Signal Processing and Speech Communication Laboratory—Prof. G. Kubin
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Exam *Adaptive Systems* on 2011/7/4

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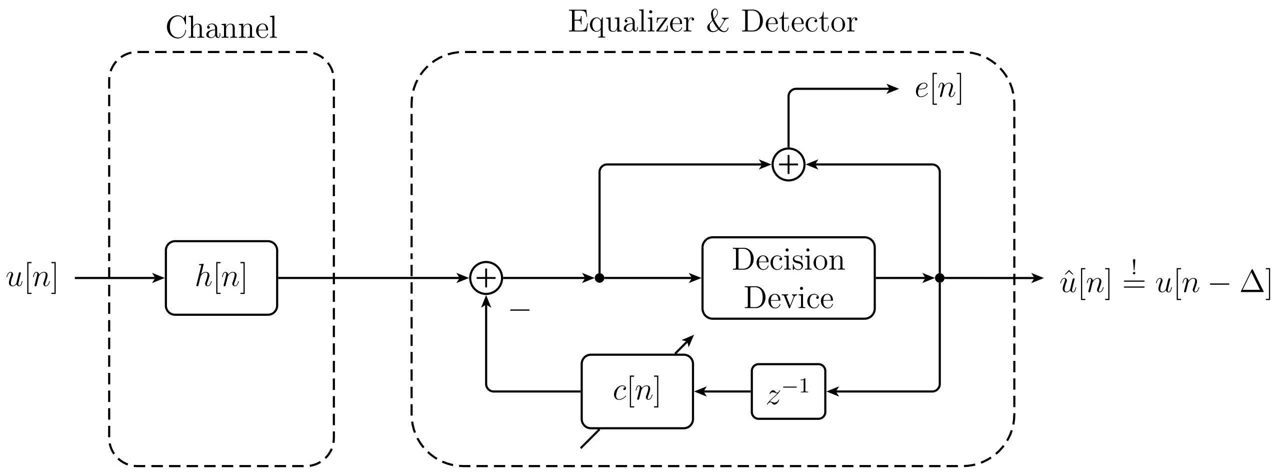
Exam duration: 3 hours
Maximum points: 100
Allowed material: mathematical formulary, calculator
You have to return this document after the exam. Best wishes for a successful exam!

Problem 1 (34 Points)

Consider the given data transmission scenario over a channel with impulse response $h[n]$. We use a decision-feedback equalizer (feedback only) in decision-directed operation mode.



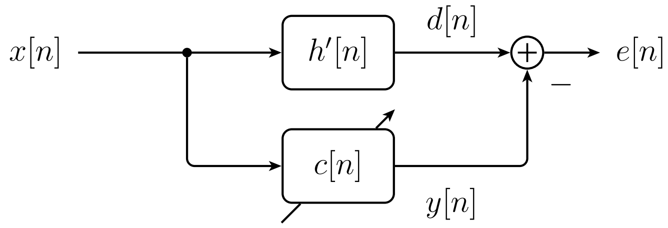
The symbols to be transmitted are $u[n] \in \{-1, 1\}$, which occur with the same probability. Consecutive symbols can be assumed to be uncorrelated. The decision device returns the sign of its input signal (zero is regarded as a positive number). The channel is given as

$$h[n] = 0.3\delta[n] + 1\delta[n-1] + 0.2\delta[n-2] - 0.8\delta[n-3].$$

(a) Consider a transmission over the given channel *without* an equalizer (decision device only). What is the ideal delay Δ that minimizes ISI (Inter Symbol Interference)? For this delay Δ, calculate the worst-case ISI and answer whether the channel's eye is open or not.

(b) Determine the sequence(s) for which the worst-case ISI occurs. Since all symbols occur with the same probability and since consecutive symbols are uncorrelated, all sequences are equally probable. An error occurs whenever the channel's eye is closed; calculate the error probability.

(c) Compare the decision-feedback equalizer to a simple system identification problem depicted below. Identify the signals $x[n]$, $y[n]$, and $d[n]$ in the schematic of the decision-feedback equalizer. If necessary, redraw the schematic to see the identities.



(d) Find the optimum solution $\mathbf{c}_{\text{opt}}$ for an N-coefficient ($N > 0$) decision-feedback equalizer in the sense of a minimum mean-squared error (MinMSE) for a general delay Δ and a general channel impulse response. **Hint:** Compute the cross-correlation vector $\mathbf{p}$ and the autocorrelation matrix $\mathbf{R_{XX}}$ and determine $\mathbf{c}_{\text{opt}}$ via the Wiener-Hopf equation.

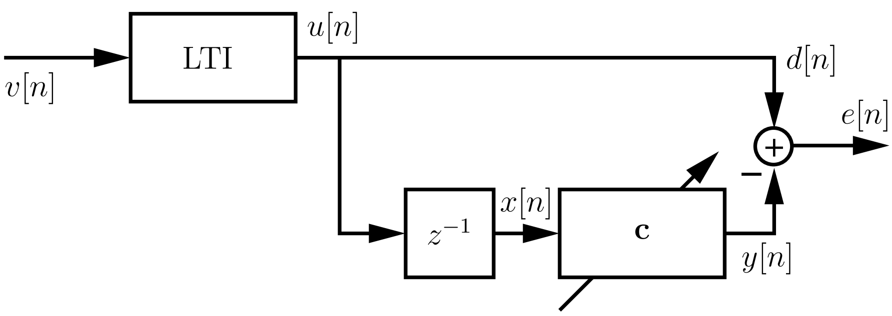
(e) What assumption do we implicitly make when we compare the decision-feedback equalizer to a system identification problem?

