

A WAVEGUIDE MODEL FOR SLAPBASS SYNTHESIS

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ABSTRACT

Starting from the waveguide model for plucked strings, a new digital signal processing model for the slapping technique on electric bassguitars is derived. The model includes amplitude limitations for the string at the frets and/or the fingerboard. These highly nonlinear elements are realized by conditional reflections which depend on the local string displacement. A model of the string dynamics for the two slapbass techniques — knocking the string with the thumb knuckle and plucking very strong with the index or middle finger — has been implemented both as MATLAB and C simulations and synthesizes sounds close to the natural instrument.

1. MODELING PLUCKED STRINGS

One of the simplest musical instruments amenable to *physical modeling* is the plucked string. The string is modeled as a linear waveguide: two waves are traveling along the string, one in either direction. As a digital representation, simple delay lines can be used. Rigid terminations yield inverted reflections of the waves (for displacement, velocity, and acceleration as state variables of the waveguide model) at the ends of the string. In a more general setting [1], these reflections are implemented by filters which can handle dispersion at the reflection and along the string, too.

The representation of a waveguide as a discrete-time delay line corresponds to spatial sampling. The waveguide section index m represents the distance x along the string. Obviously, the best choice for the spatial sampling distance is $x_s = c/f_s$, with the (phase) velocity c of the waves on the string and the temporal sampling rate f_s in the model, chosen according to the highest frequency of interest. Then, a wave travels the distance x_s on the waveguide in the time of one sampling interval. The frequency of the synthesized tone depends on the number of delay units in the delay line N as follows: $f = f_s/2N$.

The ‘Karplus–Strong’ algorithm [2] uses random initial conditions for the delay lines, corresponding to random initial displacement and velocity of the string, whereas more recent plucked-string models [1, 3, 4, 5] start with a triangular initial displacement of the string and zero velocity.

The reflection filters have low-pass characteristic whether they are FIR like in the ‘Karplus–Strong’ algorithm or IIR

filters like in the plucked-string model [1]. According to physics, the DC gain of the reflection filter at either end of the string must be -1 . The filter transfer function then controls the decay of the tone and its harmonics.

For electric guitars, a (magnetic) pickup transforms the velocity of the string in an electric signal that feeds the amplifier. For any choice of the state variable in the waveguide model (displacement, velocity, or acceleration), the pickup signal can be generated by taking the sum of the two delay line signals at the pickup position and, unless velocity is the state variable, converting it to a velocity by derivation or integration.

2. PHYSICS OF THE SLAPBASS

With the slapping technique, a brilliant and percussive sound can be produced on electric bassguitars. There are two different slapping techniques: the string is either struck (‘slapped’) with the knuckle of the thumb or it is pulled strongly away from the guitar body with the index or middle finger. In both cases, the string hits the frets during the first fundamental periods of the tone. The strongly pulled string can be modeled in the same way as the plucked string above using a triangular initial displacement. Slapping with the knuckle is very similar to striking a piano string with a hammer [6] and can be modeled as an initial velocity impulse at the striking position.

When the string excitation is so strong that it hits the frets, this results in a nonlinear limitation of the string amplitude to the space above the frets and the fingerboard. A model of this amplitude limitation requires to test which part of the string actually hits the frets or the fingerboard. The test can be implemented in a straight-forward manner if the string displacement is chosen as the state variable of the waveguides. To realize this amplitude limitation, the samples between two delay elements are reflected into the delay line that travels in the opposite direction if the sum exceeds the free distance to the fret. When the samples are inverted and the distance of the fret is added, like in figure 1, the amplitude y of the string displacement at a certain position x is limited to $y_{\text{fret}}(x)$, which describes the geometry of fingerboard and frets (the fingerboard is below the string: $y_{\text{fret}}(x) < 0$). For the discrete-time simulation the continuous space variable x is replaced by the waveguide section index m .

