

NONLINEAR ACTIVE NOISE CONTROL IN A LINEAR DUCT

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ABSTRACT

The problem in active noise control in a linear duct is examined. Essentially, a nonlinear inverse to a nonminimum phase actuator is proposed. The nonlinear inverse exploits the non-Gaussian nature of some chaotic and stochastic noise sources. The architecture of the controller is derived using Bayesian estimation theory and is shown to be a combination of a linear adaptive network and a radial basis function (RBF) or Volterra series (VS) network. Because of the nonlinear nature of the controller, the filtered-x least means square (LMS) architecture cannot be used. Hence a modified active noise controller is proposed. Simulation results demonstrate the improvements in performance achievable with the combined linear and nonlinear controller.

1. INTRODUCTION

Active noise control [1] has been successfully applied to HVAC (heating, ventilating and air conditioning) systems [2], exhaust noise and motor noise [3]. ANC, in general, is based on the principle of the destructive interference between a primary noise source and a secondary source, whose acoustic output is governed by a controller. The output of the secondary source has to be in exact antiphase with the acoustic wave produced by the primary noise source. A typical ANC system in a duct, is shown in Figure 1 and its equivalent block diagram in Figure 2. Such systems are usually based on a feedforward control strategy. The noise from the primary source travels, from left to right, as plane waves through the duct. A microphone, located upstream from the secondary source, detects the incident noise waves and supplies the controller with an input signal. The controller sends a signal to the secondary source (i.e. loudspeaker) which is in antiphase with the disturbance. A microphone, located downstream, picks up the residuals and supplies the controller with an error signal. The variance of the error signal is usually minimised by a variant of the LMS algorithm in the controller.

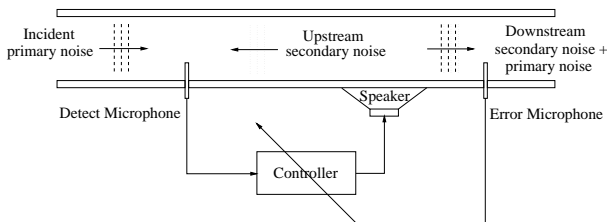


Figure 1: Feedforward control in a duct

The low-frequency noise in the duct is usually assumed

to be broadband random or periodic tonal noise. Linear adaptive signal processing techniques [1] are employed to estimate an antiphase signal. However, today it is known that many of these noise processes arise from nonlinear dynamical systems. Chaotic time series are a typical example of aperiodic time series which appear to be a stochastic process when analysed with second order statistics. Deterministic nonlinear behaviour can arise from all kinds of different physical systems [4].

The finite impulse response (FIR) controller in Figure 2 has to identify the cascaded systems $P(z)$ and $H_a^{-1}(z)$. Both systems are assumed to be linear, but nevertheless $H_a(z)$ may be minimum or nonminimum phase. If $H_a(z)$ is minimum phase its inverse is a stable infinite impulse response (IIR) filter. To approximate an IIR filter a high order FIR filter may be designed. In the case that $H_a(z)$ is nonminimum phase [5], the inverse demands a non-causal linear FIR, which is not feasible in this application. However, if the input signal $x[n]$ is stochastic non-Gaussian [6] or deterministic [4] a nonlinear inverse to the transfer function $H_a(z)$ exists and may produce an improvement in cancellation performance.

Constructing a nonlinear inverse filter to a linear filter to cancel deterministic noise has been investigated in [4]. Finding an inverse with non-Gaussian stochastic input is also encountered in numerous channel equalisation problems [6].

Section 2 describes how a nonlinear inverse to a linear filter may exist and in Section 3 the ANC system in Figure 2 is redesigned to enable nonlinear system identification. A variety of simulations and results are presented in Section 4 and Section 5 concludes this paper.

