

# APPROXIMATION OF OPTIMAL STEP SIZE CONTROL FOR ACOUSTIC ECHO CANCELLATION

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## ABSTRACT

One of the most widely used gradient-based adaptation algorithms is the so called *normalized least mean square* (NLMS) algorithm. The rate of convergence, misadjustment and noise insensitivity of the NLMS-type algorithm depend on the proper choice of the step size parameter, which controls the weighting applied to each coefficient update.

Different step size methods have been proposed to improve the convergence of NLMS-type filters, while preserving the steady-state performance. The step size methods considered here use either a step size parameter which varies with time or a separate, tap-individual step size for each filter tap. The derivation of the respective step size methods is based on different optimization criteria.

In this paper a step size parameter is proposed satisfying a combined optimization criterion leading to a time variant and individual step size parameter. The realization aspects of the new concept are discussed for an acoustic echo control application as an example.

## 1. INTRODUCTION

Adaptive filters are used extensively in communication applications such as acoustic echo control, noise reduction or channel equalization. Due to its simplicity and stability the *normalized least mean square* (NLMS) algorithm represents the most common implementation for filter adaptation.

Conventional adaptive NLMS filters use a fixed step size parameter, which can only be a compromise between the conflicting goals of fast convergence and small steady-state error. A small step size will ensure small misadjustment in steady-state and is needed for noise insensitivity in the presence of noise. On the other hand, a large step size will in general provide faster convergence and better tracking capabilities at the cost of higher excess mean-squared error in the steady-state. So the appropriate choice of the step size factor  $\alpha$  is a major issue for the adaptation process of the NLMS algorithm, which might be used for example to adapt the coefficient vector  $\underline{c}(i)$  of an echo canceller

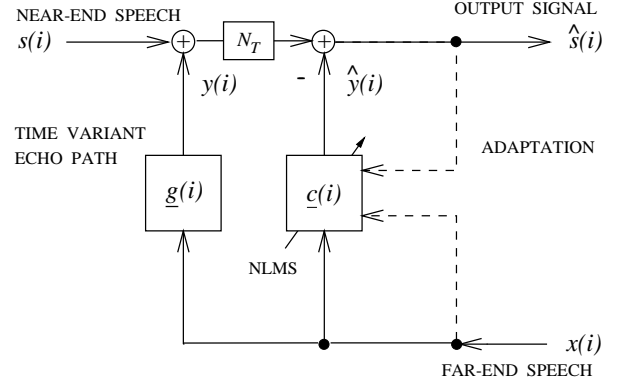


Figure 1: Acoustic echo control concept

according to Fig. 1:

$$\underline{c}(i+1) = \underline{c}(i) + \alpha \frac{\hat{s}(i) \underline{x}(i)}{\|\underline{x}(i)\|^2} \quad (1)$$

where

$$\hat{s}(i) = s(i) + [\underline{g}(i) - \underline{c}(i)]^T \underline{x}(i) \quad (2)$$

$$\underline{c}(i) = (c_0(i), c_1(i), \dots, c_k(i), \dots, c_{N-1}(i))^T \quad (3)$$

$$\underline{x}(i) = (x(i), x(i-1), \dots, x(i-N+1))^T \quad (4)$$

$$\|\underline{x}(i)\|^2 = \underline{x}^T(i) \underline{x}(i), \quad (5)$$

with  $i$  and  $k$  denoting time instant and coefficient index, respectively and  $N$  denotes the number of filter coefficients. One attempt to improve the convergence rate and noise insensitivity of the NLMS algorithm, while preserving the steady-state performance, is the application of a variable, i.e. time variant step size parameter  $\alpha(i)$  (e.g. [4,7,8]), where the step size varies for different time instants  $i$ . Based on the minimization of the expectation value

$$E\{\|\underline{d}(i+1)\|^2\} \quad \text{for } s(i) \neq 0 \quad (6)$$

an optimal time variant step size has been derived in [8] according to

$$\alpha(i) = \frac{1}{1 + \frac{E\{s^2(i)\}}{E\{x^2(i)\}} \frac{1}{E\{\|\underline{d}(i)\|^2\}}} \quad (7)$$

$$\alpha(i) = \frac{E\{x^2(i)\} E\{\|\underline{d}(i)\|^2\}}{E\{\hat{s}^2(i)\}} , \quad (8)$$

where  $\underline{d}(i)$  denotes the distance vector  $\underline{d}(i) = \underline{g}(i) - \underline{c}(i)$ . As a result of the power ratio of the two input signals  $x(i)$  and  $s(i)$  in the denominator of eq. (7) the adaptive step size  $\alpha(i)$  serves as double talk detector slowing down the adaptation process in case of  $E\{s^2(i)\} \gg E\{x^2(i)\}$ . The second term, i.e.  $1/E\{\|\underline{d}(i)\|^2\}$  balances between the conflicting goal of fast convergence and small steady-state error.

