DISCRETIZATION ISSUES FOR THE DESIGN OF OPTIMAL BLIND ALGORITHMS

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ABSTRACT

The performance and complexity of blind algorithms in a digital receiver is dependent on the prefilter prior to discretization of the received continuous time signal and the sampling rate. This paper shows that symbol spaced blind equalization algorithms are in general sub-optimal, since a matched filter cannot be used. We show that, for fractionally spaced equalizers, the prefilter can be a general low-pass filter and does not need to be matched to the unknown channel. This flexibility on choosing the prefilter can result in different discrete time models with different complexities for the signal processing algorithms to follow. As for example, a simpler whitening filter design which is needed for the success of several important blind equalization algorithms can be realized using this flexibility.

1. INTRODUCTION

Blind deconvolution algorithms involve determination of the impulse response of a system or the input signal, from the observed output signal, when neither the input signal nor the impulse response of the system is known beforehand. The need for blind deconvolution arises in application areas such as digital communication, seismic deconvolution, image restoration and so on. This paper focusses on the digital communication scenerio, where it is necessary to overcome the effects of the intersymbol interference (ISI) caused by unknown channel amplitude and phase distortion. In this case the aim of the blind deconvolution algorithm or the equalizer is to restore the transmitted information symbols from the received signal.

Many blind algorithms have been proposed to address the above problem in the past few decades [1, 2, 3, 4, 5] and these are either based on symbol spaced or fractionally spaced samples of the received continuous time signal. Higher sampling rate is associated with higher complexity and it is well known that symbol rate sampling is sufficient for various detection criteria including the maximum likelihood sequence estimation (MLSE) if the received signal is passed through a matched filter prior to sampling. Following on the theory developed for the case when the channel is known, where symbol spaced discrete channel models are well-justified, many blind equalization algorithms [3, 4, 5] based on symbol rate samples have appeared in the literature with no justification for the channel model. In contrast to the symbol spaced equalization, there have been growing interest, in recent times, in developing fractionally spaced blind equalization algorithms [1, 2]. These equalizers work on samples obtained at a rate higher than the symbol rate, usually satisfying the Nyquist sampling criteria. Although there is increase in the complexity of the A/D converter, these equalizers offer potentially significant advantages over the conventional symbol spaced equalizers in terms of lower timing phase sensitivity, reduced noise enhancement, superior identification strategies by exploiting cyclostationary information. We show here that such a fractionally spaced approach can have additional significant impacts on prefilter design and hence on blind equalization algorithms.

This paper addresses the discretization issues for blind equalization algorithms and attempts to fill the gap between the received analog signal and the discrete time samples used in the algorithm. The important conclusions of this paper are:

- Symbol spaced blind equalizers are in general sub-optimal due to the information lossy discretization process. Hence it is difficult to assess the performance of these equalizers because the

discrete channel model may not be functionally equivalent to the actual continuous time channel.

- In the context of fractionally spaced blind algorithms, the analog prefilter prior to discretization can be a general low pass filter and need not be restricted to be matched to the overall channel response or the input pulse as suggested in [6].
- The flexibility of the prefilter can lead to different discrete time models with identical *potential* optimal performance. However, the performance might be different for sub-optimal detection strategies and, therefore, represents an important design consideration.
- The equalizer structure and the algorithm need to be coupled to the choice of the prefilter. An example is a simpler whitening filter design which is necessary to implement several important blind algorithms, such as [2], proposed in the literature.

2. CONVENTIONAL SYMBOL SPACED RECEIVER

Consider a transmitted waveform

$$s(t) = \sum_n a_n g(t - nT)$$

where $\{a_n\}$ is the input data sequence, T is the symbol interval and $g(t)$ is the impulse response of the pulse shaping filter. As shown in Fig.1, the signal passes through a channel $c(t)$ and the received signal is

$$z(t) = \sum_n a_n h(t - nT) + n(t)$$

where $h(t) = c(t) \otimes g(t) = \int_{-\infty}^{\infty} c(\tau) g(t - \tau) d\tau$ is the response of the channel to the input signal pulse $g(t)$, and is assumed to be of finite length and finite energy, $n(t)$ is additive white gaussian noise with power spectral density $N_0/2$. It is well known that [3] a matched filter having an impulse response $f(t) = h^*(-t)$, where '*' denotes complex conjugation, is the front end of a receiver designed for optimum detection of the input data sequence from symbol spaced samples of the signal $z(t)$. The symbol rate samples $\{y_n\}$ of the output of the matched filter are given by

$$y_n = y(nT) = \int_{-\infty}^{\infty} z(t) h^*(t - nT) dt \qquad (1)$$

These samples $\{y_n\}$ form a set of sufficient statistics for various detection criteria including the MLSE and

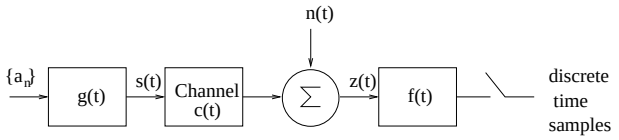

Figure 1: Block diagram showing part of a data communication system

provide discrete time stationary samples as input to the equalizer. As the noise is colored while passing through the matched filter, a noise whitening filter may be used to whiten the noise before feeding the samples to an optimal detector such as the Viterbi Algorithm [7].