2. NONLINEAR INVERSE OF A LINEAR FILTER

If the controller is to model $P(z)H_a^{-1}(z)$ when the actuator transfer function $H_a(z)$ is nonminimum phase [5], a causal linear controller will be suboptimum since it cannot, by definition, characterise the anticausal part of $H_a^{-1}(z)$. The derivation of the following mathematical expressions for the combined controller assumes that the feedback path $F(z)$ has been effectively cancelled by $\hat{F}(z)\hat{H}_a(z)$. The block diagram is shown in Figure 3. The transfer function $H_a(z)$ is assumed to be nonminimum phase with a noncausal inverse $H_a^{-1}(z)$. The transfer function $H_a^{-1}(z)$ can be written as $H_a^{-1}(z) = z^k \hat{H}_a(z)$, where k is an integer and $\hat{H}_a(z)$ is a causal transfer function. The task of the controller $H_c(z)$ is to produce an estimate of $\hat{d}[n+k]$ based

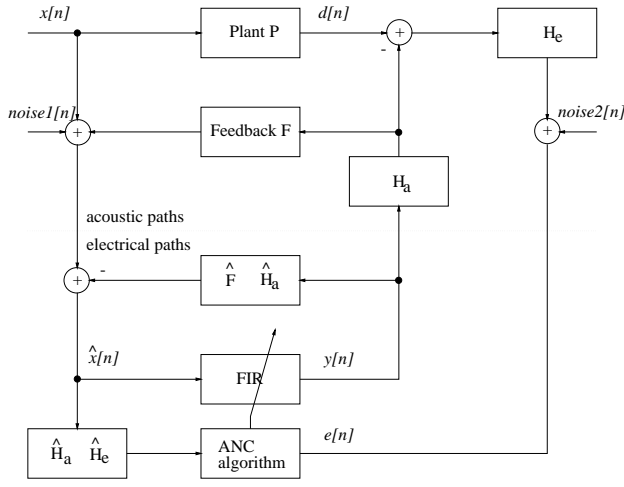


Figure 2: Block diagram of the ANC system with conventional adaptive FIR filter and feedback cancellation

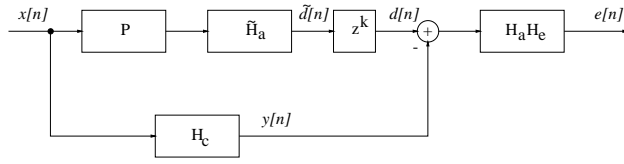


Figure 3: Block diagram of ANC system for derivation of the combined controller on the input vector $\mathbf{x}_N[n]$.

$$d[n] = \hat{d}[n+k] = f(\mathbf{x}_N[n]) \quad (1)$$

with

$$\mathbf{x}_N[n] = [x[n], x[n-1], \dots, x[n-N+1]]^T \quad (2)$$

Given \mathbf{x}_N the minimum mean square error (MMSE) estimate of $d[n]$ is obtained by the conditional expectation or Bayesian estimate :

$$\hat{d}[n] = E\{d[n]|\mathbf{x}_N\} \quad (3)$$

Apart from some special cases where $\{d[n]\}$ and $\{x[n]\}$ are jointly Gaussian, the solution of 3 is generally nonlinear. The desired signal is given by

$$d[n] = \sum_{i=0}^{N-1} h_i x[n+k-i] \quad (4)$$

where $\{h_i\}$ are coefficients of the transfer function $P(z)\hat{H}_a(z)$. Equation (4) can be expressed as two parts

$$d[n] = d_1[n] + d_2[n] \quad (5)$$

$$= \sum_{i=0}^{k-1} h_i x[n+k-i] + \sum_{i=k}^{N-1} h_i x[n+k-i] \quad (6)$$

Thus 3 can be rewritten as

$$\hat{d}[n] = E\{d_1[n] + d_2[n]|\mathbf{x}_N\} \quad (7)$$

$$= E\{d_1[n]|\mathbf{x}_N\} + E\{d_2[n]|\mathbf{x}_N\} \quad (8)$$

The second part of 8 is a linear system identification problem

$$\hat{d}_2[n] = E\left\{\sum_{i=k}^{N-1} h_i x[n+k-i]|\mathbf{x}_N\right\} \quad (9)$$

The optimum solution would be

$$d_2[n] = \sum_{i=k}^{N-1} h_i x[n+k-i] \quad (10)$$

The first part of 8 does not offer a straightforward solution

$$\hat{d}_1[n] = E\{d_1[n]|\mathbf{x}_N\} \quad (11)$$

$$= E\left\{\sum_{i=0}^{k-1} h_i x[n+k-i]|\mathbf{x}_N\right\} \quad (12)$$

$$= \sum_{i=0}^{k-1} h_i E\{x[n+k-i]|\mathbf{x}_N\} \quad (13)$$

It is usually not necessary to embed $\mathbf{x}_N[n]$ into N dimensions. For modelling reasons (curse of dimensionality) a smaller dimension $M < N$ will be more appropriate.

