Advanced Signal Processing
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Digital Subscriber Lines (xDSL):
Broadband Communication over Twisted Wire Pairs

CHANNEL EQUALIZATION

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2 Why equalization?

A transmitted signal could become unrecognizable at the receiver input due to linear amplitude and phase distortion in the channel that spreads the pulses and causes them to interfere with one another (ISI, intersymbol interference) [6, 1, 5].

For no ISI Nyquist condition I must be fulfilled:

$$f(kT_s) = \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{if } k \neq 0 \end{cases} \qquad f(t) = g(t) * c(t) * h(t)$$

or in the frequency domain

$$\sum_{n=-\infty}^{\infty} F(f + F_s n) = 1.$$

In general this condition will be violated. An equalizer must be employed to minimize the distortion. This equalizer can be derived under a zero-forcing (ZF) criterion, which means that the distortion is removed (equalizer is the inverse filter) leaving the noise unconstrained. In general this leads then to a noise enhancement at the receiver input. The alternative minimum mean-square error (MMSE) criterion minimizes the combined influence of distortion and noise and has therefore a much more practical importance.

Dividing the available bandwith into many smaller frequency regions where the channel distortion becomes quite neglectable also can minimize the effect of ISI.

If the linear distortion introduced by the system is not known a priori or is time-variant, an adaptive equalizer must be used, that adapts to the unknown or changing conditions. Well working adaptation algorithms can be found in the literature [8][4].

Fig.1 shows a simplified data communication systems with the impulse responses of the transmit filter ($g(t)$), the channel ($c(t)$), the receive filter ($h(t)$) and the equalizer ($q[k]$).

The model of the channel includes the noise-source $n(t)$, which is normally assumed to be a white gaussian signal. The dashed connection from the input via the delay to the output has no physical meaning, it should symbolize the known training sequence at the receiver-side needed for adaptation.

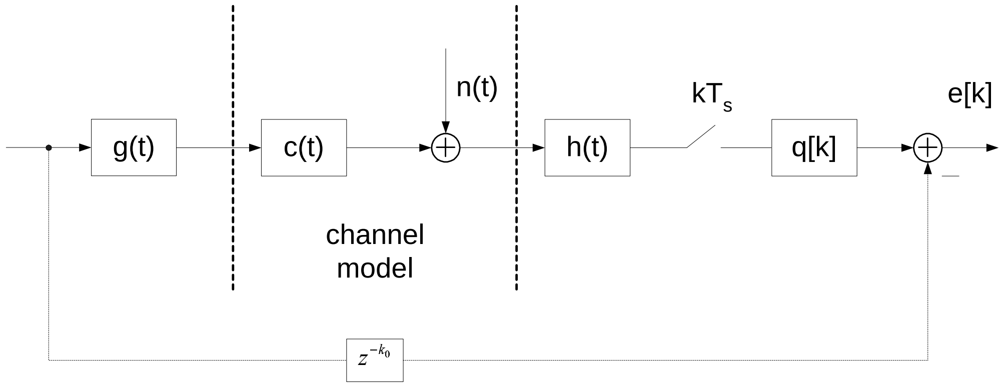

Figure 1: Data communication system

3 Linear equalizer

Using the zero-forcing criterion we have to compute an inverse filter for the equalizer:

$$F(j\omega) = G(j\omega) \cdot C(j\omega) \cdot H(j\omega)$$

$$Q(j\omega) = \frac{1}{F(j\omega)}$$

Implementing such a filter in the analog domain is rather difficult or impossible. Therefore we will use digital filters:

$$\{f[k]\} * \{q[k]\} = \{\delta[k - k_0]\}$$

with

$$f[k] = \begin{cases} f_k & 0 \leq k \leq m \\ 0 & \text{otherwise.} \end{cases} \qquad q[k] = \begin{cases} q_k & 0 \leq k \leq n \\ 0 & \text{otherwise} \end{cases}$$

The convolution gives a sequence of length $l = n + m + 1$. In matrix form for e.g. $m = 3$ and $n = 4$ we get:

$$
\underbrace{\begin{bmatrix} f_0 & & & & \\ f_1 & f_0 & & \mathbf{0} & \\ f_2 & f_1 & f_0 & & \\ f_3 & f_2 & f_1 & f_0 & \\ & f_3 & f_2 & f_1 & f_0 \\ & & f_3 & f_2 & f_1 \\ & \mathbf{0} & & f_3 & f_2 \\ & & & & f_3 \end{bmatrix}}_{\mathbf{F}} \underbrace{\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}}_{\mathbf{q}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{d}}
$$

The solution of this set of equations in a mean square error sense is

$$
\mathbf{q} = (\mathbf{F}^T\mathbf{F})^{-1}\mathbf{F}^T\mathbf{d}
$$

and gives the zero-forcing feedforward equalizer (ZF-FFE), also known as linear equalizer (ZF-LE).

