Signal Processing and Speech Communication Laboratory—Prof. G. Kubin
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Exam *Adaptive Systems* on 2009/5/29

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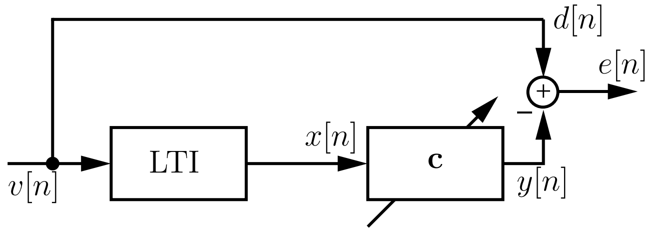
Exam duration: 180 minutes
Maximum points: 100
Allowed material: mathematical formulary, calculator
You have to return this document after the exam. Good luck!

Problem 1 (35 Points)

Consider the following inverse-modeling problem:



The adaptive transversal filter with N coefficients $\mathbf{c}$ (i.e., causal FIR filter) should converge towards the inverse (or at least an MSE-optimal approximation of the inverse) of the LTI system with impulse response $h[n]$ and transfer function $H(z)$. The signal $v[n]$ is zero-mean white noise with $\sigma_v^2 = 1$.

(a) Answer what properties the LTI system must satisfy in order to enable the adaptive filter to exactly identify the systems's inverse.

(b) Assume the LTI system (with input $v[n]$ and output $x[n]$) has the following purely recursive difference equation:

$$x[n] = v[n] + 0.5 \cdot x[n-1] + 0.1 \cdot x[n-2].$$

Calculate the auto-correlation sequence $r_{xx}[k]$ for $k = 0, 1, 2$. Note, use at least 4 decimal places for all further computations (or don't round at all).

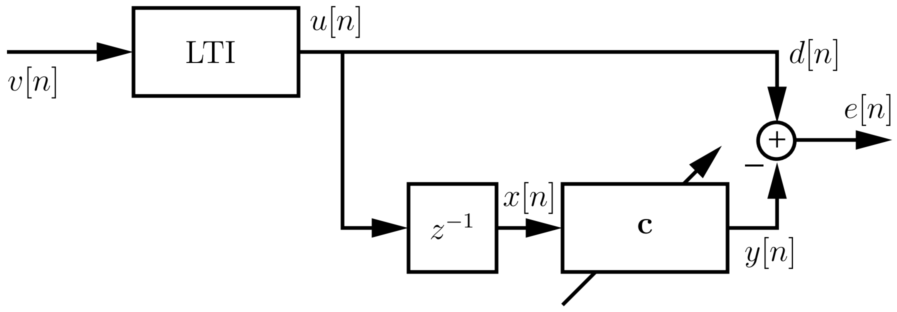
(c) Assume $N = 3$. Write down the auto-correlation matrix $\mathbf{R_{xx}}$ of the tap-input vector $\mathbf{x}[n] = \big[x[n],\, x[n-1],\, x[n-2]\big]^\mathsf{T}$ and the cross-correlation vector $\mathbf{p}$ between the signal $d[n]$ and the tap-input vector.

(d) For $N = 3$, compute the MSE-optimal coefficient vector $\mathbf{c}$ of the adaptive filter.

(e) Compute the MSE-optimal coefficient vector $\mathbf{c}$ of the adaptive filter for $N = 2$.

Problem 2 (30 Points)

A prediction-error filter can be considered as a blind equalizer for AR processes.



The first 3 samples of the auto-correlation sequence of $u[n]$ are given as $r_{uu}[0] = 1.0$, $r_{uu}[1] = 0.\bar{5}$, and $r_{uu}[2] = 0.3\bar{7}$ where the bars denote repeating decimals.

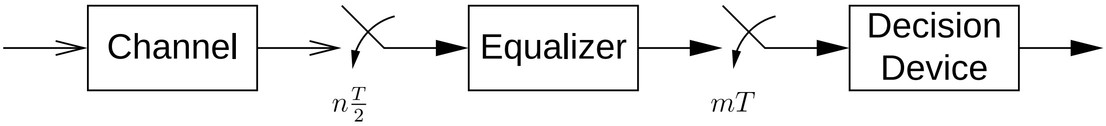
(a) For a predictor with $N = 1$ coefficient, find the MSE-optimal coefficient of the adaptive filter.

(b) For a predictor with $N = 2$ coefficients, find the MSE-optimal coefficient vector $\mathbf{c}$ of the adaptive filter.

(c) Assume, $u[n]$ is an AR process of 2nd order. Write down the process-generator difference equation and calculate the variance σ_v^2 of the white-noise input $v[n]$.

(d) Give similarities and differences between the approaches in Problem 1 and Problem 2. What happens when $v[n]$ is not white?

Problem 3 (35 Points)



Consider the $\frac{T}{2}$-fractionally-spaced equalizer illustrated above where the received signal is sampled at a rate twice the symbol rate. The decision device is synchronized with the *even-indexed* samples (at which actual symbols are sended).

The discrete-time FIR description of the communication channel is

$$H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3} = 1 + 3\, z^{-1} + 1\, z^{-2} + 1/3\, z^{-3}$$

where the unit delay z^{-1} corresponds to $\frac{T}{2}$.

(a) Design a *least-squares equalizer* with 2 coefficients

$$C(z) = c_0 + c_1 z^{-1}$$

such that the cascade of channel and equalizer approximates $H(z)C(z) \approx 1$ (delay-free, perfect equalization).

(b) Repeat the last task to design an equalizer with 3 coefficients

$$C(z) = c_0 + c_1 z^{-1} + c_2 z^{-2}.$$

(c) Consider the symbols to be transmitted are $u[m] \in \{-1, 1\}$. For the two solutions obtained above, determine whether the cascade of channel and equalizer enables an ISI-free transmission or not. In case there is ISI, determine whether the cascade's eye is open or closed. (Hint: at what rate does the decision device operate?)