To obtain the pickup signal, the time derivative of the string displacement (i.e., the sum of the samples in the de-

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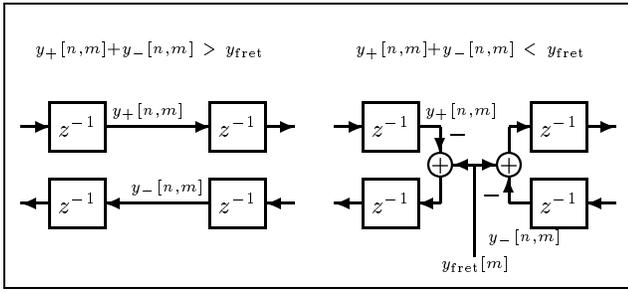


Figure 1. Model of string amplitude limitation as displacement conditional reflection. $y_+[n, m]$ and $y_-[n, m]$ are the forward and backward traveling displacement waves, respectively, where n is the time index and m the waveguide section index.

lay lines) has to be computed at the pickup position. In the model, a simple discrete-time approximation for the derivative is used: an FIR filter with frequency response $1 - z^{-1}$.

A magnetic pickup cannot be supposed to sense the velocity of just one point along the string and simulations show unrealistic high frequency components in this case. One remedy is to put a lowpass filter at the output. On the other hand, one can use spatially distributed pickup models to take into account the nonfinite extensions of the magnetic field of the pickup. A gaussian weighted tapping of the sum in the delay lines centered at the pickup position provides an appropriate damping at high frequencies.

3. SIGNAL PROCESSING IMPLEMENTATION

The complete model for the slabpass consists of a waveguide with amplitude limitation, two reflection filters at the ends of the waveguide and a pickup subcircuit, and it is shown in figure 3. The model has no explicit input, the only driving energy source are the (forced) initial conditions of the delay elements which are set according to the initial displacement and velocity of the string for each sound to be synthesized. For plucking the string, a triangular initial displacement is set up which is achieved by initializing both delay lines with a triangular function of half the total amplitude (state variable is displacement). The equal values in both delay lines correspond to zero initial velocity (see [1]). On the other hand, when the string is slapped a zero initial displacement and a velocity pulse at the slapping position are induced. So the sum of the initial delay line values must be zero and the difference is set proportional to the cumulative sum of the velocity pulse (corresponding to displacement being the integral of velocity). A plot of initial displacement and velocity for a plucked and a slapped string is shown in figure 2. To avoid unrealistic, over-idealized initial conditions, the edges of the initial velocity impulse and the corner of the triangular initial displacement are rounded.

The ‘amplitude limiting’ two-ports, which contain the reflection according to figure 1, are inserted into the waveguide delay lines in this part which corresponds to the fingerboard. They can be inserted between every delay element or only at a wider spacing. The former enables a detailed modeling of the fingerboard geometry while the latter reduces computational effort. We use a linear function for

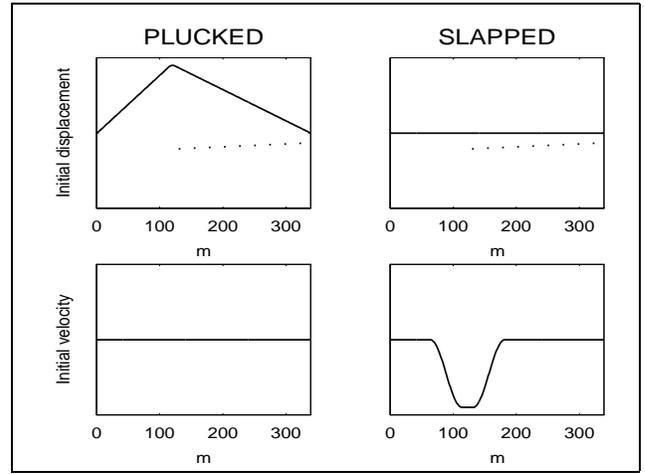


Figure 2. Initial conditions for the plucked and the slapped bassguitar. Initial position and initial velocity over the waveguide section index m are shown for sound C at 65.4 Hz. In the plots of initial position the dotted line shows the position of the fingerboard $y_{fret}[m]$.

$y_{fret}(x)$ with the distance between the strings initial position and the fingerboard at the body end and at the head end as parameters.

The computational cost is mainly determined by the number of ‘limiting’ blocks: Each block uses one addition to determine the sum of the displacement waves then the sum is compared to the limiting value and, in case limitation takes place, another two additions are necessary. If the fingerboard extends over 3/4 of the length of the string there are approximately 800 unit delays of the waveguides in the area of the fingerboard for the lowest tone (E1 at 41.2 Hz) on a bassguitar¹. Neglecting the operations for the reflection filters, the distributed pickup and the output filter and the waveguide indexing, a number of 3200 operations per sampling interval is necessary. This number can be reduced by placing ‘limiting’ blocks only at a wider spacing in the area of the fingerboard — e.g. at the positions of the frets. A linear spacing with $n_i = 8, i = 3 \dots k$ brings a reduction in total computational cost of nearly the factor 8, but does not affect the perceived sound quality. With the above parameters, a PC with 486DX2 processor at 66 MHz takes approximately twice real time for synthesizing (using a C-program without specific optimization).

