Signal Processing and Speech Communication Laboratory—Prof. G. Kubin
Graz University of Technology

Exam *Adaptive Systems* on 2006/7/3

Name MatrNr. StudKennz.

Exam duration: 120 minutes
Maximum points: 100
Allowed material: mathematical formulary, calculator
You have to return this document after the exam! Good luck!

Problem 1 (33 Points)

Consider the following identification problem:

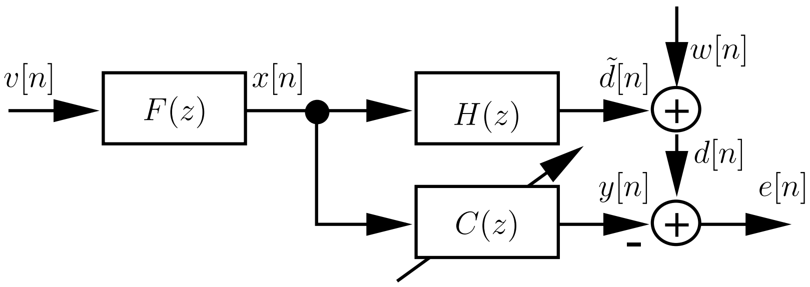


Figure 1: Adaptive system with correlated and uncorrelated white noise.

The white Gaussian noise process $v[n]$ with zero mean and variance $\sigma_v^2 = 2$ is filtered by the FIR filter $F(z)$, which has an impulse response of $f[n] = -0.2\delta[n] + 1.1\delta[n-1]$ (moving average (MA) process). The resulting output $x[n]$ is used to identify the system $H(z)$, which has an impulse response of $h[n] = 0.5\delta[n-1]$, by the adaptive filter $C(z)$ with 3 coefficients, i.e., $\mathbf{c} = [c_0, c_1, c_2]^T$. The desired output $d[n]$ is corrupted by the additive white Gaussian noise $w[n]$ with zero mean and variance $\sigma_w^2 = 0.01$.

(a) Determine the values of the auto-correlation matrix $\mathbf{R} = E\left\{\mathbf{x}[n]\mathbf{x}[n]^T\right\}$.

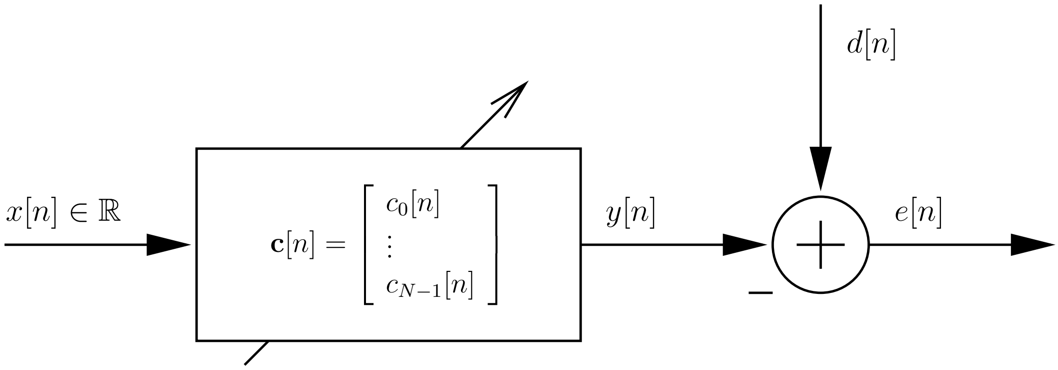
(b) Determine the values of the cross-correlation vector $\mathbf{p} = E\left\{d[n]\mathbf{x}[n]\right\}$.

(c) Determine the optimal coefficient vector $\mathbf{c}_{opt}$ for the filter $C(z)$ in the sense of the Wiener solution.

(d) Determine the minimum mean-squared error (MMSE) J_{min} of the system.

Problem 2 (33 Points)

Consider the following linear filtering problem:



The auto-correlation sequence of $x[n]$ and the cross-correlation between $x[n]$ and $d[n]$ are assumed to be known (we can build the auto-correlation matrix $\mathbf{R}$ and the cross-correlation vector $\mathbf{p}$). The following adaptation equation (*coefficient-leakage gradient search*) is used to adapt $\mathbf{c}[n]$

$$\mathbf{c}[n] = (1 - \mu\alpha)\mathbf{c}[n-1] + \mu(\mathbf{p} - \mathbf{R}\mathbf{c}[n-1]),$$

where α is the leakage parameter ($0 < \alpha \ll 1$) and μ is the step-size parameter.

(a) Assume convergence, i.e., $\mathbf{c}[n] = \mathbf{c}[n-1] = \mathbf{c}_\infty$. Where does this algorithm converge to.

