MINIMISATION OF THE MAXIMUM ERROR SIGNAL IN ACTIVE CONTROL *

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ABSTRACT

This paper deals with Multiple Input Multiple Output systems for active control of acoustic signals. These systems are used when the acoustic field is complex and therefore a number of sensors are necessary to estimate the sound field and a number of sources to create the cancelling field. A steepest descent iterative algorithm is applied to minimise the p-norm of a vector composed by the output signals of a microphone array. The existing algorithms deal with the 2-norm of this vector. This paper describes a general framework that covers the existing systems and then it focuses on the ∞-norm minimisation algorithm. The minimax algorithm based on the ∞-norm minimises the output signal which has the greatest power. It is shown by means of simulations using measured data from a real room that the minimax algorithm leads to a more uniform final noise field than the existing algorithms.

1. INTRODUCTION

In recent years adaptive signal processing has been developed and applied to the expanding field of active noise control (ANC) [1]. The objetive of ANC at low frequencies in rooms and enclosures is normally to achieve a global control of the sound field. Therefore multichannel active control methods must be used. The sound field in the enclosure is measured by means of multiple sensors. The signals recorded by these sensors will be termed error signals. These error signals are then passed to a digital controller that adjusts the sound generated by a number of secondary sources following a certain minimisation strategy. The most common strategy consists in minimising the sum of the squares of the measured signals. Such strategy together with an adaptive FIR filtering squeme leads to an algorithm called multiple error LMS which has been studied [2] [3] and applied to some practical cases. The best known applications are the control of "boom" noise in cars [4] and the control of propeller-induced noise in flight cabin interiors [5].

The minimisation of the sum of the squares produces a residual acoustic field in the enclosure that can have large differences among the level values [6] at different locations within the enclosure. In most applications a more uniform acoustic field is desired. This difference of levels could be easily perceived by a person walking inside of a room. In order to get a more uniform acoustic field a weighted squared error strategy of minimisation was used in [2].

The analytical framework is developed for a general single frequency acoustic model. To test the algorithm with real data, a small rectangular reverberant room was chosen.

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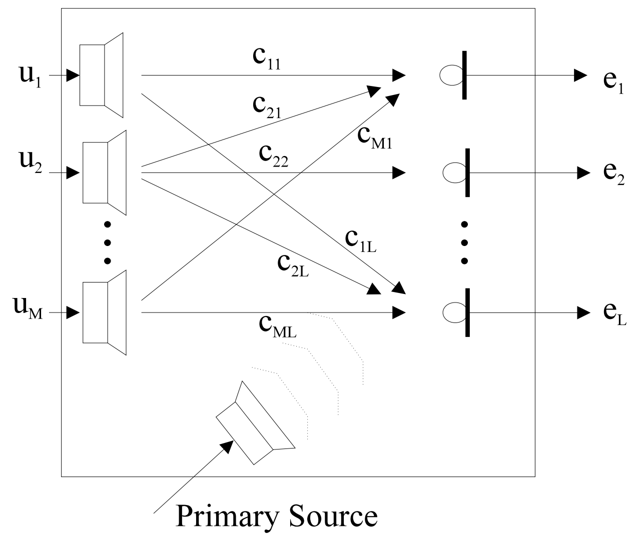

Figure 1. Sketch of the acoustic model. The loudspeakers are fed by their complex strengths denoted by u_m. The complex pressure signals picked up by the sensors are denoted by e_l and they are composed the disturbance due to the primary source and the control signals. The system responses between the secondary sources strengths and the control signals at the sensors are named c_{lm}.

2. ACOUSTIC MODEL

A frequency domain model of an acoustic system is considered. The system under study is assumed to be linear and is excited in the steady state at a single frequency. In this case, the amplitude and phase of each signal (in the steady state) can be described by a complex number. Therefore the complex pressure in a sensor output, e_l, can be expressed as the sum of the contributions from a number of sources. In general, $l \in \{1 \dots L\}$ and there are assumed to be $M+1$ sources: one primary source and M secondary sources.