3. PREFILTER FOR SYMBOL SPACED BLIND EQUALIZERS

In the context of blind channel equalization, the channel is not known *a priori* and hence the matched filter cannot be implemented perfectly. Suppose we use an arbitrary prefilter $f(t)$ and sample the signal at the symbol rate. The signal just prior to discretization is,

$$\begin{aligned} y(t) &= z(t) \otimes f(t) \\ &= \sum_n a_n x(t - nT) + n(t) \otimes f(t) \end{aligned} \qquad (2)$$

where $x(t) = h(t) \otimes f(t)$ is the overall pulse response. The symbol spaced samples, therefore, become

$$y(kT) = \sum_n a_n x(kT - nT) + n(t) \otimes f(t) \mid_{t=kT} . \qquad (3)$$

The blind channel equalization algorithms then work on the samples $y(kT)$ to equalize the effective channel $x(\cdot)$. It should be noted that the discretization (3) of equation (2) is information lossy [7] if $f(t) \neq h^*(-t)$ and the performance of the equalizer becomes dependent on the choice of $f(t)$. As an example, consider a zero-forcing linear equalizer. For communication systems with excess bandwidth, that is with nonzero frequency components for $|f| > \frac{1}{2T}$, the equalizer must have a transfer function

$$G(f) = \frac{1}{\sum_l H(f + \frac{l}{T}) F(f + \frac{l}{T})} \quad , \qquad |f| \leq \frac{1}{2T}$$

The noise power at the output of the equalizer becomes proportional to

$$\int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{\sum_m |F(f + \frac{m}{T})|^2}{|\sum_l H(f + \frac{l}{T}) F(f + \frac{l}{T})|^2} df. \qquad (4)$$

It is easy to see from (4) that the prefilter frequency response $F(f)$ becomes dependent on the channel response $H(f)$ to achieve maximum SNR under the zero-forcing condition. This dependence is because of the overlap in the folded spectrum due to undersampling. As a result, a sub-optimal choice of $f(t)$ can result in serious degradation of the performance of the equalizer. On the other hand, if this filter is chosen to be the matched filter, the noise power at the output of the equalizer becomes proportional to

$$\int_{-\frac{1}{2T}}^{\frac{1}{2T}} \frac{1}{\sum_l |H(f+\frac{l}{T})|^2} df$$

and the dependence on the prefilter disappears. Since this requires knowledge of the channel, therefore, it is not feasible to rely on a matched filter for blind equalization.

4. PREFILTER FOR FRACTIONALLY SPACED BLIND EQUALIZERS

In this section, we consider the prefilter for fractionally spaced blind equalizers. Assume that $h(t)$ is band limited to $|f| \leq \frac{1}{2T'}$, where $T' = rT$ with $r < 1$, typically $r = \frac{1}{2}$ for systems with less than 100% excess bandwidth. Since $h(t)$ is bandlimited, using the sampling theorem, we can write,

$$h(t) = \sum_{m=-\infty}^{\infty} h_m \mathrm{sinc}(\frac{1}{T'}(t - mT'))$$

for a suitable choice of coefficients $\{h_m\}$. Therefore, the matched filter output samples $\{y_n\}$ given by equation (1) can be written as

$$\begin{aligned} y_n &= \int_{-\infty}^{\infty} z(t) \sum_{m=-\infty}^{\infty} h_m^* \mathrm{sinc}(\frac{1}{T'}(t - nT - mT')) \\ &= \sum_{m=-\infty}^{\infty} h_m^* \int_{-\infty}^{\infty} z(t) \mathrm{sinc}(\frac{1}{T'}(t - (\frac{n}{r} + m)T'))dt \end{aligned} \tag{5}$$

Let p_m and q_m be two arbitrary sequences such that

$$h_m = p_m \otimes q_m \tag{6}$$

Using equation (6) in (5) we can write

$$y_n = \sum_k p_k^* [z(t) \otimes q^*(-t)]_{t=(\frac{n}{r}+k)T'} \tag{7}$$

where

$$[z(t) \otimes q^*(-t)]_{t=(\frac{n}{r}+k)T'} = \int_{-\infty}^{\infty} z(t) q^*(t - (\frac{n}{r} + k)T'))dt \tag{8}$$

and

$$q(t) = \sum_{m=-\infty}^{\infty} q_m \mathrm{sinc}(\frac{1}{T'}(t - mT'))$$

is the bandlimited analog filter corresponding to the sequence q_m satisfying (6). It is clear from (7) that the Nyquist rate samples at rate $\frac{1}{T'}$ at the output of the filter $f(t) = q^*(-t)$ can be used to generate the sufficient statistics $\{y_n\}$. Hence, these samples themselves are sufficient statistics and the algorithm can operate on these discrete time samples. Thus the front end filter $f(t)$ can be a general filter $q^*(-t)$ satisfying (6). However, that filter should not have nulls in the bandwidth of interest.

