Signal Processing and Speech Communication Laboratory—Prof. G. Kubin
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Exam *Adaptive Systems* on 2007/3/24

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Exam duration: 120 minutes
Maximum points: 100
Allowed material: mathematical formulary, calculator
You have to return this document after the exam! Good luck!

Problem 1 (33 Points)

Consider the following identification problem: The white Gaussian noise process $v[n]$ with

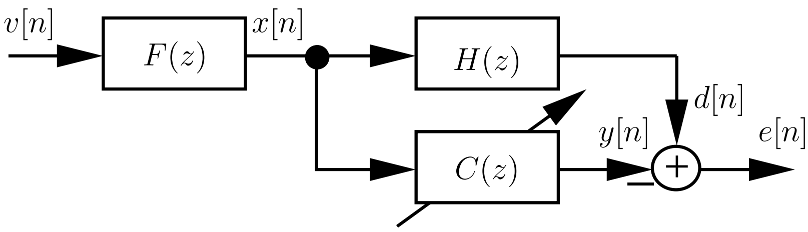


Figure 1: Adaptive system with correlated and uncorrelated white noise.

zero mean and variance $\sigma_v^2 = 1$ is filtered by the FIR filter $F(z)$, which has an impulse response of $f[n] = -0.1\delta[n] + 1\delta[n-1]$ (moving average (MA) process). The resulting output $x[n]$ is used to identify the system $H(z)$, which has an impulse response of $h[n] = 0.9\delta[n] + 0.5\delta[n-1]$, through the adaptive filter $C(z)$.

(a) Determine the values of the autocorrelation $r[k]_{vv} = E\{x[n]x[n-k]\}$ for $k = 0, 1, 2$.

(b) Assume that the adaptive filter $C(z)$ has 2 coefficients, i.e., $\mathbf{c} = [c_0, c_1]^T$. Determine the Wiener solution $\mathbf{c}_{opt}$.

(c) Assume that the adaptive filter $C(z)$ has 3 coefficients, i.e., $\mathbf{c} = [c_0, c_1, c_2]^T$. Again, determine the Wiener solution $\mathbf{c}_{opt}$.

(d) Assume that the adaptive filter $C(z)$ has 1 coefficient, i.e., $\mathbf{c} = [c_0]^T$. Again, determine the Wiener solution $\mathbf{c}_{opt}$.

Problem 2 (33 Points)

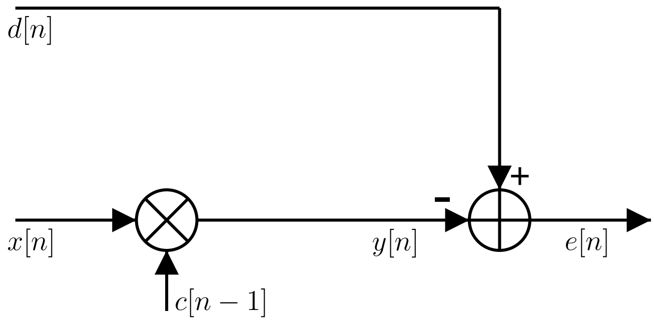
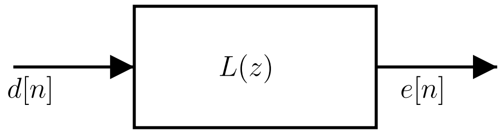


Figure 2: Adaptive gain factor.

Consider the adaptive system shown in Figure 2.

(a) Write down the LMS algorithm to adapt $c[n]$ and find a transfer function relationship $L(z) = \frac{E(z)}{D(z)}$ for the system with input $d[n]$ and output $e[n]$ for $x[n] = -3$ (Hint: What is the relation between $y[n]$ and $c[n-1]$? Furthermore, the relation $y[n] = d[n] - e[n]$ might be useful).

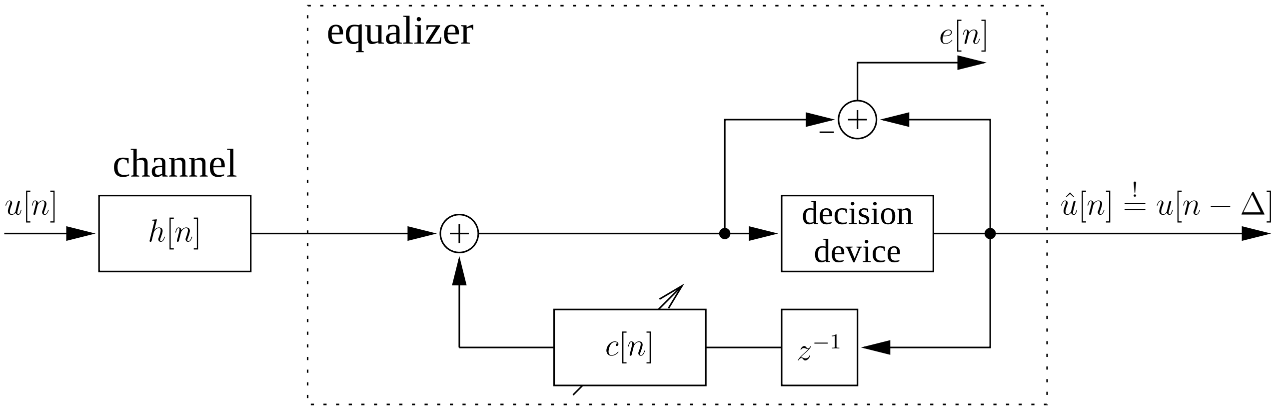
(b) Calculate the poles and zeros of $L(z)$ and plot them in the z-plane.



(c) Determine the upper and lower bound on μ for the stability of $L(z)$.

Problem 3 (34 Points)

Consider the given data transmission scenario with a channel with impulse response $h[n]$ and a decision-feedback equalizer with two coefficients $c[n] = c_0\delta[n] + c_1\delta[n-1]$.



The symbols to be transmitted are $u[n] \in \{-1, 1\}$, which occur with the same probability. Consecutive symbols can be assumed to be independent. The decision device returns the sign of its input signal (zero is regarded as a positive number). The channel is known as

$$h[n] = -0.4\delta[n] + 0.6\delta[n-1] + \delta[n-2] - 0.4\delta[n-3].$$

(a) Consider a transmission over the given channel *without* an equalizer (decision device only). What is the ideal delay Δ to minimize ISI (Inter Symbol Interference), i.e., regarding the impulse response of the channel, after which delay Δ should the decision device make a decision? For this ideal delay Δ, calculate the worst-case ISI, i.e., the worst input signal combination, and answer whether the decision device can always decide correctly or not.

(b) Find the optimum solution for an N-coefficient ($N > 0$) decision-feedback equalizer in the sense of a minimum mean-squared error (MMSE) for a general delay Δ. (Hint: Consider that the input signal $u[n]$ is a white-noise signal which gives a simple autocorrelation matrix $\mathbf{R}_{uu}$ with very special properties.)

(c) What is the ideal delay Δ for the 2-coefficient $\mathbf{c} = [c_0, c_1]^T$ decision-feedback equalizer to minimize ISI? Calculate again the worst-case ISI and answer whether the equalizer can open the channel's eye (can determine the correct symbol in the worst case).