$$\hat{d}_1[n] = \sum_{i=0}^{k-1} h_i E\{x[n+k-i]|\mathbf{x}_M\} \quad (14)$$

$E\{x[n+k-i]|\mathbf{x}_M\}$ is an optimal predictor of $x[n+k-i]$. To predict $x[n+k-i]$ a RBF network or a VS may be implemented. Thus the prediction of $x[n+k-i]$ can be estimated by

$$E\{x[n+k-i]|\mathbf{x}_M\} \doteq \sum_{j=1}^L \mu_{ij} \Phi_j(\mathbf{x}_M[n]) \quad (15)$$

where L is the number of the nonlinear kernels $\{\Phi_j\}$ and linear weights $\{\mu_{ij}\}$. Therefore an estimate of $d_1[n]$ can be found as

$$\hat{d}_1[n] = \sum_{i=0}^{k-1} h_i \sum_{j=1}^L \mu_{ij} \Phi_j(\mathbf{x}_M[n]) \quad (16)$$

and merging the two linear layers $\{h_i\}$ and $\{\mu_{ij}\}$ together, (16) becomes

$$\hat{d}_1[n] = \sum_{j=1}^L w_j \Phi_j(\mathbf{x}_M[n]) \quad (17)$$

finally an estimate of $d[n]$ can be found as

$$E\{d[n]|\mathbf{x}_N\} \doteq \sum_{j=1}^L w_j \Phi_j(\mathbf{x}_M[n]) + \sum_{i=k}^{N-1} h_i x[n+k-i] \quad (18)$$

as the input vector $\mathbf{x}_N[n]$ does not contain any future values k may be set to zero. The linear combiner will have the form

$$y[n] = [\Phi_1, \Phi_2, \dots, \Phi_L, x[n], x[n-1], \dots, x[n-N+1]] \begin{bmatrix} w_1, w_2, \dots, w_L, h_0, h_1, \dots, h_{N-1} \end{bmatrix}^T \quad (19)$$

The linear weights $\{w_i\}$ and $\{h_i\}$ in (19) are estimated by a least squares algorithm.

The linear system identification part in (19) requires an input signal $\mathbf{x}_N[n]$ which is broadband. As long as the noise excites all the frequencies necessary to identify the linear system, the linear part does not require any assumptions about the distribution or nonlinear deterministic characteristics in the input signal $\mathbf{x}_N[n]$.

The nonlinear prediction part, though, requires that there is some nonlinear mapping between adjacent input samples.

If the noise is nonlinear and deterministic, i.e. a chaotic time series, it is possible to predict the noise in the short term [4]. As the function f in $x[n+k] = f(\mathbf{x}_M[n])$ is generally nonlinear to generate chaos, it is necessary to use a nonlinear model for the k-step prediction task. Chaotic time series which arise from dynamical systems are clearly non-Gaussian but are also deterministic. It is possible to embed the finite dimensional manifold θ of the dynamical system with the methods of delays [4]. As the system evolves through time the measured values of the time series in the tapped delay line describe a trajectory in an embedded state space. This embedded attractor does not occupy the whole state space. This is in strong contrast to some stochastic processes, which fill out more or less the whole state space.

3. NONLINEAR CONTROL OF LINEAR PLANT

The acoustic delay, modelled by $H_e(z)$, can cause stability problems in the control algorithm. To circumvent the stability problems the linear adaptive FIR controller in Figure 2 is usually trained by the filtered-x LMS algorithm. The derivation of the filtered-x LMS algorithm may be found in [7]. In [7] the authors point out that the filtered-x LMS algorithm will only converge if the adaptation process is slow and that the controller is linear. In the filtered-x LMS algorithm both the input $x[n]$ and plant output $e[n]$ are filtered implicitly before being presented to an adaptive filter algorithm. While this filtering process or linear mapping is effectively cancelled if the system is linear, it distorts observations of a nonlinear system.

To circumvent these restrictions the ANC system was redesigned to allow a nonlinear system identification. The block diagram is shown in Figure 4. $P(z)$ is the transfer function of the duct plant between the detection microphone and the control source. $H_a(z)$ is the transfer function of the actuator, which also represents a driving unit and a reconstruction filter for the D/A conversion. The acoustic path, between the control source and error microphone, and the error microphone itself is given as the transfer function $H_e(z)$. The acoustic feedback path $F(z)$ in the duct is neglected for the two following reasons. For the first it is assumed that the off-line modelled filter $\hat{F}(z)\hat{H}_a(z)$ eliminates the feedback and secondly that uni-directional microphones are used. The noise, caused by turbulence in the duct on the microphones, is assumed to be zero.