(b) Transform the given adaptation equation in a way that it shows the recursive adaptation of the misalignment vector, which is defined here as $\mathbf{v}[n] = \mathbf{c}[n] - \mathbf{c}_\infty$. Hint: substitute for $\mathbf{p}$ using the result from **(a)**.

(c) Decouple the modes of the in **(b)** transformed adaptation equation by using a unitary transform $\mathbf{q}[n] = \mathbf{Q}^H\mathbf{v}[n]$, where $\mathbf{R} = \mathbf{Q}\Lambda\mathbf{Q}^H$.

(d) Write the decoupled equation from **(c)** as individual exponential sequences and derive a condition on μ to ensure convergence towards $\mathbf{c}_\infty$, i.e., specify the range $\mu_{min} < \mu < \mu_{max}$.

Problem 3 (34 Points)

Consider the given channel equalization task:

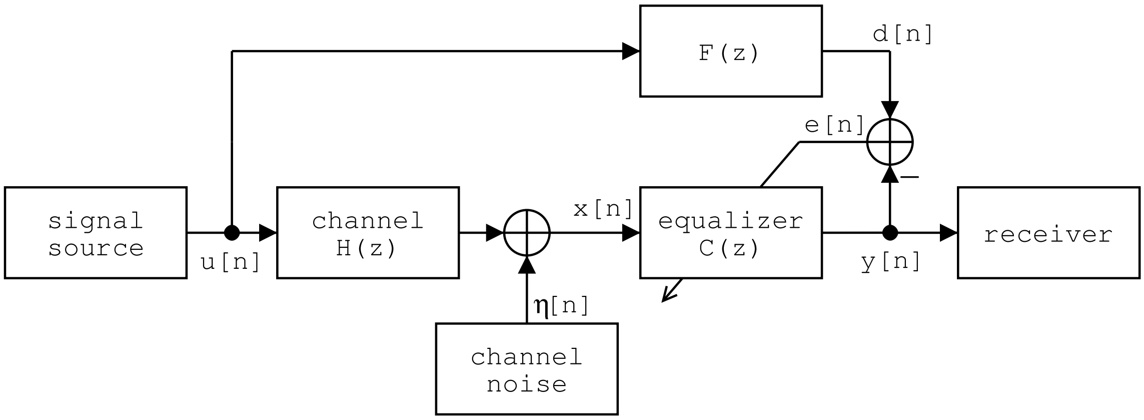


Figure 2: Channel equalization

The transmitted symbols $u[n] \in \{-1, 1\}$ are assumed to be a white noise process with zero mean and variance $\sigma_u^2 = 1$. The additive white noise is *uniformly* distributed with zero mean and variance $\sigma_\eta^2 = \frac{0.15^2}{3}$. The receiver consists of a simple decision device, which returns 1 for $y[n] \geq 0$ and -1 otherwise. The impulse response of the channel is given by $h[n] = 0.3\delta[n] - 0.3\delta[n-1] + 0.95\delta[n-2] + 0.5\delta[n-3]$. The equalizer consists of two coefficients, i.e., the impulse response is $c[n] = c_0\delta[n] + c_1\delta[n-1]$.

(a) Consider a transmission over the given channel *without* equalizer and channel noise (receiver consists only of a decision device). What is the ideal delay $F(z) = z^{-\Delta}$ to minimize ISI (Inter Symbol Interference), i.e., regarding the impulse response of the channel, after which delay Δ should the decision device make a decision? Calculate the worst-case ISI for this ideal delay Δ, i.e., the worst input symbol combination. State whether the decision device can *always* decide correctly or not.

(b) Write down the design equation for the channel equalizer $C(z)$ in the minimum mean-squared error sense (MMSE), which considers the channel impulse response $h[n]$ as well as the variance of input symbols $u[n]$ and the noise $\eta[n]$, and determine the entries of the matrices $\mathbf{H}$, $\mathbf{H}^T\mathbf{H}$, and $\left(\mathbf{H}^T\mathbf{H} + \mathbf{I}\frac{1}{SNR}\right)$.

(c) The optimal coefficient vector regarding the desired response $F_1(z) = z^{-2}$ is $\mathbf{c}_{1,opt} = [0.7297, -0.2783]^T$ and the optimal coefficient vector regarding the desired response $F_2(z) = z^{-3}$ is $\mathbf{c}_{2,opt} = [0.322, 0.6849]^T$. Determine the overall impulse responses $\hat{f}_1[n]$ and $\hat{f}_2[n]$ of the channel impulse response and the corresponding channel equalizers $\mathbf{c}_{1,opt}$ and $\mathbf{c}_{2,opt}$, respectively.

(d) Determine the inter symbol interference (ISI) for both channel equalizers (without noise). Does the decision device in the receiver always decide correctly? Regarding the noise gain, which solution would you choose? Justify your answers.