The reflection filters can be rather simple: an ideal reflection ($R_L(z) = -1$) at the left end and a single-pole low-pass filter $R_R(z)$ at the right end is used in our model. In absence of amplitude limitation, the location of this pole—or, in general, the frequency characteristics of the reflection filters—controls the decay of the generated tone and its harmonics. By only varying the pole location, a variety of timbres can be produced, from fast decaying acoustic guitar sounds to electric guitar timbres with long sustain.

Especially in a steel string the dispersion of waves along the string is notably strong [6]. The stiffness of the string leads to a higher phase velocity at high frequencies and causes noninteger harmonics. With proper phase charac-

¹If a sampling rate of $f_s = 44.1\text{kHz}$ is used.

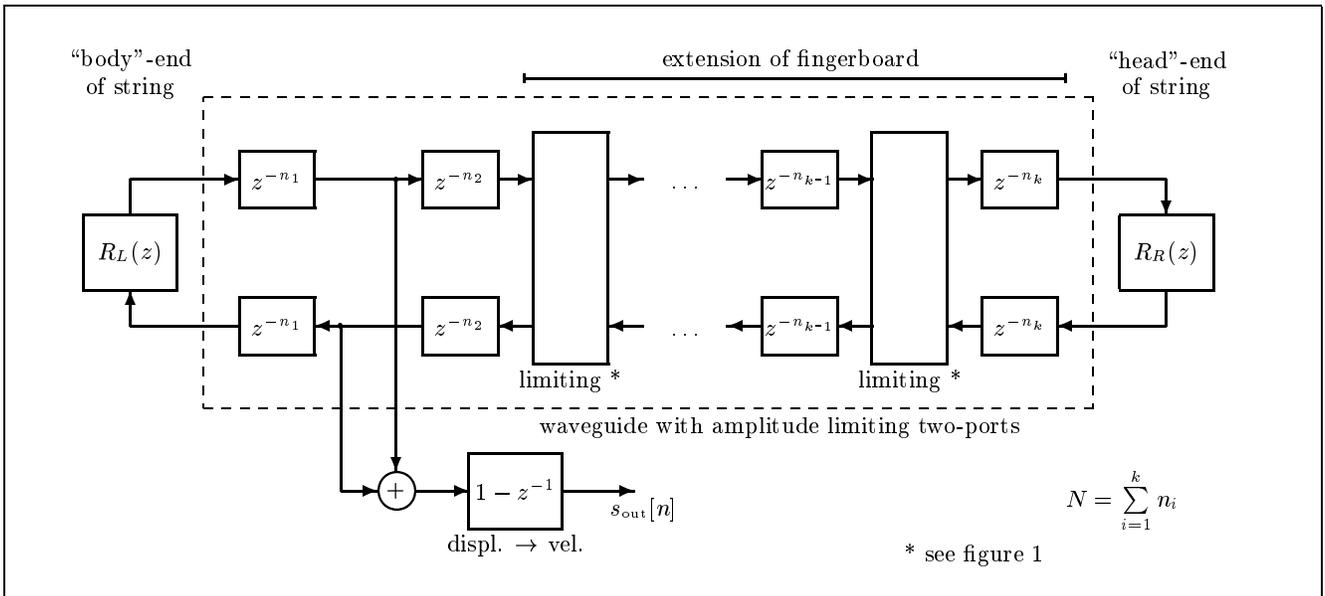


Figure 3. Complete digital simulation model for the slabass. The two-ports labeled ‘limiting’ are implemented according to figure 1. For accurate modeling of the fingerboard surface, the waveguide sections with amplitude limitation (about 2/3 of the length of the string) should have unit delay, i.e., $n_3 = n_4 = \dots = n_k = 1$. Reasonable reduction in computational cost is obtained by reducing the number of ‘limiting’ blocks and placing them at proper positions — e.g. uniformly spaced — by allowing $n_3, \dots, n_k > 1$.

teristics of the reflection filters this effect can be taken into account in the model. Experiments with a few reflection filters with lower group delay at high frequencies than at low frequencies did however not result in an audible change so we kept the simple reflection filters described above.

