical Urban - TU



- Hilly Terrain - HT



- Equalizer Test - EQ



Measured PDP

Example

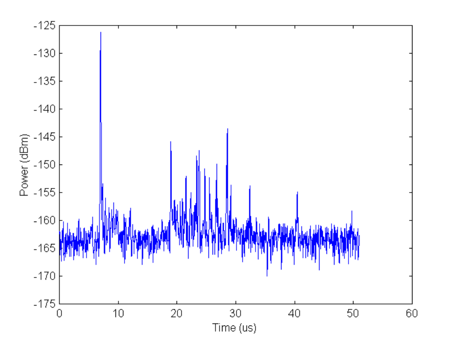


Figure: Measured power delay profile. Source: *Institute for Telecommunication Sciences, Boulder, Colorado*

GSM/EDGE Equalizer Design

- $f_r(r), n(R), d(\rho)$ have to be calculated by numerical integration.

Rural Area

- Esentially flat channel (∼"one tap only").
- Zeros of $h(t)$ mainly influenced by transmit- and receiver input filter.
- $h_T(t)$: GMSK pulse, as standardized for EDGE.
- $h_R(t)$: Reciever designer's choice. In the paper: squared-root raised cosine (SRC) with $\alpha = 0.3$.

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GSM/EDGE Equalizer Design (cont'd)

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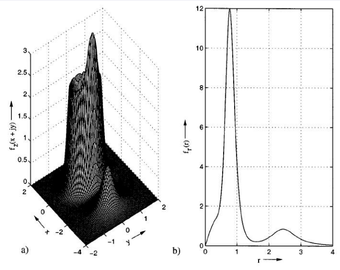

Figure: $f_z(z)$ and $f_r(r)$ for $L = 7$ [Schober01]

- $f_z(z)$ not rotational symmetric.
- Most zeros inside $|z| < 1$, some in area $|z| \geq 2$, however $\Rightarrow$
- DFE resp. DDFSE/RSSE performance increasable.
- Only 1 zero inside $0.9 \leq |z| \leq 1.11 \Rightarrow$ truncation of $h[n]$ to $L = 3$ possible.

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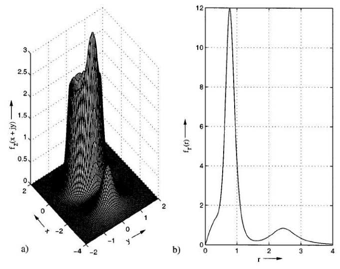

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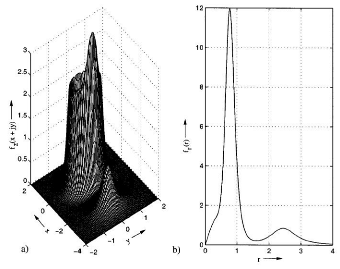

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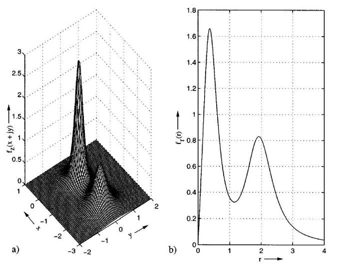

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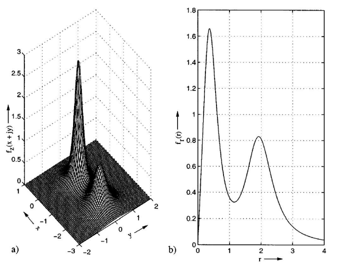

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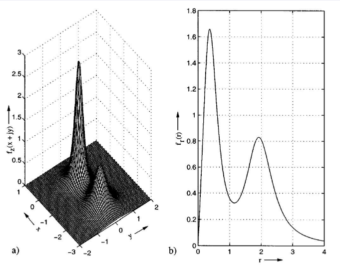

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Equalizer Test

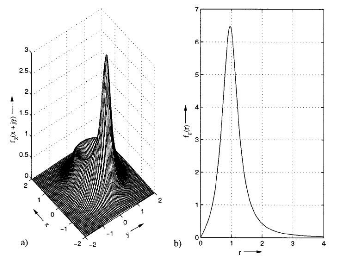
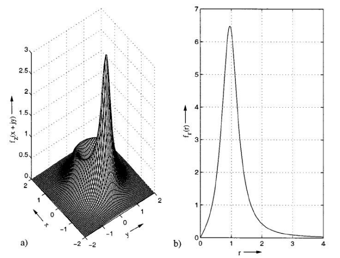


Figure: $f_z(z)$ and $f_r(r)$ for $L = 6$ [Schober01]

- $f_z(z)$ not circ. symm.
- Correlated taps - many zeros near unit circle.
- On average number of zeros inside and outside $|z| = 1$ equal $\Rightarrow$ Prefilter.
- 1.2 zeros inside $0.9 \leq |z| \leq 1.11 \Rightarrow$ truncation of $h[n]$ to $L = 3$ possible.

GSM/EDGE Equalizer Design (cont'd)

Equalizer Test



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Summary

- Overview on MPC impulse responses and distribution of zeros.
- Suitable equalizer concepts.
- Equalizer design for statistically described MPCs.

The End.

The End

Thank you for your attention!

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