4 MSE-feedforward equalizer

Using the mean-square error defined by

$$
MSE = \min E[e_k^2] = E[(d_{k-k_0} - \mathbf{X}_k^T\mathbf{q})^2]
$$

where $\mathbf{X}_k^T = \begin{bmatrix} x_k & x_{k-1} & \dots & x_{k-n} \end{bmatrix}$ is the received signal vector for an equalizer order of n, results in optimal equalizer coefficients q_{opt}:

$$
\mathbf{q}_{opt} = (E[\mathbf{X}_k\mathbf{X}_k^T])^{-1}E[d_{k-k_0}\mathbf{X}_k].
$$

With $\mathbf{R} = E[\mathbf{X}_k\mathbf{X}_k^T]$ (input correlation matrix) and $\mathbf{P} = E[d_{k-k_0}\mathbf{X}_k]$ (cross correlation between desired response and the input components) the above result can be simplified:

$$
\mathbf{q}_{opt} = \mathbf{R}^{-1}\mathbf{P}
$$

Summary

- ZF-FFE forces the ISI to zero at the slicer input
  only optimal in a noisefree environment
  in practice a noise-enhancement occurs

- MSE-FFE minimizes the variance of the slicer error

5 Adaptive equalizer

In a hardware implementation, equalizer coefficients are identified iteratively most often with the LMS algorithm due to its simplicity:

$$\mathbf{q}_{k+1} = \mathbf{q}_k + \mu \mathbf{X}_k (d_{k-k_0} - \mathbf{X}_k^T \mathbf{q}_k).$$

For stability reason the step-size parameter μ should satisfy the following condition:

$$\mu < \frac{2}{\mathrm{Tr}\{\mathbf{R}\}}.$$

For stationary inputs this condition is equivalent to

$$\mu < \frac{2}{(n+1)\sigma_x^2},$$

where σ_x^2 is the variance of the input signal and n is the order of the equalizer.

The convergence behaviour of an adaptive equalizer with different values for the step-size parameter μ is shown in Fig.2. The higher μ, the smaller the settling time, but the higher the misadjustment.

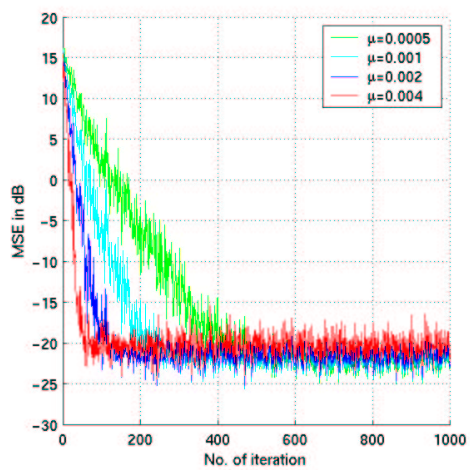

Figure 2: Convergence of an adaptive equalizer

6 Fractionally spaced equalizer

Baud rate equalizers are very sensitive to the sampling timing phase and suffer from the aliasing effect. The performance of a baud rate equalizer depending on the timing phase is shown in Fig.3.

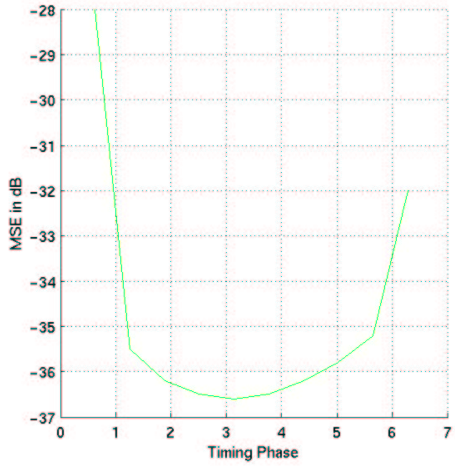

Figure 3: MSE for a 2 mile 26 AWG twisted pair loop

A fractionally spaced equalizer as depicted in Fig.4 could be used to avoid these problems. Most often the oversampling ratio (OSR) is 2. A severe disadvantage of this scheme is that the ADC works at OSR-times the baudrate (higher power dissipation).