Equation (1) describes the relationship between the source strengths and the error sensor outputs at a frequency ω_o.

$$e_l(\omega_o) = d_l(\omega_o) + \sum_{m=1}^{M} c_{lm}(\omega_o) u_m(\omega_o) \qquad (1)$$

In the last expression d_l is the disturbance due to the primary source measured at sensor l. The strength of the mth secondary source is represented by u_m. The complex numbers c_{lm} are the system response between the lth error sensor and the mth secondary source, these complex coeficients are the ratio between the signal measured at the *lth* sensor and the *mth* secondary source strength. For notational simplicity, the excitation frequency ω_o is supressed in the rest of the text. Equation (1) can then be written in the following vector form.

$$e_l = d_l + \mathbf{c}_l \mathbf{u} \tag{2}$$

The column vector $\mathbf{u} = [u_1, \ldots, u_M]^T$ contains the secondary source strengths and it is called the secondary source strength vector. The row vector $\mathbf{c}_l$ components are the system responses between each of the secondary sources and the *lth* error sensor. An equivalent development for a single secondary source can be found in [7].

3. ALGORITHM DERIVATION

3.1. The cost function

The error signal vector contains the disturbance which is to be minimised. A family of algorithms is defined. Each algorithm belonging to this family will minimise a different measure of this error vector. The error criterion, also called the cost function, is defined as the p-norm of the error vector. Functions related to the norm are very often used in minimisation problems because they are convex functions. The definition of this cost function is given by the equation (3). Similar cost function definitions can be found in [8] and [9] with the purpose of adaptive filtering of non-gaussian stable signals. This adaptive algorithm is used in [10] for the active control of impulsive noise. All these papers deal with p-norm cost functions with $1 < p < 2$. In general

$$J_p = \|\mathbf{e}\|_p = \sqrt[p]{\sum_{l=1}^{L} |e_l|^p} \tag{3}$$

where $1 < p < \infty$ and $|e_l|^2 = e_l^* e_l$. Equation (3) defines a family of cost functions depending on the values of p. Different values of the parameter p lead from the sum of the squares, $p = 2$, to the maximum measured signal in the limit with p tending to infinity. In case of $p = 2$ the p-norm cost function takes the following form,

$$J_2 = \|\mathbf{e}\|_2 = \sqrt{\sum_{l=1}^{L} |e_l|^2} \tag{4}$$

Instead of J_2, the squared version of this cost function, J_2^2 is commonly used in optimisation problems since both functions share the same extreme values. Minimising J_2^2 by means of a steepest descent method leads to the MELMS algorithm, [1] [2] [3]. The MELMS algorithm updates the secondary strength vector using the following recursion [6],

$$\mathbf{u}(n+1) = \mathbf{u}(n) - \mu \sum_{1=l}^{L} \mathbf{c}_l^H e_l(n) \tag{5}$$

where the parameter μ is called the convergence parameter and $(\cdot)^H$ denotes the conjugate transpose of the chosen vector. It is assumed that the system under control achieves its steady state before the next secondary strength vector update.

Our study focuses on the limiting case, $p \to \infty$. For the limiting case, it is obtained the expression bellow,

$$J_\infty = \lim_{p\to\infty} J_p = \max_l |e_l| = |e_b| \tag{6}$$

where e_b corresponds to the error signal with largest absolute value at each iteration.

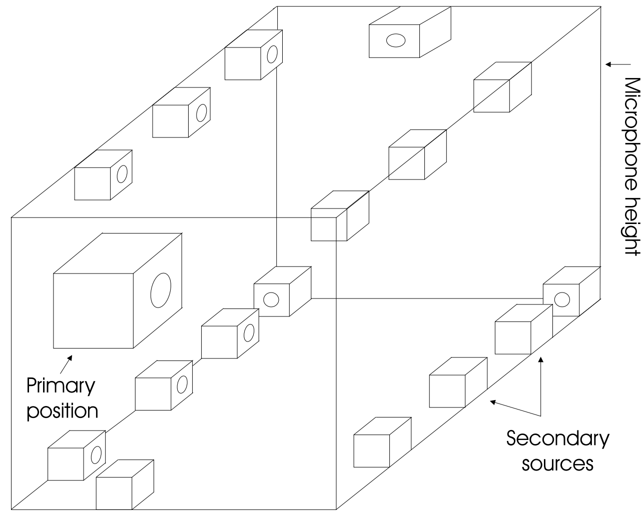

Figure 2. The experimental enclosure showing the layout of the loudspeakers and microphones used for the laboratory experiments. The microphones are suspended from the ceiling.

