

# Digital Subscriber Lines (xDSL) Twisted-pair channel modeling

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# Talk overview

- Introduction to DSL (digital subscriber line)
- Twisted-pair channel modeling
- Transceiver front-end noise
- Channel capacity
- Hybrid circuits

# Types of DSL

- ISDN (integrated services digital networks)
  - Started 1985 at AT&T
  - Used bandwidth : 80 kHz
  - Basic rate ISDN transports 160 kb/s
  - loop length up to 18 kft (5.5 km)







# Subscriber loop environment

- connects central office (CO) and end user
- designed for 4 kHz voice band
- ringing voltage up to 140 volts
- Plain Old Telephone Service (POTS), analog phones
- different wire gauge, installations (aerial,...)

# Bridged taps

- open wire connected in parallel to loop
- used for reaching future customers
- unterminated → reflections
- → signal loss, distortions
- 80% of US loops have bridged taps



# Loading coils

- series inductor in twisted pair loop
- used for extending beyond 18 kft
- flattens frequency response across voice band
- low pass filtering
- blocks xDSL signals
- 15% of US loops have loading coils

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# Two-port modeling

- yields accurate results for DSL <20 MHz
- loop consists of cascade of different wire types

$$(1) \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \Phi \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

# Two-port modeling

with

$$(2) \quad T(f) = \frac{V_2}{V_1} = \frac{1}{A + B \frac{I_2}{V_2}}$$

and  $V_2/V_1 = Z_2 = Z_L$  the load impedance yields

$$(3) \quad T(f) = \frac{Z_L}{AZ_L + B}$$

# Two-port modeling

Transfer function  $H(f)$

$$(4) \frac{V_L(f)}{V_s(f)} = H(f) = \frac{V_L(f) V_2(f)}{V_2(f) V_S(f)} = \frac{Z_1}{Z_1 + Z_S} T(f)$$

with  $Z_1 = V_1/I_1$ , input impedance of two-port

# Equivalent circuit

deriving the ABCD parameters

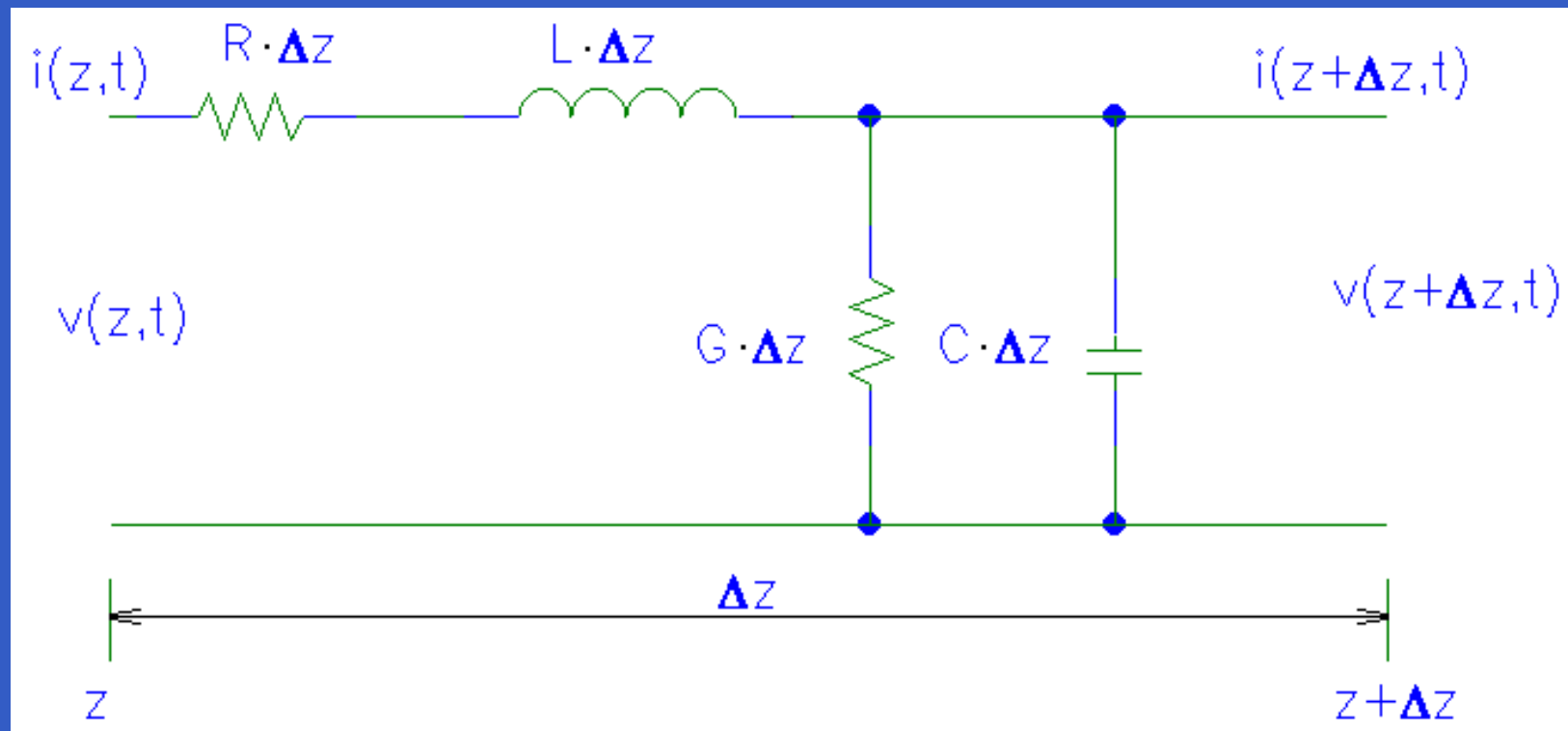


Figure 1: The transmission line equivalent circuit.