Calculating the coefficients we can setup the same equations as for the baud rate equalizer:

$$\underbrace{\begin{bmatrix} f_0 & & & & \\ f_{1/2} & f_0 & & \mathbf{0} & \\ f_1 & f_{1/2} & f_1 & & \\ f_{3/2} & f_1 & f_{1/2} & f_1 & \\ f_2 & f_{3/2} & f_1 & f_{1/2} & f_1 \\ & f_2 & f_{3/2} & f_1 & f_{1/2} \\ & & f_2 & f_{3/2} & f_1 \\ & \mathbf{0} & & f_2 & f_{3/2} \\ & & & & f_2 \end{bmatrix}}_{\mathbf{F}} \underbrace{\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}}_{\mathbf{q}} = \underbrace{\begin{bmatrix} 0 \\ d_1 \\ 0 \\ d_2 \\ 1 \\ d_3 \\ 0 \\ d_4 \\ 0 \end{bmatrix}}_{\mathbf{d}}$$

Since the Nyqusit condition doesn't prescribe any value for d_i, the even equations can be skipped and we get a reduced set of equations:

$$\underbrace{\begin{bmatrix} f_0 & & & & \\ f_1 & f_{1/2} & f_1 & & \\ f_2 & f_{3/2} & f_1 & f_{1/2} & f_1 \\ & & f_2 & f_{3/2} & f_1 \\ & & & & f_2 \end{bmatrix}}_{\overline{\mathbf{F}}} \underbrace{\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}}_{\mathbf{q}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}}_{\overline{\mathbf{d}}}$$

Now it's possible to find an m in such a way, that the number of constraints is equal

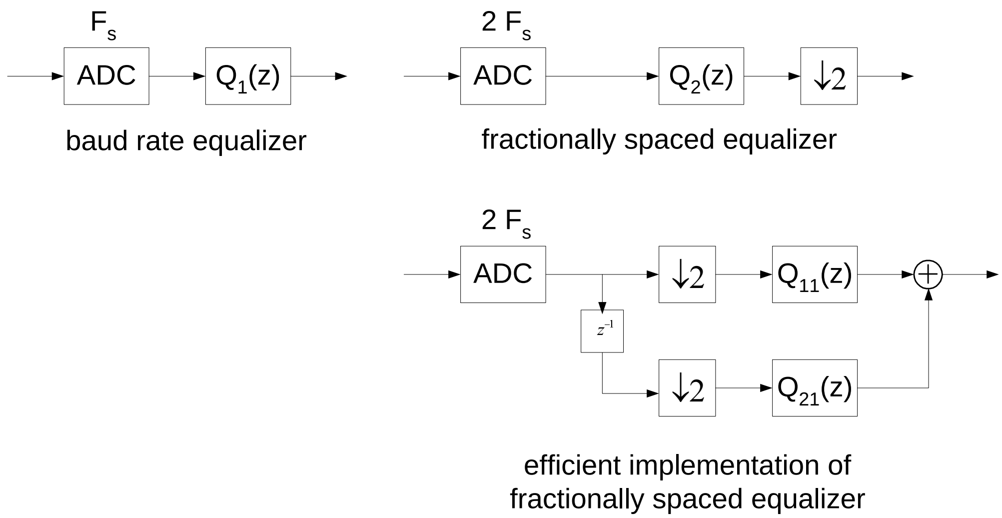

Figure 4: Baud rate equalizer, fractionally spaced equalizers

to the number of free equalizer coefficients:

$$\lfloor \frac{n+m+1}{2} \rfloor = n+1$$

Note, that in the case of the baud rate equalizer, the number of constraints is always higher than the number of unknowns and therefore no exact solution exists. If the matrix $\overline{\mathbf{F}}$ has a rank of $n+1$, the solution can easily be found via matrix inversion:

$$\mathbf{q} = (\overline{\mathbf{F}})^{-1}\overline{\mathbf{d}} \quad \text{(zero forcing solution).}$$

In a time-varing application an adaptive algorithm must be used to find the optimal coefficients. Again, the LMS algorithm is a good choise from a practical point of view. Note, that in a finite precision implementation the fractionally spaced equalizer can become unstable when coeffieecient values exceed the hardware limitation. A tap leakage algorithm with the following relationship

$$\mathbf{q}_{k+1} = (1-\alpha)\mathbf{q}_k + \mu \mathbf{X}_k (d_{k-k_0} - \mathbf{X}_k^T \mathbf{q}_k)$$

should be employed. The value α is usally smaller than < 0.1.

7 Decision feedback equalizer (DFE)

The noise enhancement problem using a feedforward equalizer can be avoided by employing a nonlinear equalizer named decision feedback equalizer (DFE). Fig.5 shows the principle structure. Solving the difference equation

$$x_k = f_0 d_k + f_1 d_{k-1} + f_2 d_{k-2} + f_3 d_{k-3} + \dots$$

for d_k, gives the following expression

$$d_k = \frac{1}{f_0} x_k - \frac{1}{f_0}(f_1 d_{k-1} + f_2 d_{k-2} + f_3 d_{k-3} + \ldots),$$

which is basically a recursive filter.