(f) What is the best delay Δ for a decision-feedback equalizer with 2 coefficients $\mathbf{c} = [c_0, c_1]^T$ to minimize ISI? For this delay, give the optimum equalizer coefficients. Calculate the worst-case ISI and answer whether the equalizer can open the channel's eye or not.

(g) Finally, assume that only a decision-feedback equalizer with one coefficient is available, i.e., $\mathbf{c} = c_0$. Can this equalizer open the channel's eye? If not, how could you modify the schematic of the decision-feedback equalizer such that a single-coefficient equalizer suffices to open the channel's eye?

Problem 2 (32 Points)

A prediction-error filter can be considered as a blind equalizer for AR processes.



The first 3 samples of the auto-correlation sequence of $u[n]$ are given as $r_{uu}[0] = 1.0$, $r_{uu}[1] = 0.\bar{5} = \frac{5}{9}$, and $r_{uu}[2] = 0.3\bar{7} = \frac{17}{45}$ where the bars denote repeating decimals.

(a) For a predictor with $N = 1$ coefficient, find the MSE-optimal coefficient of the adaptive filter.

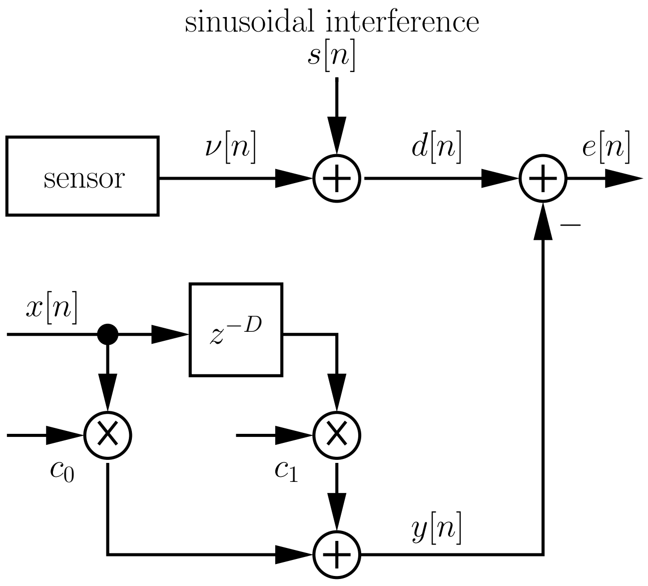
(b) For a predictor with $N = 2$ coefficients, find the MSE-optimal coefficient vector $\mathbf{c}$ of the adaptive filter.

(c) Assume, $u[n]$ is an AR process of 2nd order. Write down the process-generator difference equation and calculate the variance σ_v^2 of the white-noise input $v[n]$.

(d) Next, assume that the coefficient vector $\mathbf{c}$ ($N = 2$) is updated constantly, such that it is MSE-optimal. Would the coefficients change if $v[n]$ is not white? How would they change? What would happen to the coefficients if $v[n]$ is white, but the LTI system is an FIR filter?

Problem 3 (34 Points)

Consider the following adaptive *sinusoidal-interference canceler*:



The input signal is given by

$$x[n] = \cos\left(2\pi \frac{f_{ac}}{f_s} n + \varphi\right).$$

The frequency $f_{ac} = 50$ Hz, and the sampling frequency $f_s = 2400$ Hz. The phase offset φ is considered as a random variable uniformly distributed over $(-\pi, \pi]$. The delay in the adaptive transversal filter is $D \in \mathbb{N}$. The sensor signal $\nu[n]$ is unknown, but it is uncorrelated with $x[n]$. The sinusoidal interference is

$$s[n] = \frac{1}{2}\cos\left(2\pi \frac{f_{ac}}{f_s} n + \varphi + \frac{\pi}{4}\right).$$

(a) Determine the auto-correlation sequence $r_{xx}[k]$ of the above given $x[n]$ as well as the auto-correlation matrix $\mathbf{R_{xx}}$ of the tap-input vector $\mathbf{x}[n] = \big[x[n],\, x[n-D]\big]^\mathsf{T}$. (Do not just give the final result for $r_{xx}[k]$, but carefully show how to find the result by means of integration.)

(b) Determine the eigenvalues λ_i of $\mathbf{R_{xx}}$, and give the condition number $\kappa = \frac{\lambda_{max}}{\lambda_{min}}$. Find the (smallest) delay $D \in \mathbb{N}$ that minimizes κ. Explain, what theoretical and practical advantages such a selection of D can bring.

(c) Determine the cross-correlation vector $\mathbf{p}$ between the signal $d[n]$ and the tap-input vector $\mathbf{x}[n] = \big[x[n],\, x[n-D]\big]^\mathsf{T}$.

(d) Determine the Wiener-Hopf solution for the two coefficients c_0 and c_1. (Take the D found before or use a general D if you have not found one before.)

(e) Finally, set $D = 1$. Compute the auto-correlation matrix $\mathbf{R_{xx}}$, its inverse, and the two coefficient c_0 and c_1. What difference do you observe?