The limitation amplitude in the two-ports along the waveguide (y_{fret} in figure 1) has a much more significant influence on the slabass sound. To achieve the typical timbre, the wave must be limited at least during the first period at the last fret at the body end of the fingerboard, but in the natural instrument a limitation takes place at other frets, too, and produces additional noise pulses in the following periods. The limitation amplitude has to be reduced towards the end of the fingerboard (the ‘first’ fret)—but not down to zero, or the oscillation is damped in a few periods.

4. SIMULATION RESULTS

The output signals of this model have the following characteristic features that are also found in natural slabass signals (figures 4–5):

In the first period corresponding to the nominal frequency, additional fast reflections (which are numbered as 1, 2, and 3 in figure 4) are present. These reflections originate from two different sources: first, the reflection at the body end of the fingerboard (the last fret) when the string amplitude is limited there. The delay of this reflection (number 1 in figure 4) corresponds to the length between the end of the string at the guitar body and the end of the fingerboard. The reflections 2 and 3 in figure 4 can be explained from the subsequent recoil of the string at the knuckle as they are not seen in the natural signal in figure 5 where the string is plucked but not struck. A reflection pattern similar to the one in the real signal in figure 4 oc-

curs for a string on a piano that is struck with the piano hammer [6]: after hitting the string the hammer does not immediately bounce back to its resting position but stays in vicinity to the string’s initial position and can be seen as another conditional reflection point. In contrary to the reflections at the frets, the reflections at the hammer are not reflections at a rigid termination and only temporary, because the hammer is pushed back in its resting position by the string. At present, such secondary excitations are not included in our model (cf. the simulated signal in figure 4).

In the first two periods, an additional broadband noise pulse is generated that is due to the limiting at the other frets along the fingerboard (‘fret noise’). This effect is particularly pronounced in the example shown in figure 5. After roughly the second period the shape of the time-domain signal remains nearly periodic, with a decaying envelope that is caused by the dispersion of the reflection filters. Apart from this decay, both the signal components due to the fingerboard reflection and due to the fret noise recur almost periodically.

The spectrograms in figure 6 show that the correspondence between the natural and synthetic signals is better than one might guess from inspecting the time-domain waveforms in figure 4 alone. The initial broadband pulse and the pattern due to the amplitude limitation evolve over the first few periods and then are repeated unchanged except for faster decaying high frequency components. The artificial signal shows a somewhat more regular structure, both in the time and the frequency direction. This may be caused by the missing reflections at the knuckle which introduce new phase relations in the real signal or by non-linear effects in the string itself due to its strong excitation.

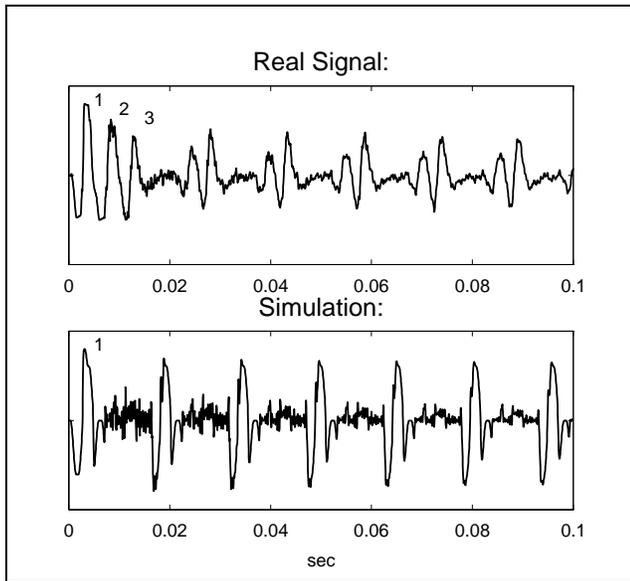


Figure 4. The slapped bassguitar (struck by knuckle). Comparison between natural and simulated signal for sound C at 65.4 Hz.