In an alternative approach the statistical characteristics of room impulse responses are considered within the so called exponentially weighted step size parameter [5], where different step sizes are applied for different coefficients  $c_k(i)$ , where  $c_k(i)$  denotes the  $k^{\text{th}}$  component of vector  $\underline{c}$  at time instant  $i$ . The derivation is based on the minimization of

$$E\{d_k^2(i+1)\} \quad \text{for } s(i) = 0 \quad (9)$$

and yields

$$\alpha_k(i) = \frac{N E\{d_k^2(i)\}}{E\{\|\underline{d}(i)\|^2\}} . \quad (10)$$

The resulting algorithm provides a higher rate of convergence, however, since it was developed for  $s(i) = 0$ , it remains sensitive to noise.

Within the following section a new step size parameter satisfying a combined optimization criterion is introduced. Section 3 contains the detailed discussion of its realization aspects and in section 4 the performance results for time variant room impulse responses  $\underline{g}(i)$  are presented and analyzed.

## 2. OPTIMAL STEP SIZE CONTROL

The optimal — variable and individual — step size parameter for an NLMS-driven adaptation algorithm is obtained if the different optimality criteria given in eq. (6) and (9) are combined minimizing

$$E\{d_k^2(i+1)\} \quad \text{for } s(i) \neq 0 . \quad (11)$$

The application of the combined optimality criterion results in a new step size parameter, which equals the product of the variable and individual step size parameter

$$\alpha_k^{\text{opt}}(i) = \alpha(i) \cdot \alpha_k(i) \quad (12)$$

introduced in eq. (7) and (10), respectively. The corresponding proof has been presented in [3].

Consequently, with this in the sense of eq. (11) optimal step size parameter the adaptation process as well as the noise insensitivity are improved, which will be underlined by the simulation results in section 4.

## 3. REALIZATION ASPECTS

In principle for the implementation of the new combined convergence factor  $\alpha_k^{\text{opt}}(i)$  the step size parameters  $\alpha(i)$  and  $\alpha_k(i)$  have to be approximated. However, due to the combined implementation there are some additional critical aspects to be considered.

### 3.1. Approximation of the time variant step size

In [8] the optimal time variant step size given by eq. (8) is approximated by

$$\alpha(i) \approx \hat{\alpha}(i) = \frac{\|\underline{x}(i)\|^2 \sum_{n=0}^{N_T-1} c_n^2(i)}{N_T \hat{s}^2(i)} \quad (13)$$

$$\text{with } E\{x^2(i)\} \approx \frac{\|\underline{x}(i)\|^2}{N}$$

$$\text{and } E\{\hat{s}^2(i)\} \approx \bar{s}^2(i) = (1 - \gamma) \sum_{n=0}^i \gamma^{(i-n)} \hat{s}^2(i) ,$$

$$0 < \gamma < 1 .$$

The unknown system distance is given by extrapolating the system distance estimation by the introduction of  $N_T$  additional equalization coefficients  $c_n(i)$  and a delay of the echo path by  $N_T$  samples according to Fig. 1 leading to

$$\frac{E\{\|\underline{d}(i)\|^2\}}{N} \approx \frac{\|\underline{d}(i)\|^2}{N} \approx \frac{\sum_{n=0}^{N_T-1} c_n^2(i)}{N_T} . \quad (14)$$

This step size method has been further developed by [7] and [4] reacting on changes of the unknown system  $\underline{g}(i)$  and achieving robustness in noisy environment, respectively. All modifications proposed in [4,7] have been applied within  $\hat{\alpha}(i)$ . Since they are not leading to any additional measures due to the combination with the individual step size, they are not further discussed.

### 3.2. Approximation of the individual step size

For the implementation of the individual step size parameter in a first step  $\alpha_k(i)$  is approximated by an exponential decay curve

$$\alpha_k(i) \approx \alpha_k = \text{const} \cdot e^{-\left(6.9 \frac{T_S}{T_R}\right)k} \quad (15)$$

for all  $k \in \{0, 1, \dots, N-1\}$ , where  $T_S$  denotes the sampling interval and  $T_R$  the reverberation time of the room (see Fig. 2).

In a second step, the exponential decay curve can further be approximated by discrete steps  $\hat{\alpha}_k$  according to Fig. 2, which in practice reduces significantly the computational complexity without major effect on the performance.

