Signal Processing and Speech Communication Laboratory—Prof. G. Kubin
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Exam *Adaptive Systems* on 2011/9/26

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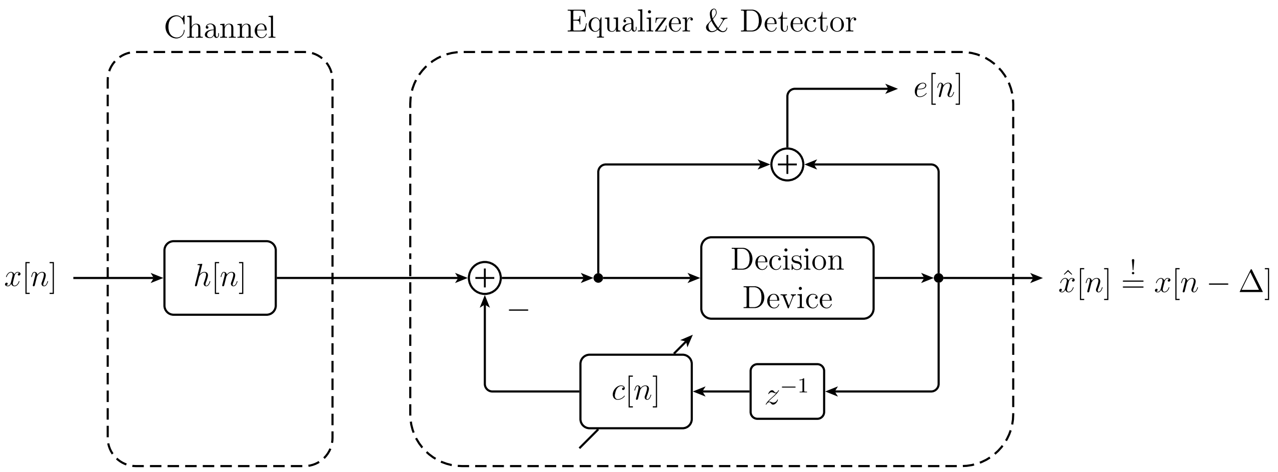
Exam duration: 3 hours
Maximum points: 100
Allowed material: mathematical formulary, calculator
You have to return this document after the exam. Best wishes for a successful exam!

Problem 1 (34 Points)

Consider the given data transmission scenario over a channel with impulse response $h[n]$. We use a decision-feedback equalizer (feedback only) in decision-directed operation mode.



The symbols to be transmitted are $x[n] \in \{-1, 1\}$, which occur with the same probability. Consecutive symbols can be assumed to be uncorrelated. The decision device returns the sign of its input signal (zero is regarded as a positive number). The channel is given as

$$h[n] = \delta[n] + 0.9\delta[n-1] + 0.8\delta[n-2] - 0.4\delta[n-3].$$

(a) Consider a transmission over the given channel *without* an equalizer (decision device only). What is the ideal delay Δ that minimizes ISI (Inter Symbol Interference)? For this delay Δ, calculate the worst-case ISI and answer whether the channel's eye is open or not.

(b) Determine the sequence(s) for which the worst-case ISI occurs. Since all symbols occur with the same probability and since consecutive symbols are uncorrelated, all sequences are equally probable. An error occurs whenever the channel's eye is closed; calculate the symbol error probability.

(c) Derive the mean-squared error (MSE) $E\{e^2[n]\}$ as a function of the equalizer coefficient vector $\mathbf{c}$. Use a general delay Δ, a general impulse respone, and assume that the coefficient vector has length N.

(d) Find the optimum solution $\mathbf{c}_{\text{opt}}$ by minimizing the MSE from the previous task. What is the minimum MSE for the optimum solution? **Hint:** Insert $\mathbf{c}_{\text{opt}}$ in the expression you obtained for $E\{e^2[n]\}$.

