Signal Processing and Speech Communication Laboratory—Prof. G. Kubin
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Exam *Adaptive Systems* on 2008/5/16

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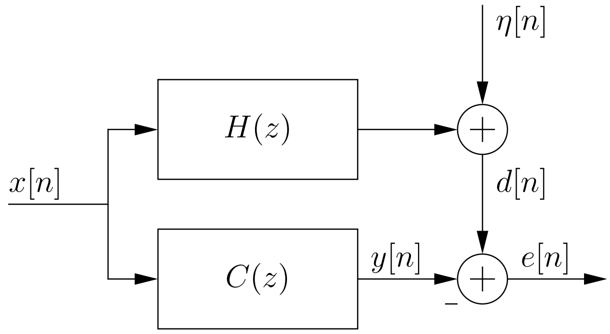
Exam duration: 120 minutes
Maximum points: 100
Allowed material: mathematical formulary, calculator
You have to return this document after the exam. Good luck!

Problem 1 (34 Points)



The signal $x[n]$ with zero mean and variance $\sigma_x^2 = 10$ is used to identify the system $H(z)$ with the impulse response $h[n] = 0.5\delta[n] - 0.3\delta[n-1] + 0.1\delta[n-2]$. The adaptive transversal filter $C(z)$ has only 2 coefficients $\mathbf{c} = [c_0, c_1]^T$. The additive noise $\eta[n]$ has zero mean and variance $\sigma_\eta^2 = 0.2$ and is **not fully uncorrelated** with $x[n]$:

$$E\{\eta[n]x[n-m]\} = \begin{cases} 1, & m = 1 \\ 0, & m \neq 1 \end{cases} .$$

(a) For a general auto-correlation sequence $r_{xx}[k]$, derive the optimal coefficients $\mathbf{c}_{opt}$ of the adaptive filter in the sense of a minimum mean-squared error: $\mathbf{c}_{opt} = \operatorname{argmin}_{\mathbf{c}} E\{|e[n]|^2\}$.

(b) Assume $x[n]$ to be **white** noise with zero mean and variance $\sigma_x^2 = 10$. Calculate the Wiener solution $\mathbf{c}_{opt}$, and calculate the minimum mean-squared error (MinMSE), i.e., $E\{|e[n]|^2\}$ when the adaptive filter $C(z)$ operates in its optimum.

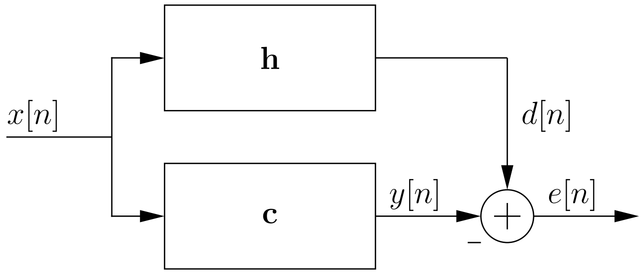
(c) Now, assume that the input signal $x[n]$ is **not white** and has the auto-correlation sequence

$$r_{xx}[k] = \begin{cases} 10, & k \text{ even} \\ 0, & k \text{ odd} \end{cases} .$$

Again, determine the Wiener solution $\mathbf{c}_{opt}$ and the minimum mean-squared error (MinMSE).

Problem 2 (33 Points)

Consider a *system identification problem* where the orders of both the adaptive and the unknown transversalfilter are 1 (2 coefficients). The output of the unknown system $d[n]$ can be assumed to be accessible without an additive noise source.



The statistics of the input signal are known as

$$\mathbf{R}_x = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix}.$$

The *Gradient Method*

$$\mathbf{c}[n] = \mathbf{c}[n-1] + \mu \left(\mathbf{p} - \mathbf{R}_x \mathbf{c}[n-1]\right)$$

with $\mu = 1/2$ is used to adapt the coefficients of the adaptive filter. The coefficients of the unknown system are $\mathbf{h} = [2,\ 1]^T$.

(a) Simplify the adaptation algorithm by substitution for $\mathbf{p} = E\{\mathbf{x}[n]d[n]\}$ according to the given system identification problem. Additionally, introduce the misalignment vector $\mathbf{v}[n]$ and rewrite the adaptation algorithm such that $\mathbf{v}[n]$ is adapted.

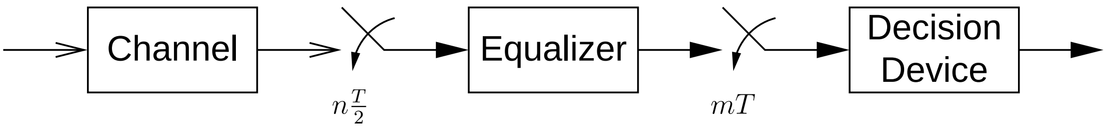
(b) The coefficients of the adaptive filter are initialized with $\mathbf{c}[0] = [0,\ -1]^\mathsf{T}$. Find an expression for $\mathbf{v}[n]$ either analytically or by calculation of some (three should be enough) iteration steps. Do the components of $\mathbf{v}[n]$ show an exponential decay? If yes, determine the corresponding time constant.

(c) Repeat the previous task when the coefficients are initialized with $\mathbf{c}[0] = [0,\ 3]^\mathsf{T}$. Do the components of $\mathbf{v}[n]$ show an exponential decay? If yes, determine the corresponding time constant.

(d) Repeat the previous task when the coefficients are initialized with $\mathbf{c}[0] = [0,\ 0]^\mathsf{T}$. Do the components of $\mathbf{v}[n]$ show an exponential decay? If yes, determine the corresponding time constant.

(e) Explain, when an exponential decay of the components of $\mathbf{v}[n]$ can be observed although the input signal $x[n]$ is not white.

Problem 3 (33 Points)



Consider the $\frac{T}{2}$-fractionally-spaced equalizer illustrated above where the received signal is sampled at a rate twice the symbol rate. The decision device is synchronized with the *even-indexed* samples (at which actual symbols are sended).

The discrete-time FIR description of the communication channel is

$$H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3} = 1 + 3\,z^{-1} + 1\,z^{-2} + 1/3\,z^{-3}$$

where the unit delay z^{-1} corresponds to $\frac{T}{2}$.

(a) Design a least-squares equalizer with 2 coefficients

$$C(z) = c_0 + c_1 z^{-1}$$

such that the cascade of channel and equalizer approximates $H(z)C(z) \approx 1$ (delay-free, perfect equalization).

(b) Repeat the last task to design an equalizer with 3 coefficients

$$C(z) = c_0 + c_1 z^{-1} + c_2 z^{-2}.$$

(c) Consider the symbols to be transmitted are $u[m] \in \{0, 1\}$. For the two solutions obtained above, determine whether the cascade of channel and equalizer enables an ISI-free transmission or not. In case there is ISI, determine whether the cascades eye is open or closed. (Hint: at what rate does the decision device operate?)