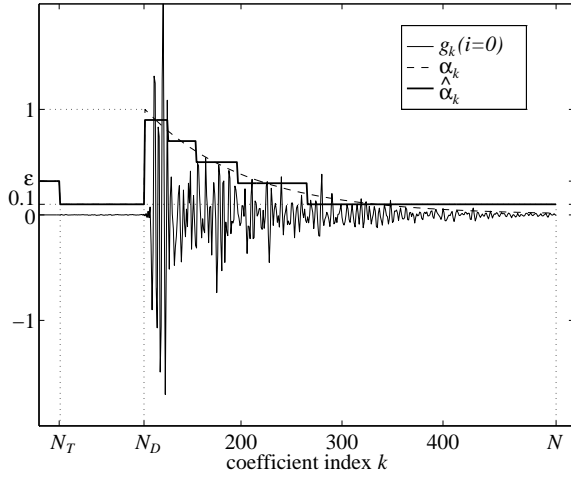


Figure 2: Room impulse response  $g_k(i=0)$ , exponential decay curve  $\alpha_k$  and its approximation  $\hat{\alpha}_k$  with discrete steps

### 3.3. Approximation of the optimal step size

The approximation of the optimal step size parameter is obtained combining the results from paragraph 3.1 and 3.2 in form of

$$\hat{\alpha}_k^{\text{opt}}(i) = \hat{\alpha}(i) \cdot \hat{\alpha}_k. \quad (16)$$

Now, for the implementation of  $\hat{\alpha}_k$  some additional aspects have to be considered. First, the parameter  $\varepsilon$  is introduced, which weights the first  $N_T$  filter coefficients according to  $\hat{\alpha}_k = \varepsilon$  for  $0 \leq k < N_T$ , see also Fig. 2. Since the system distance estimation is based on these  $N_T$  coefficients (see eq. (14)), weighting factor  $\varepsilon$  represents a critical parameter. In order to determine parameter  $\varepsilon$  the identity

$$\frac{\sum_{n=0}^{N_T-1} \varepsilon^2 + \sum_{n=N_T}^{N_D-1} 0.1^2 + \sum_{n=N_D}^{N-1} e^{-2\left(6.9 \frac{T_S}{T_R}\right)(n-N_D)}}{N} = \frac{\sum_{n=0}^{N_T-1} \varepsilon^2}{N_T} \quad (17)$$

is applied analogously to eq. (14), which yields

$$\varepsilon = \sqrt{\frac{1}{N-N_T} \left[ N_D \cdot 0.1^2 + \sum_{n=N_D}^{N-1} e^{-2\left(6.9 \frac{T_S}{T_R}\right)(n-N_D)} \right]}. \quad (18)$$

Secondly, the empirical factor 0.1 is introduced, which represents a lower limit of  $\hat{\alpha}_k$  during the idle time, i.e.  $\hat{\alpha}_k = 0.1$  for  $N_T \leq k < N_D$ .

The exponential decay curve, i.e.  $N_D$  and  $T_R$ , needed for the evaluation of  $\hat{\alpha}_k$  can be determined applying a special initialization phase, where a perfect sequence is used as optimal excitation signal for the NLMS algorithm [1]. Parameter  $N_T$  is a constant value and set to  $N_T = 20$  with respect to [7].

The final equation for the new adaptation process is given by

$$\underline{c}(i+1) = \underline{c}(i) + \hat{\alpha}(i) \frac{\hat{s}(i)}{\|\underline{x}(i)\|} \frac{\mathbf{A} \underline{x}(i)}{\|\mathbf{A} \underline{x}(i)\|} \quad (19)$$

with the diagonal matrix  $\mathbf{A} = \text{diag}(\hat{\alpha}_0, \hat{\alpha}_1, \dots, \hat{\alpha}_{N-1})$  and the application of a modified normalization according to [6]. Matrix  $\mathbf{A}$  enables the individual adaptation of the filter coefficients  $c_k(i)$  with respect to the characteristics of the room impulse response, while with  $\hat{\alpha}(i)$  the negative influence of signal  $s(i)$  is reduced. Approximating the exponential decay curve  $\alpha_k$  with a few discrete steps the computational overhead due to the last fraction in eq. (19) can significantly be reduced, while the performance is mainly not affected.