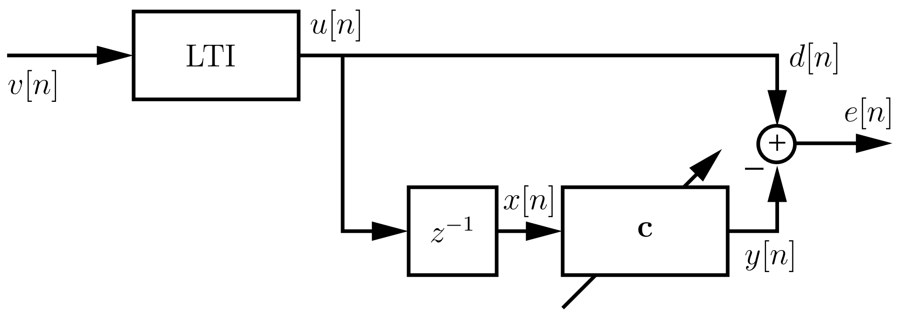
(e) What is the best delay Δ for a decision-feedback equalizer with 2 coefficients $\mathbf{c} = [c_0, c_1]^T$ to minimize ISI caused by the given impulse response $h[n]$? For this delay, give the optimum equalizer coefficients. Calculate the worst-case ISI and answer whether the equalizer can open the channel's eye or not.

(f) Sketch the impulse response of the subsystem from $x[n]$ to the input of the decision device *after* equalization. Compare it to the original impulse response. What has changed?

(g) Finally, assume that only a decision-feedback equalizer with one coefficient is available, i.e., $\mathbf{c} = c_0$. Can this equalizer open the channel's eye?

Problem 2 (32 Points)

A prediction-error filter can be considered as a blind equalizer for AR processes.



The first 3 samples of the auto-correlation sequence of $u[n]$ are given as $r_{uu}[0] = 1.0$, $r_{uu}[1] = 0.\bar{5} = \frac{5}{9}$, and $r_{uu}[2] = 0.3\bar{7} = \frac{17}{45}$ where the bars denote repeating decimals.

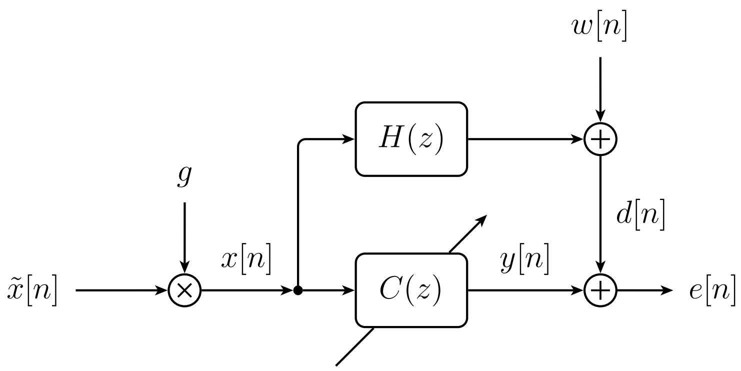
(a) For a predictor with $N = 1$ coefficient, find the MSE-optimal coefficient of the adaptive filter.

(b) For a predictor with $N = 2$ coefficients, find the MSE-optimal coefficient vector $\mathbf{c}$ of the adaptive filter.

(c) Assume, $u[n]$ is an AR process of 2nd order. Write down the process-generator difference equation and calculate the variance σ_v^2 of the white-noise input $v[n]$.

(d) Next, assume that the coefficient vector $\mathbf{c}$ ($N = 2$) is updated constantly, such that it is MSE-optimal. Would the coefficients change if $v[n]$ is not white? How would they change? What would happen to the coefficients if $v[n]$ is white, but the LTI system is an FIR filter?

Problem 3 (34 Points)



The *projection algorithm*

$$\mathbf{c}[n] = \mathbf{c}[n-1] + e[n]\frac{\mathbf{x}[n]}{||\mathbf{x}[n]||^2}$$

should be used to identify the system $H(z)$ with the impulse response $h[n] = h_0\delta[n]$. The input signal $\tilde{x}[n]$ is a zero-mean, white noise signal with unit variance. The adaptive filter $C(z)$ is of zeroth order, i.e., $\mathbf{c}[n] = c_0[n]\delta[n]$. The additive noise signal $w[n]$ is also zero mean and white with variance σ_w^2. It can be assumed that $\tilde{x}[n]$ and $w[n]$ are uncorrelated.

(a) Using the schematic and the formula of the projection algorithm, find an expression for the coefficient $c_0[n]$.

(b) Determine the Wiener-Hopf solution $\mathbf{c}_{\text{opt}}$ and use it in the previous result to determine the misalignment vector $\mathbf{v}[n] = \mathbf{c}[n] - \mathbf{c}_{\text{opt}}$.

(c) Compute the expected value and the variance of $\mathbf{v}[n]$. What would be a requirement for perfect identification?

(d) Describe the influence of the gain factor g regarding the misalignment vector. Are large or small values of g preferable, and why?