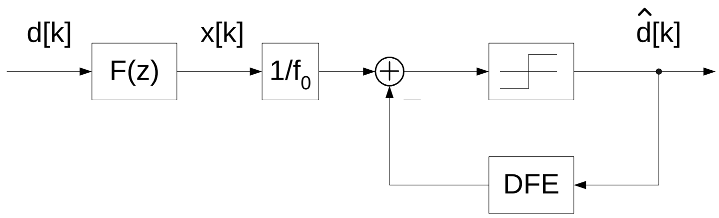

Figure 5: Decision feedback equalizer

For best performance we have to use both an FFE and DFE as can be seen in the next section. Fig.6 shows the basic schematic.

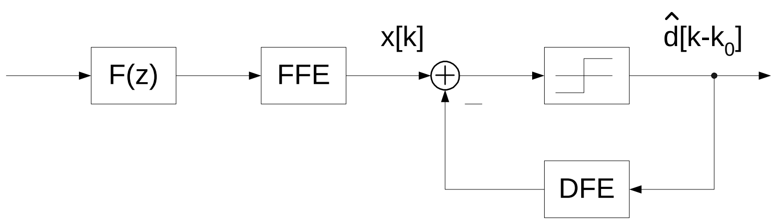

Figure 6: FFE-DFE

8 Equalizer performance comparison

Tab.1 shows the SNR at the equalizer output for different equalizers and different input noise levels. The highest SNR is achieved by using a fractionally spaced feedforward equalizer in combination with a decision feedback equalizer.

noiselevel	SNR at input of receiver	SNR at output of FFE	Frac.	DFE	Frac. + DFE
dBm/Hz	dB	dB	dB	dB	dB
-110	20.9	18.3	20.1	18.4	22.7
-120	30.9	27.9	30.0	28.3	32.7
-130	40.9	36.0	39.6	38.0	42.7

Table 1: Receiver input and equalizer output SNRs

9 Tomlinson precoding

The DFE was derived under the assumption that all decisions are correct. When the slicer makes a single wrong decisions this error is propageted due to the recursive structure of the DFE. The Tomlinson precoding method can be used to avoid the DFE error propagation problem. In this method the DFE (see Fig.7) is moved into the transmitter to filter the original data symbols. In addition, a modulo operation is necessary in the feedback loop to make the operation stable.

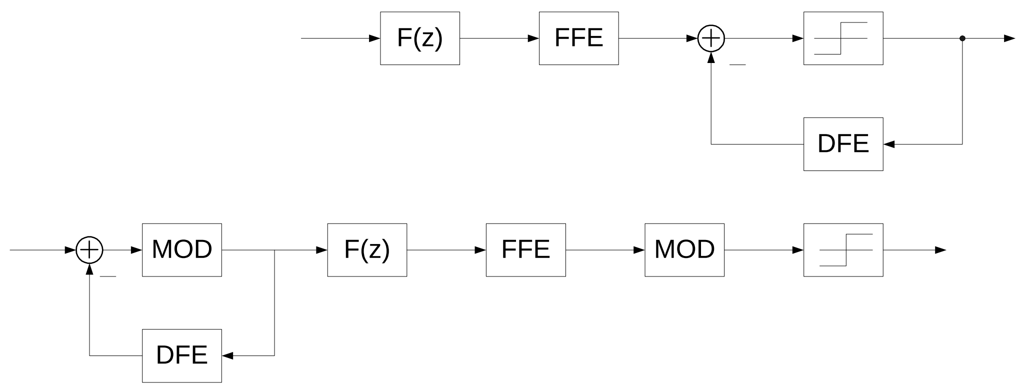

Figure 7: Comparison between a DFE system and a Tomlinson precoding system

10 Equalizers for discrete multitone systems (DMT)

A short introduction to DMT-based systems is given in [3, 2]. In a DMT system the transmission channel is divided into a number of parallel subchannels. An efficient partitioning technique uses FFT/IFFT to demodulate/modulate the data signal.

A direct consequence of finite FFT/IFFT size is interblock interference (IBI), the tail of the previous block multicarrier symbol will corrupt the beginning of the current block. To mitigate the effect of IBI, a technique known as cyclic prefix [7], or cyclic extension is applied to the modulated block multicarrier symbol. Since the length of the cyclic prefix must be equal to the length of the channel impulse response and, unfortunately, the channel impulse response can be rather long, we would waste a non neglectable part of the channel capacity. Therefore a time domain equalizer (TEQ) is used to reduce the length of cyclic prefix. Additionally, the received signal is passed through a frequency domain equalizer (FEQ) to flatten the overall channel freqency response.

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