## 4. SIMULATION RESULTS

In order to investigate the performance of the proposed algorithm computer simulations were performed for an acoustic echo control application with a time variant room impulse response  $\underline{g}(i)$ , (see also [2]). Fig. 3 shows the results of different step size methods in terms of *Echo Return Loss Enhancement* ERLE( $i$ ) and *System Distance* D( $i$ ) defined by

$$\frac{\text{ERLE}(i)}{\text{dB}} = 10 \lg \frac{E\{y^2(i)\}}{E\{(\hat{y}(i) - y(i))^2\}} \quad (20)$$

$$\frac{D(i)}{\text{dB}} = 10 \lg \frac{\|\underline{g}(i) - \underline{c}(i)\|^2}{\|\underline{g}(i)\|^2}. \quad (21)$$

Parameter  $\zeta(i)$  indicates the intensity of time variance of  $\underline{g}(i)$ , i.e. with increasing parameter  $\zeta(i)$  the room impulse response becomes more time variant. The input signals  $x(i)$  and  $s(i)$  are included to mark the different phases of talker activity.

In the periods, where only  $s(i)$  is active, the adaptive step size  $\hat{\alpha}(i)$  serves as double talk detector slowing down the adaptation process. This leads to a more or less constant course of the system distance. Consequently, the adaptation process reacts robust to the interference influence of signal  $s(i)$ . Furthermore, Fig. 3 outlines that especially in the transition from the variant impulse response ( $\zeta(i)$  large referring to fast movements of the near-end talker) to an invariant situation ( $\zeta(i)$  small referring to a stationary near-end talker) the new step size method benefits from the combination with the individual step size parameter resulting in a higher rate of convergence. Finally, it can be concluded from Fig. 3 that the effect due to the use of  $\hat{\alpha}_k$  instead of  $\alpha_k$  is minimal.

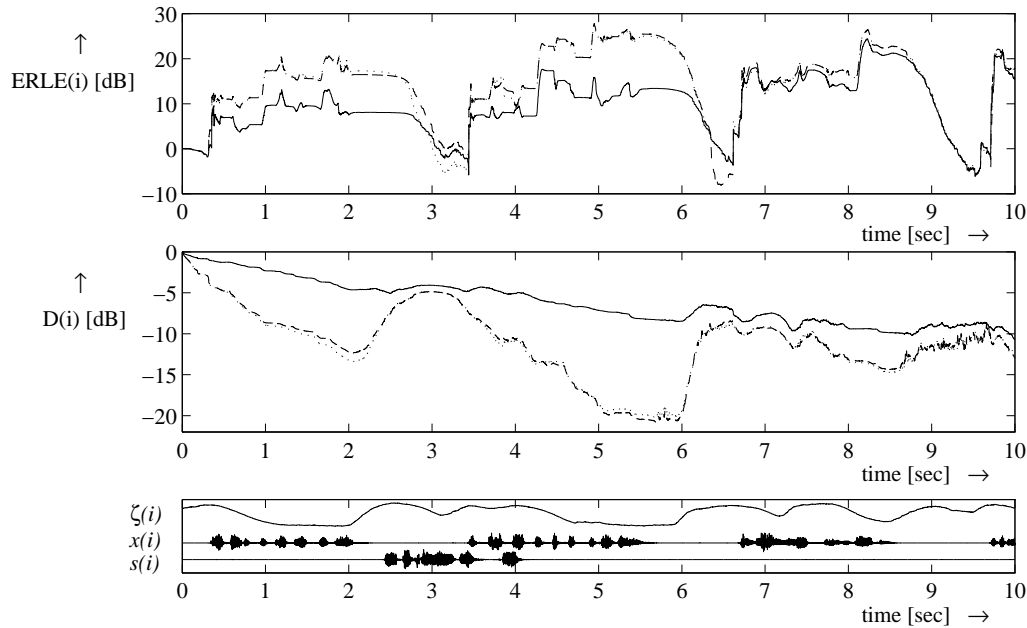


Figure 3: Comparison of *Echo Return Loss Enhancement*  $ERLE(i)$  and *System Distance*  $D(i)$  for a time variant simulation (see [2]) for different step size parameters ( $\zeta(i)$  indicates the intensity of time variance of  $\underline{g}(i)$ )

- NLMS algorithm with  $\hat{\alpha}(i)$
- - - NLMS algorithm with  $\alpha_k \cdot \hat{\alpha}(i)$
- ..... NLMS algorithm with  $\hat{\alpha}_k \cdot \hat{\alpha}(i) = \hat{\alpha}_k^{opt}(i)$

## 5. CONCLUSIONS

In this paper the approximation of the optimal – time variant and tap-individual – step size parameter for an NLMS-driven adaptation algorithm was presented and its practical aspects discussed in detail. The combination of the two step size methods also supposes the improvements achieved for each approach. Consequently, the new combined step size method complements the characteristics of the two individual approaches in such a way that it reacts robust in case of double talk situations and additionally benefits from a higher rate of convergence taking a-priori knowledge of the room impulse response into account. So, especially in the presence of an interference signal or in case of a room impulse response varying with time significant improvements are obtained without major increase of the computational complexity.

## 6. REFERENCES

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