To get back to the actual error signal $e[n]$ it is necessary to build the transfer function $(\hat{H}_a(z)\hat{H}_e(z))^{-1}$ into the error path. The inverse of $H_a(z)H_e(z)$ is modelled off-line, using only the loudspeaker as a white noise source. Fortunately it is possible to use an inverse modelling delay to estimate the inverse more accurately. To compensate the inverse modelling delay the signals $x[n]$ and $y[n]$ are delayed by the same delay z^{-m} . The signals $y[n-m]$ and $e'[n-m]$ are added

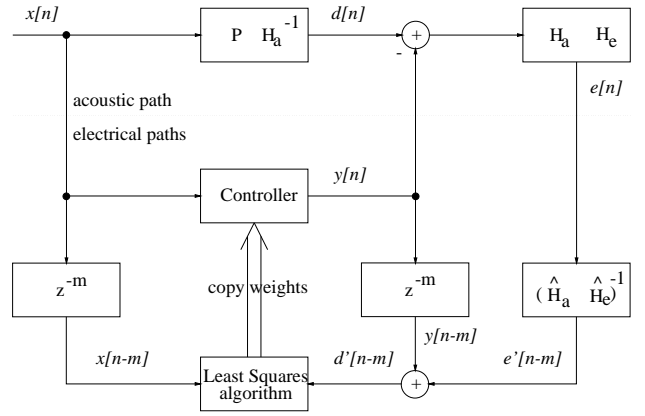


Figure 4: Block diagram of ANC system used in simulations up to form the desired signal $d'[n-m]$. The delayed input signal $x[n-m]$ and the delayed desired signal $d'[n-m]$ are supplied to a least squares algorithm. Another advantage in using this scheme, as shown in Figure 4, is that more sophisticated least squares algorithms (Householder transformations, SVD) can be implemented, which are especially useful when dealing with a nonlinear controller and a coloured input $\mathbf{x}_N[n]$.

4. SIMULATIONS AND RESULTS

The following simulations are based on the block diagram in Figure 4. Six different input noise signals $x[n]$ with zero mean and variance $\sigma_x^2 = 1.0$ are used: White Gaussian noise, white uniform noise, coloured Gaussian noise, Logistic chaotic noise [8], Lorenz chaotic noise [9] and Duffing chaotic noise [10]. The coloured Gaussian noise is the white Gaussian noise filtered by a 2nd order lowpass IIR filter with a normalised cut-off frequency of 0.05. The coloured Gaussian noise is normalised to have zero mean and unit variance.

Two different nonlinear models were investigated. The first one is a truncated and modified Volterra filter with only quadratic and cubic terms. The second one is a normalised Gaussian radial basis function (NRBF) network [6]. Both of the nonlinear models are linear in their parameters and are, therefore, easy to train, compared to neural networks like multilayer perceptron (MLP) networks. For comparison in performance three different controllers were investigated.

- Linear FIR filter with $N = 10$ coefficients
- Combined linear ($N = 10$) and Volterra (quadratic, cubic) filter
- Combined linear ($N = 10$) and NRBF ($L_{Logistic} = 337, L_{Lorenz} = 159, L_{Duffing} = 126$) network

The input vector $\mathbf{x}_M[n]$ has a dimension of $M = 3$ in Table 1. In Tables 2 - 4 the dimension is $M = 5$. The main acoustic plant $P(z)$ is a 4th order FIR filter [11].

$$P(z) = z^{-2} - 0.3z^{-3} + 0.2z^{-4}$$

The controller $H_c(z)$ does not depend on the error path transfer function $H_e(z)$. Therefore, only one error path $H_e(z)$ is used, $H_e(z) = z^{-5}$. Three actuators $H_a(z)$ with different phase characteristics were chosen. Their transfer functions are shown in the Tables 1 - 4.

The inverses of all combinations of $H_a(z)H_e(z)$ are estimated by exciting white noise through $H_a(z)H_e(z)$ and training an adaptive 32nd order FIR filter. The inverse modelling delay z^{-m} is $m = 16$ for $H_{a1}(z)H_e(z)$ and $H_{a3}(z)H_e(z)$ and $m = 5$ for $H_{a2}(z)H_e(z)$.