Since the above result holds independent of the equalizer structure employed, fractionally spaced equalizers, therefore, can have an additional advantage in terms of flexibility of the prefilter. The prefilter can now be coupled to the blind algorithm design problem and can be exploited to find the best possible equalizer structure in terms of performance and complexity of implementation.

5. EXAMPLE

We present here an example showing the realization of a noise whitening filter for fractionally spaced equalizers. If the prefilter has zeros close to the unit circle, the whitening filter design may be difficult to achieve, and the white noise model as assumed in several blind algorithms such as [2] may not be valid. We consider two low-pass filters, one of which has zeros very close to the unit circle, and the other is a minimum phase filter with none of the zeros near the unit circle. We have chosen a minimum phase filter so that the combination of the prefilter and the noise whitening filter does not lead to additional all-pass filter terms. We call the two filters as filter 1 and filter 2 respectively. The frequency response of the filters are shown in Fig.2. Although the impulse response of the noise whitening filter corresponding to filter 1 does not converge even after hundreds of samples, filter 2 does not give rise to such problem. This can be observed from the impulse response of the noise whitening filter corresponding to filter 2 shown in Fig.3. Hence if filter 1 happens to be the pulse shaping filter, we can prefer filter 2 to filter 1 for the prefilter and thus realize the whitening filter. It is to be mentioned that we have not explored all

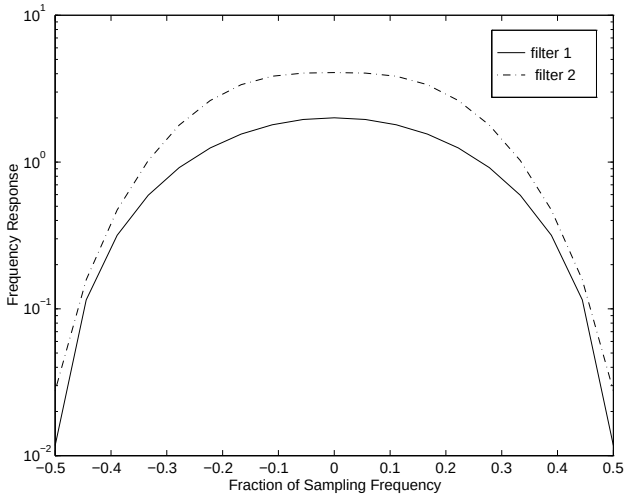

Figure 2: Frequency response of the two filters.

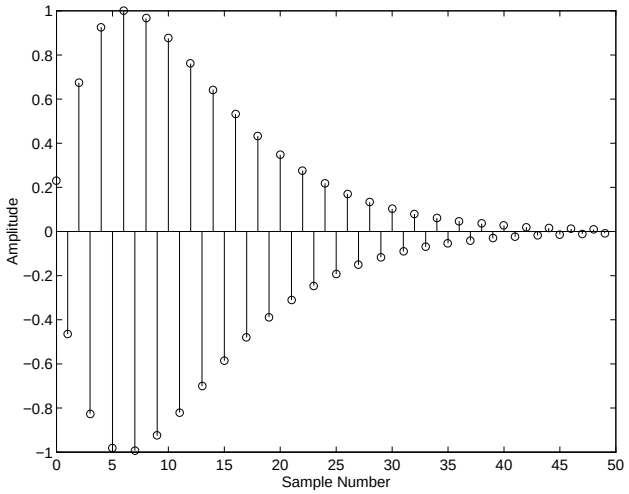

Figure 3: Impulse response of the noise whitening filter.

the possible prefilters to get the best solution for realizing the whitening filter. Also, this example is not exhaustive in terms of all the possible implications the prefilter can have for the design of blind equalization algorithms.

6. CONCLUSION

In this paper we have shown that symbol spaced blind equalization algorithms are sub-optimal due to the information lossy discretization process, and hence the performance of these algorithms are difficult to characterize. To achieve optimality, these blind equalizers must have a filter matched to the overall unknown channel impulse response prior to the sampler which is not possible to realize. On the other hand, fractionally spaced equalizers can have a more general prefilter. This can result in different discrete time models with identical potential optimal performance but with different possible implications with regard to the design of blind algorithms. The equalizer structure and the algorithm, therefore, cannot be decoupled from the choice of the prefilter. We have shown an example of a whitening filter design using this flexibility. We have not explored all the possible implications that the choice of the prefilter might have for the design of the blind algorithms and, therefore, this needs further investigation.

7. REFERENCES

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