3.2. Minimax Algorithm Derivation

A recursive steepest descent type algorithm can be defined using the expressions of the gradient vector and the cost function with p tending to infinity [6] [7]. The complex derivatives rules in [11] have been followed here. The recursion will be,

$$\mathbf{u}(n+1) = \mathbf{u}(n) - \alpha \mathbf{c}_b^H e_b(n) \tag{7}$$

where the parameter α is named the algorithm convergence parameter. Equation (7) corresponds to the recursion called *the Minimax Algorithm* in this paper. The secondary source strength vector is updated using only one of the error signals. The algorithm behaves as if it is trying to minimise in each iteration the largest error signal power in a least squares sense. We are, however, using only one error signal in each iteration of the algorithm. A similar iterative algorithm working on single channel data in the time domain can be found in [12]. Information about the potential computational savings and implementation of such algorithms can be found in [13].

As pointed above, only one of the measured signals is used in the algorithm calculations at any one time in equation (7). Therefore the computational load is reduced as compared with the load given by (5). The scanning error algorithm [14] can also use only one signal in each algorithm iteration. This algorithm minimises the error signals in turn (individually or in groups), in a least squares sense. However, it is shown in [6] [14] that this algorithm produces the same final acoustic field than the sum of squares.

4. SIMULATIONS WITH REAL DATA

Data from an acoustic system working in a real room have been used to test the behavior of the minimax algorithm and compare it with MELMS. The system responses and primary field were measured in a small enclosure. The enclosure measured about 6m x 2.2m x 2.1m and had a reverberation time of about 1/3 second in the low frequency range. The excitation frequency was 88 Hz and the active noise control system is composed of $L = 32$ microphones and $M = 16$ secondary loudspeakers. The positions of the loudspeakers in the room are shown in figure 2. The 32 microphones are uniformly distributed at the height shown in the figure. The measurements of the system responses and

	ATTENUATION(dB)	
	sum of squares	maximum level
2-norm	32.22	32.57
minimax	29.72	36.38

Table 1. Attenuation after control. The attenuation of the sum of the squares of the error signals and the attenuation of the maximum squared error signal are shown for both criterions of minimisation.

the primary field in the enclosure were then used in simulations in which either $\|\mathbf{e}\|_2^2$ or $\|\mathbf{e}\|_\infty$ was minimised using the steepest descent method.

Table 1 shows the attenuation obtained after convergence of the simulated algorithm using either the 2-norm of the error vector criterion or the ∞-norm. The difference of levels before and after control is measured for the sum of squares of errors and for the maximum square error. In spite of the fact that it is not designed to minimise the sum of the squared errors, the minimax algorithm still achieves a reasonable attenuation of this quantity. The minimax algorithm however, achieves almost 4dB more attenuation in the maximum error than the least squares algorithm. It is important to note that after 2-norm minimisation there can be several error signals with powers over the minimax maximum level. This fact implies that there can exist zones in the room with relatively high level of noise when the 2-norm is used. The presence of zones with low level noise compensates this effect and achieves the minimum value in the sum of the squares.

The zones of low and high noise level after convergence of both algorithms can be more easily seen in the figures 3 4 and 5. Each pair of integer values of the x and y axes of these figures locates an error sensor in the room. The z axes value at these coordinates represents the level of noise.

Figure 6 illustrates the evolution of the maximum error signal level for the minimax algorithm and the MELMS algorithm. The data used in the figure 6 are obtained from simulations using the system responses and primary field measured in the enclosure. The maximum value of the convergence parameter μ was chosen in the curve for the MELMS algorithm. The minimax algorithm gives a convergence which has piecewise exponential decay. The MELMS algorithm curve is made up from multiple exponential decays [3]. The speed of convergence of the minimax algorithm increases with the convergence parameter but then so does the midsadjustment of the maximum error signal final level. This relationship is further discussed in [6] [7].

5. CONCLUSIONS

In this paper a new criterion of minimisation has been applied to active noise control. This criterion has been used to develop an iterative algorithm that minimises the maximum value of each of the error signals and which can be implemented in practice. The algorithm uses only one error signal in each iteration so it needs less computational effort than the MELMS algorithm which uses all the error signals in each iteration. However, the new algorithm has to find out which of the error signals has the largest level although there exist efficient methods for this calculation.

The minimisation of the maximum level of the error signals has been shown to improve the uniformity in the final acoustic field. The simulations have been carried out for a fixed positions of the loudspeakers and the microphones in the room.