# Transmission line equation

$$(5) \quad -\frac{\partial v(z, t)}{\partial z} = Ri(z, t) + L\frac{\partial i(z, t)}{\partial t}$$

$$(6) \quad -\frac{\partial i(z, t)}{\partial z} = Gv(z, t) + C\frac{\partial v(z, t)}{\partial t}$$

# Time harmonic equations

with

$$(7) v(z, t) = \Re [V(z)e^{j\omega t}], \quad i(z, t) = \Re [I(z)e^{j\omega t}]$$

$$(8) \quad -\frac{dV(z)}{dz} = (R + j\omega L)I(z), \quad -\frac{dI(z)}{dz} = (G + j\omega C)V(z)$$



# Time harmonic equations

combining gives

$$(9) \quad \frac{d^2 V(z)}{dz^2} = \gamma^2 V(z), \quad \frac{d^2 I(z)}{dz^2} = \gamma^2 I(z)$$

with

$$(10) \quad \gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{ZY}$$

- frequency dependent propagation constant  $\gamma$
- attenuation constant  $\alpha$
- phase constant  $\beta$

# Time harmonic equations

Solutions are

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}.$$

Characteristic impedance

$$(12) \quad Z_0 = \frac{V_0^+}{I_0^+} = \frac{V_0^-}{I_0^-} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

# Matrix form

for a given wire length  $d$

$$\begin{bmatrix} V(0) \\ I(0) \end{bmatrix} = \begin{bmatrix} \cosh(\gamma d) & Z_0 \sinh(\gamma d) \\ \frac{1}{Z_0} \sinh(\gamma d) & \cosh(\gamma d) \end{bmatrix} \begin{bmatrix} V(d) \\ I(d) \end{bmatrix} \quad (13)$$

# Characteristic functions

with  $V(d)/I(d) = Z_L$

$$(14) \quad T = \frac{1}{\cosh(\gamma d) + \left(\frac{Z_0}{Z_L}\right) \sinh(\gamma d)}$$

and input impedance  $V(0)/I(0)$

$$(15) \quad Z_1 = Z_0 \frac{Z_L + Z_0 \tanh(\gamma d)}{Z_0 + Z_L \tanh(\gamma d)}$$

# Characteristic functions

$$(16) \quad H(f) = \frac{Z_1}{Z_1 + Z_S} T(f)$$

and therefore

$$(17) \quad H(f) = \frac{Z_0 / \cosh(\gamma d)}{Z_S \left[ \frac{Z_0}{Z_L} + \tanh(\gamma d) \right] + Z_0 \left[ 1 + \frac{Z_0}{Z_L} \tanh(\gamma d) \right]}$$

# Bridged taps

$$(18) \quad \Phi_2 = \begin{bmatrix} 1 & 0 \\ 1/Z_{bt} & 1 \end{bmatrix}$$

when terminated with open circuit

$$(19) \quad Z_{bt} = Z_{0t} \frac{\cosh(\gamma d)}{\sinh(\gamma d)}$$

# Loading coils

$$(20) \quad \Phi_{coil} = \begin{bmatrix} 1 & j\omega L_{coil} \\ 0 & 1 \end{bmatrix}$$

when terminated with open circuit

$$(21) \quad Z_{bt} = Z_{0t} \frac{\cosh(\gamma d)}{\sinh(\gamma d)}$$

# Overall two-port

$$(22) \quad \Phi = \Phi_0 \Phi_1 \cdots \Phi_N$$

and the source voltage divider is

$$(23) \quad \Phi_0 = \begin{bmatrix} 1 & Z_S \\ 0 & 1 \end{bmatrix}$$



# RCLG are frequency dependent

Approximate measurement fittings valid up to 20 MHz:

- resistance  $R(f) = a + b\sqrt{f}$
- inductance  $L(f) = c + \frac{d}{\sqrt{f}}$
- conductance  $G(f) \propto f$
- Capacitance is approximately constant

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# Transceiver front-end noise

- Near end Crosstalk (NEXT)
- Far end Crosstalk (FEXT)
- Impulse noise (CO switching,...)

# NEXT/FEXT

- NEXT: Crosstalk between a receiving path and a transmitting path a DSL transceiver at the same end of two different subscriber loops
- FEXT: crosstalk between a receiving path and a transmitting path a DSL transceiver at opposite ends of two different subscriber loops
- can be modeled by Unger model

# Useful links

- Online book about DSL at [www.paradyne.com](http://www.paradyne.com)
- ADSL - forum [www.adsl.com](http://www.adsl.com)
- Html-bases documentation of the talk  
<http://snape.inw.tu-graz.ac.at/talks/dsl/>