The least squares algorithm is a block least squares algorithm using the Householder transformation. The normalised mean square error (NMSE) is computed, after convergence, as follows :

$$\text{NMSE} = 10 \log_{10} \left(\frac{\sigma_e^2}{\sigma_d^2} \right) \text{ dB}$$

where σ_e^2 is the variance of $e[n]$ and σ_d^2 is the variance of $d[n]$.

In the simulation where the input signal $x[n]$ is white Gaussian, white uniform or coloured Gaussian a purely linear controller achieves the best result. The results are provided in Table 1. The results for the white noise processes are provided as a basis for comparison since no linear or nonlinear predictive component is possible. When coloured Gaussian noise is used linear prediction improves the performance for the maximum phase actuator $H_{a1}(z)$ and nonminimum phase actuator $H_{a3}(z)$. The results for $H_{a2}(z)$ with a coloured input demonstrates a marked improvement in performance. At first sight, this may appear surprising. The actuator is minimum phase and thus no predictive component is required. However this is not a straightforward system identification task. The achievable NMSE is dependent on both the spectrum of the input signal and the transfer function of the actuator. Therefore there is no reason to expect the elements of column 4 in Table 1 to be the same as in the columns 2 and 3.

The situation changes dramatically when the input signal $x[n]$ is non-Gaussian and deterministic. Both combined linear and nonlinear controllers in Tables 3 and 4 achieve far better performances in conjunction with the maximum phase actuator $H_{a1}(z)$ and the nonminimum phase actuator $H_{a3}(z)$ compared to the linear controller in Table 2. The results for the Duffing equation are initially surprising especially with respect to the minimum phase actuator $H_{a2}(z)$. However the architecture of (18) only provides a FIR approximation to $H_{a2}^{-1}(z)$. The additional terms could be provided through backward prediction using the embedding vector $\mathbf{x}_N[n]$. The architecture is flexible enough to provide this prediction implicitly without interference from the user. This is a topic of current investigation.

5. CONCLUSIONS

A new active noise canceller architecture has been developed from the perspective of Bayesian estimation theory. This architecture exploits the non-Gaussian nature of the noise source to alleviate the effects of nonminimum phase actuator transfer functions. Simulation results have demonstrated that this architecture offers a significant performance improvement with respect to traditional linear controllers.

Actuator $H_a(z)$	Linear		
	Gauss	Uniform	Colour
$H_{a1}(z) = 0.5 + z^{-1}$	-11	-11	-43
$H_{a2}(z) = 1.0 + 0.5z^{-1}$	-41	-41	-74
$H_{a3}(z) = 1.0 + 1.5z^{-1} - z^{-2}$	-12	-12	-45

Table 1: NMSE in dB using a linear controller and stochastic noise for $x[n]$

Actuator $H_a(z)$	Linear		
	Logistic	Lorenz	Duffing
$H_{a1}(z) = 0.5 + z^{-1}$	-10.8	-34.6	-70
$H_{a2}(z) = 1.0 + 0.5z^{-1}$	-41	-64.8	-99.7
$H_{a3}(z) = 1.0 + 1.5z^{-1} - z^{-2}$	-12	-35.6	-66.6

Table 2: NMSE in dB using a linear controller and chaotic noise for $x[n]$

Actuator $H_a(z)$	Linear + Volterra		
	Logistic	Lorenz	Duffing
$H_{a1}(z) = 0.5 + z^{-1}$	-16.8	-50.5	-91
$H_{a2}(z) = 1.0 + 0.5z^{-1}$	-41	-65.1	-105
$H_{a3}(z) = 1.0 + 1.5z^{-1} - z^{-2}$	-17.9	-50.5	-72.8

Table 3: NMSE in dB using a combined linear and nonlinear controller and chaotic noise for $x[n]$

Actuator $H_a(z)$	Linear + NRBF		
	Logistic	Lorenz	Duffing
$H_{a1}(z) = 0.5 + z^{-1}$	-28.5	-55.8	-89
$H_{a2}(z) = 1.0 + 0.5z^{-1}$	-40.5	-65.6	-105.2
$H_{a3}(z) = 1.0 + 1.5z^{-1} - z^{-2}$	-29.6	-53.5	-75

Table 4: NMSE in dB using a combined linear and nonlinear controller and chaotic noise for $x[n]$

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