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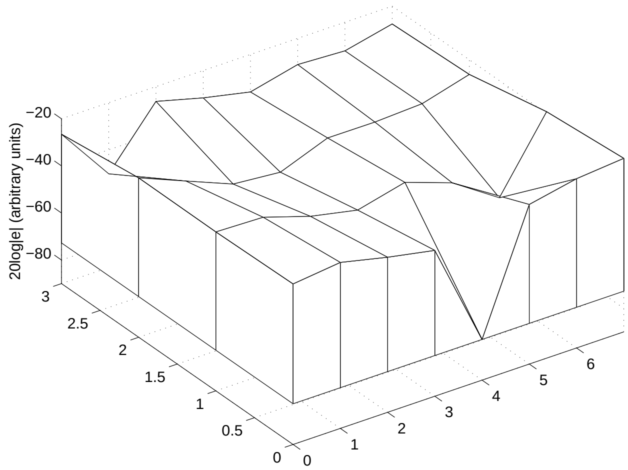

Figure 3. Original error signal levels at the sensors. The horizontal axes represent the row number (0 to 3) and the column number (0 to 7) of the sensors in the microphone array.

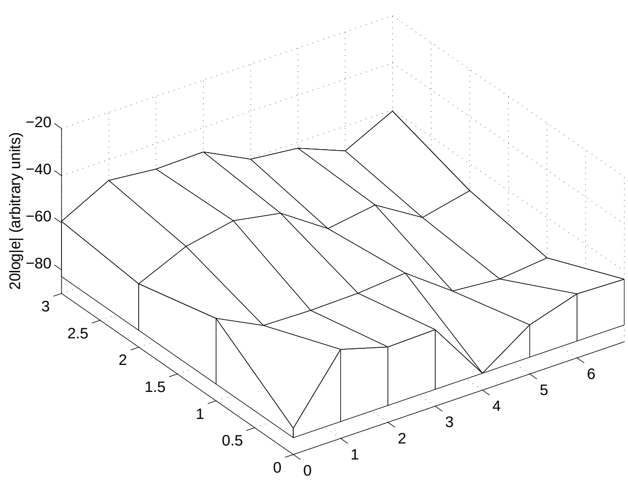

Figure 4. Error signal levels at the sensors after minimisation of the sum of squared signals (MELMS). The horizontal axes represent the row number (0 to 3) and the column number (0 to 7) of the sensors in the microphone array.

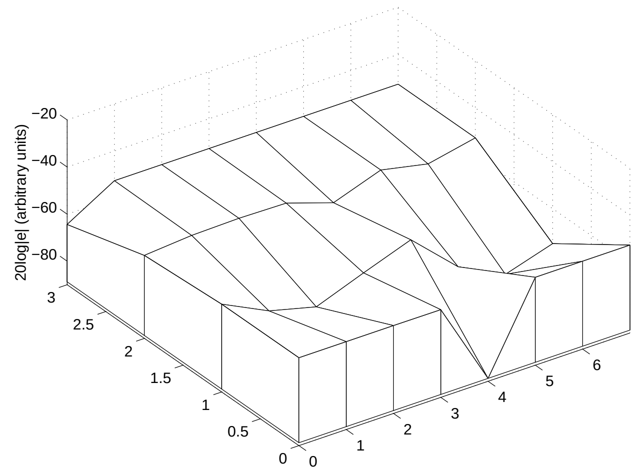

Figure 5. Error signal levels at the sensors after minimisation of the maximum value (MINIMAX). The horizontal axes represent the row number (0 to 3) and the column number (0 to 7) of the sensors in the microphone array.

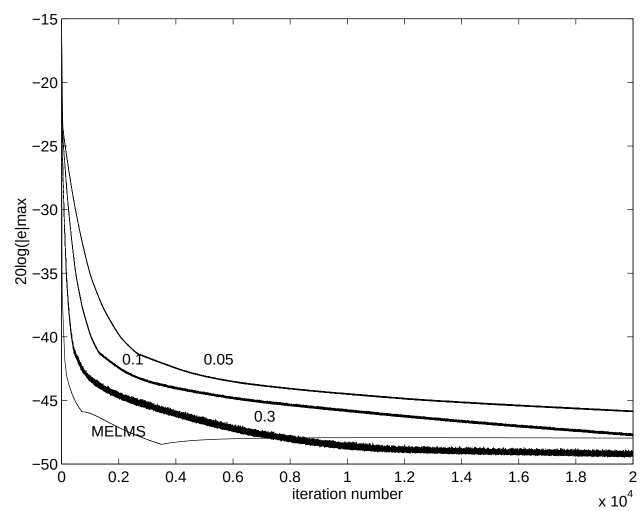

Figure 6. Evolution of the maximum error signal level for the MINIMAX algorithm with increasing values of the convergence parameter, $\alpha = 0.05, 0.1, 0.3$, and for the MELMS algorithm with the maximum allowed convergence parameter.