Signal Processing and Speech Communication Laboratory—Prof. G. Kubin
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Exam *Adaptive Systems* on 2006/11/25

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Exam duration: 120 minutes
Maximum points: 100
Allowed material: mathematical formulary, calculator
You have to return this document after the exam! Good luck!

Problem 1 (33 Points)

Consider the following identification problem:

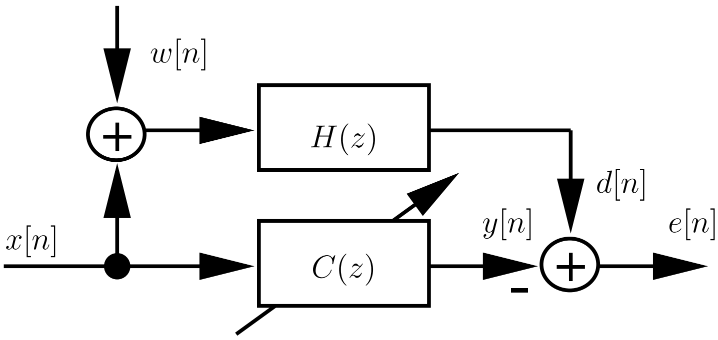


Figure 1: Adaptive system

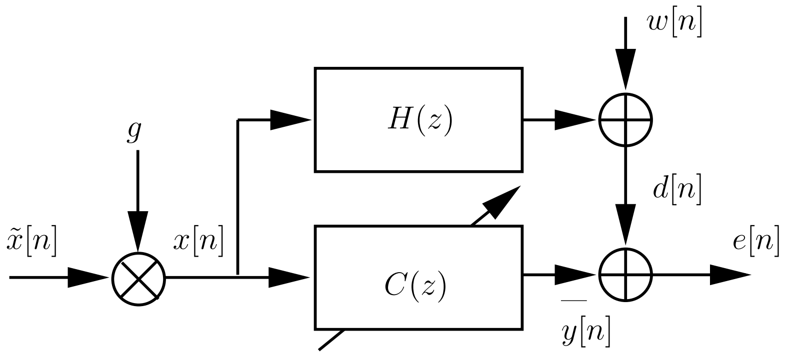
Assume that the process $w[n]$ and the process $x[n]$ are statistically independent.

(a) Derive the Wiener solution $\mathbf{c}_{opt}$ for the adaptive filter $C(z)$.

(b) Derive the minimum mean-square error (MMSE) J_{min} of the system.

(c) Assume that $w[n]$ is a white-noise process with zero mean and variance σ_w^2=0.01, $H(z)$ is a FIR filter with $H(z) = 1 + 0.5z^{-1} + 0.5z^{-2}$, and the adaptive filter $C(z)$ is of order 2, i.e., consists of 3 coefficients. Determine J_{min} for this configuration.

Problem 2 (33 Points)



The projection algorithm should be used to identify the system $H(z)$ with the impulse response $h[n] = h_0\delta[n]$. The input signal $x[n]$ is the white-noise signal $\tilde{x}[n]$ with zero mean and variance $\sigma_x^2 = 1$ multiplied by the gain factor g. The adaptive transversal filter $C(z)$ is of order 0, i.e., $c[n] = c_0\delta[n]$. The desired signal $d[n]$ consists of the filter output and an additive white-noise signal $w[n]$ with zero mean and variance σ_w^2. Please notice that $x[n]$ is the input signal for the system identification.

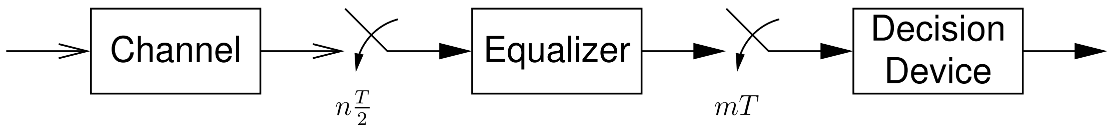
(a) Write down the *projection algorithm* for the given system.

(b) Express the coefficient $c_0[n]$ of the projection algorithm in relation to the impulse response $h[n]$ and the noise $w[n]$ (Hint: Express the error $e[n]$ by the input signal $x[n]$, the filter coefficients of the impulse responses $h[n]$ and $c[n]$, and the noise $w[n]$).

(c) Determine the misalignment vector $\mathbf{v}[n] = \mathbf{c}[n] - \mathbf{c}_{opt}$ of the projection algorithm for the given system, where $\mathbf{c}_{opt}$ is the vector of the Wiener solution. Use the result from (b).

(d) Describe the influence of the gain g (larger, equal, and smaller 1) regarding the misalignment vector $v[n]$.

Problem 3 (34 Points)



Consider the $\frac{T}{2}$-fractionally-spaced equalizer illustrated above where the incoming signal is sampled at a rate twice the symbol rate. The decision device is synchronized with the *even-indexed* samples.

The discrete-time FIR description of the communication channel is

$$H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} = 1 + 2\,z^{-1} + 3/4\,z^{-2}$$

where the unit delay z^{-1} corresponds to $\frac{T}{2}$.

(a) Design a minimum MSE equalizer of order 1 (2 coefficients)

$$C(z) = c_0 + c_1 z^{-1}$$

so that the cascade of channel and equalizer yields $H(z)C(z) = 1$ (delay-free perfect equalization).

(b) Repeat the last task to design a minimum MSE equalizer that yields $H(z)C(z) = z^{-2}$ (perfect equalization with delay).

(c) Calculate the noise gain of both solutions. Which solution would you prefer and why?