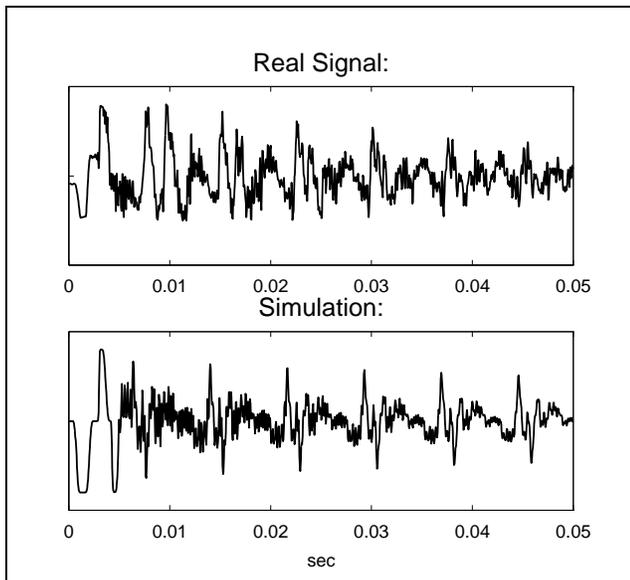


Figure 5. The 'heavily plucked' bassguitar (string initially pulled strongly away from rest). Comparison between natural and simulated signal for sound c at 130.8 Hz.

The high quality of the synthesized sounds is confirmed by informal listening experiments.

5. CONCLUSION

A new model of the slapping technique for electric bassguitars has been presented. To the authors' knowledge, no such model has been documented in the archival literature before. The model includes the nonlinear limitation effects at the frets and the fingerboard. It is implemented as a waveguide simulation where, unlike most other guitar models, the string displacement is used as the state variable.

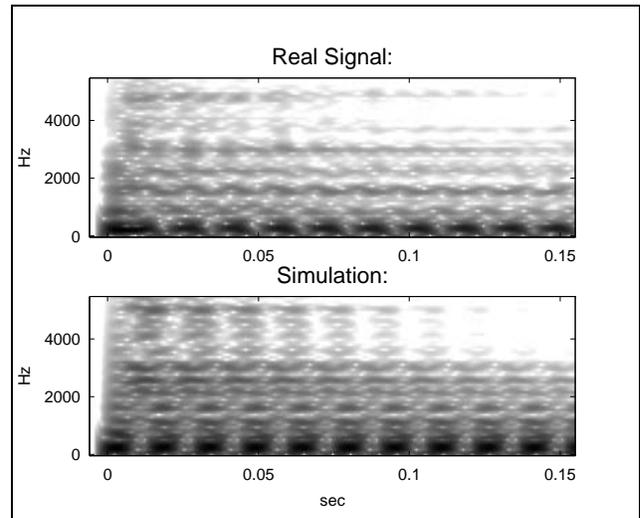


Figure 6. Spectrograms of signals in figure 4.

The model offers an efficient signal processing implementation and produces realistic sounds. As such, it is suitable for applications in multimedia PC's and virtual instruments [7].

REFERENCES

- [1] J. O. Smith III, "Physical modeling using digital waveguides," *Computer Music J.*, vol. 16, no. 4, pp. 74-91, 1992.
- [2] K. Karplus and A. Strong, "Digital synthesis of plucked string and drum timbres," *Computer Music J.*, vol. 7, no. 2, pp. 43-55, 1983.
- [3] M. Karjalainen and U. K. Laine, "A model for real-time sound synthesis of guitar on a floating-point signal processor," in *Proc. Int. Conf. Acoust. Speech Sign. Process.*, (Toronto, ON), pp. 3653-3656, May 1991.
- [4] M. Karjalainen, V. Välimäki, and Z. Jánosy, "Towards high-quality sound synthesis of the guitar and string instruments," in *Proc. International Computer Music Conference (ICMC'93)*, (Tokyo, Japan), pp. 56-63, 1993.
- [5] M. Karjalainen *et al.*, "Modeling and synthesis of string instruments," WWW document <http://www.hut.fi/TKK/Akustiikka/stringmodels/strings.html>, Acoustics Laboratory, Helsinki University of Technology, Espoo (Finland), Mar. 1995.
- [6] A. Askenfelt and E. V. Jansson, "From touch to string vibrations. III: String motion and spectra," *J. Acoust. Soc. Am.*, vol. 93, no. 4, Pt. 1, pp. 2181-2196, 1993.
- [7] J. Guerard and X. Boutillon, "Bowing virtual strings," in *Proc. Int. Conf. Acoust. Speech Sign. Process.*, (Atlanta, GA), pp. 1756-1759, May 1996.