Signal Processing and Speech Communication Laboratory—Prof. G. Kubin
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Exam *Adaptive Systems* on 2010/12/17

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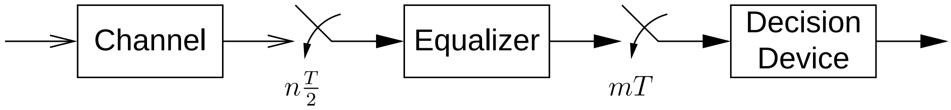
Exam duration: 3 hours
Maximum points: 100
Allowed material: mathematical formulary, calculator
You have to return this document after the exam. Good luck!

Problem 1 (34 Points)



Consider the $\frac{T}{2}$-*fractionally-spaced equalizer* illustrated above where the incoming signal is sampled at a rate twice the symbol rate. The decision device is synchronized with the even-indexed samples. The discrete-time FIR description of the communication channel for the high sampling rate is

$$H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3} = 1/2 + 1\, z^{-1} + 1/2\, z^{-2} + 1/4\, z^{-3}.$$

(Note, the unit delay z^{-1} corresponds to $\frac{T}{2}$ here.)

(a) Assume the transmitted symbols to be ± 1 and consider the receiver without the equalizer (replace it by a straight line). For the given channel, can the open-eye condition be met? Hint: which samples of the channel's impulse response influence the transmitted, T-spaced symbols?

(b) Calculate the 3 coefficients of the $\frac{T}{2}$-fractionally-spaced equalizer

$$C(z) = c_0 + c_1 z^{-1} + c_2 z^{-2}$$

such that the cascade of the given channel and the equalizer is (or approximates) $H(z)C(z) = 1$, i.e., enables a delay-free and ISI-free transmission.

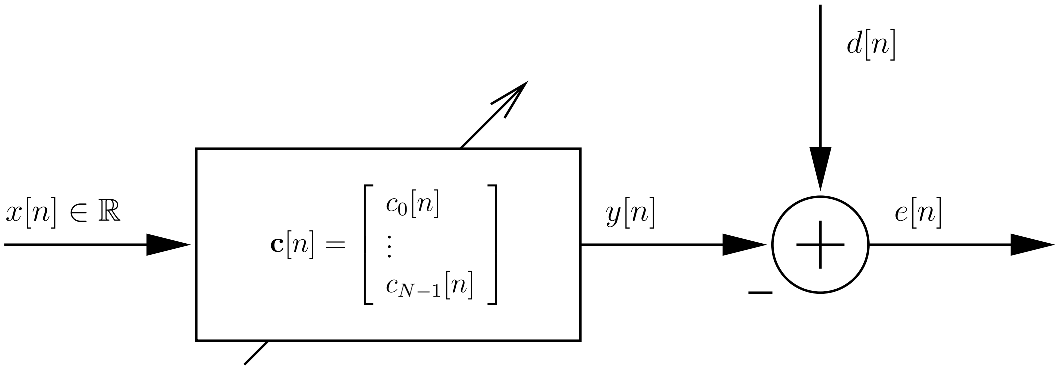
(c) Calculate the 3 coefficients of the $\frac{T}{2}$-fractionally-spaced equalizer such that the cascade is (or approximates) a delay of 1 symbol.

(d) Calculate the 3 coefficients of the $\frac{T}{2}$-fractionally-spaced equalizer such that the cascade is (or approximates) a delay of 2 symbols.

(e) Consider the channel to be noisy. Compute the noise gains of the three equalizers of the previous tasks. Which one of the three equalizers should be chosen?

Problem 2 (32 Points)

Consider the following linear filtering problem:



The auto-correlation sequence of $x[n]$ and the cross-correlations between $x[n-k]$ and $d[n]$ are assumed to be known (we can build the auto-correlation matrix $\mathbf{R_{xx}}$ and the cross-correlation vector $\mathbf{p}$). The following adaptation rule (*coefficient-leakage gradient search*) is used to adapt $\mathbf{c}[n]$

$$\mathbf{c}[n] = (1 - \mu\alpha)\mathbf{c}[n-1] + \mu(\mathbf{p} - \mathbf{R_{xx}}\mathbf{c}[n-1]),$$

where α is the leakage parameter ($0 < \alpha \ll 1$) and μ is the step-size parameter.

(a) Assume convergence. Where does this algorithm converge to?

(b) Transform the given adaptation rule in a way such that it adapts the misalignment vector (defined as $\mathbf{v}[n] = \mathbf{c}[n] - \mathbf{c}_\infty$). Hint: substitute for $\mathbf{p}$ using the result from **(a)**.

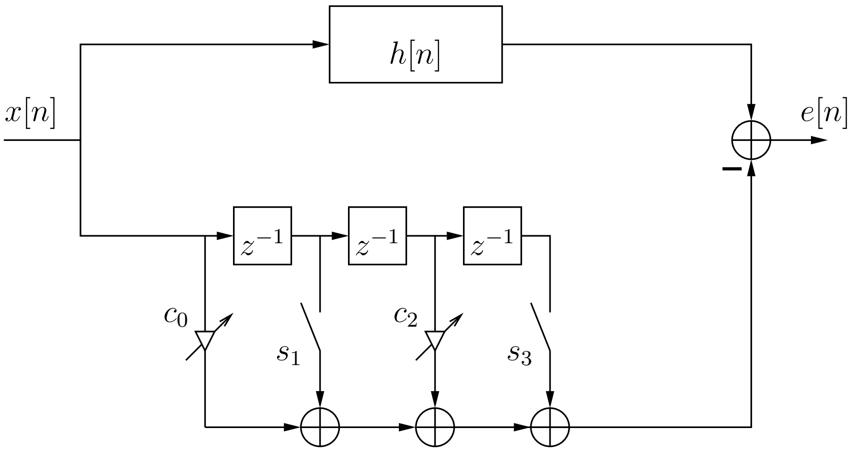
(c) Decouple the adaptation rule of **(b)** into a set of scalar adaptation expressions by using a unitary transform $\mathbf{q}[n] = \mathbf{Q}^H\mathbf{v}[n]$.

(d) Write the decoupled equation from **(c)** as individual exponential sequences and derive a condition on μ to ensure convergence towards $\mathbf{c}_\infty$, i.e., specify the range $\mu_{min} < \mu < \mu_{max}$ (assume α to be given).

(e) Compute the worst-case convergence time constant τ_{max}.

Problem 3 (34 Points)

Consider the system identification problem shown below. Note, the states of the switches are not optimized but given: we either use them both opened or both closed. The LTI system to be identified has the following impulse response vector $\mathbf{h} = [h_0, h_1, h_2, h_3]^\mathsf{T}$.



(a) Both switches are open. Determine the MinMSE solution for the two coefficients $\mathbf{c} = [c_0, c_2]^\mathsf{T}$ for a general auto-correlation sequence $r_{xx}[k]$.

(b) Both switches are closed now. Determine the MinMSE solution for the two coefficients for this scenario.

(c) Now, the input signal is a sinusoidal: $x[n] = \cos(\theta n + \varphi)$ with φ a uniformly distributed random phase. Derive the auto-correlation sequence $r_{xx}[k]$. Are there frequencies θ such that the MinMSE solution is $c_0 = h_0$ and $c_2 = h_2$ (regardless the states of the switches)? If yes, determine such a frequency.