BARANKIN BOUND FOR SOURCE LOCALIZATION IN SHALLOW WATER

Joseph Tabrikian and Jeffrey L. Krolik

Department of Electrical and Computer Engineering,
Duke University, Durham, NC 27708-0291, USA
joseph@ee.duke.edu

ABSTRACT

Matched-field methods are known to have a severe ambiguity problem. In low signal-to-noise-ratios (SNR's), where the estimator cannot distinguish between the ambiguity function peak near the true source location and ambiguous ones, its mean square error deviates radically from the Cramer-Rao lower bound (CRLB). In this paper, the Barankin bound for the source localization problem in an uncertain shallow water environment is derived. In particular, a method of selection of the test-points for evaluation of the bound is presented. The bound is evaluated using a "general mismatch" benchmark scenario. The results presented here predict the threshold SNR below which the performance degrades dramatically. Channel uncertainties in the benchmark scerario are shown to increase this threshold SNR by as much as 3dB.

1. INTRODUCTION

This paper concerns source localization with a vertical line array in a complex multipath ocean channel, as illustrated in Fig. 1. Localization algorithms which use full wave acoustic propagation models of complex multipath conditions, known as matched-field methods, are effective at higher signal-to-noise-ratios (SNR's) but suffer from severe ambiguities at lower SNR's. Further, in the presence of unknown channel parameters the ambiguity problem is even more significant.

A common tool for evaluating the achievable performance of a parameter estimation algorithm is the Cramer Rao Lower Bound (CRLB) [1], [3] and [6]. The use of the CRLB is usually justified by appealing to an asymptotic theorem which asserts that the CRLB can be closely approached by the maximum-likelihood estimator under asymptotic conditions: i.e. "sufficiently large" (SNR) and/or observation time. Actually, in low SNRs where the estimator is prone to ambiguous estimates, the mean square error of unbiased estimators deviates radically from the CRLB as the SNR is reduced, exhibiting a threshold phenomenon [6]. Thus below the threshold, the CRLB is no longer useful. The threshold SNR for a given processor is an important measure of its performance. Moreover, it is most important to be able to establish the non-asymptotic optimality of an algorithm. In this paper,we use the Barankin bound [2] to predict the achievable performance of any unbiased localization algorithm in non-asymptotic conditions. In particular, an algorithm for which the threshold SNR is similar to that of the Barankin bound is presented. In [8] the Barankin bound is studied for different scenarios and it is compared to the non-asymptotic performance of some well known localization algorithms.

This paper is organized as follows: In section 2, the problem is defined and formulated. Section 3 presents the Barankin bound for this problem. Section 4 describes techniques for reducing the amount of computations required to calculate the bound. Section 5 presents the results using computer simulations

2. PROBLEM FORMULATION

Consider a point source at depth z_o and range r_o which radiates a monochromatic signal at angular frequency ω in a time invariant shallow-water waveguide. The acoustic field is sampled by a vertical array of N sensors. The depth of the ith sensor from the upper surface is denoted by z_i. The sensor locations are assumed to be known. Fig. 1 depicts the environmental configuration and the source-array geometry. The environmental scenario is one of the more complex benchmark cases used in the May 1993 NRL Workshop on Acoustic Models in Signal Processing [5].

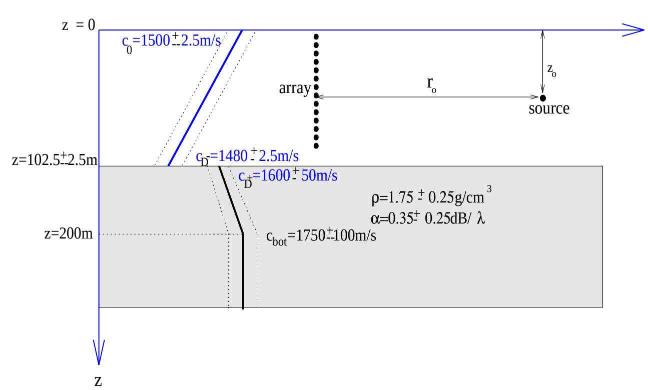

Figure 1. The NRL workshop "genlmis" scenario configuration

Using a normal mode model, the complex amplitude of the received signal at sensor i can be expressed by (see e.g. [9]):

$$y_i = b \sum_{m=1}^{M} \phi_m(z_i)\phi_m(z_o)\frac{e^{j\kappa_m r_o}}{\sqrt{\kappa_m r_o}} + n_i \ , \quad i = 1, \cdots, N \ , \ (1)$$

where $\phi_m(\cdot)$ and κ_m are the modal eigenfunction and horizontal wavenumber of the mth mode, respectively. M is the number of the propagating modes in the channel, and n_i stands for the additive noise complex amplitude at the ith sensor. The received noise at the sensors is assumed to be zero-mean, Gaussian, with known covariance matrix, $\mathbf{R_n}$.

The modal eigenfunctions, $\phi_m(z_i)$, and horizontal wavenumbers, κ_m, depend on environmental parameters which describe the bathymetry, geo-acoustic properties of the bottom, and sound-speed in the water column. In practice, these parameters are not precisely known. The uncertain environmental parameters considered in this paper, are shown in Fig. 1. The objective here is to estimate the source location parameters (r_o, z_o) from the measurement vector $\mathbf{y} \triangleq [y_1, \cdots, y_N]^T$, when the complex signal amplitude b is unknown, and the environment is uncertain. The vector of unknown parameters, $\mathbf{\Theta}$ includes the source range and depth, the real and imaginary parts of the signal complex amplitude, and the vector of environmental parameters.

3. THE BARANKIN BOUND

The Barankin bound for mean square error of any unbiased estimator of $\mathbf{\Theta}$ from the measurement vector $\mathbf{y}$ is given by [2]:

$$cov\{\hat{\mathbf{\Theta}}\} \geq \mathbf{T}\left(\mathbf{B} - \mathbf{1}\mathbf{1}^T\right)^{-1}\mathbf{T}^T \tag{2}$$

where $\hat{\mathbf{\Theta}}$ is the unbiased estimate of $\mathbf{\Theta}$. The matrix $\mathbf{T}$ is defined as

$$\mathbf{T} \triangleq [(\mathbf{\Theta}_1 - \mathbf{\Theta}) \ \ (\mathbf{\Theta}_2 - \mathbf{\Theta}) \cdots (\mathbf{\Theta}_K - \mathbf{\Theta})] \ . \tag{3}$$

To maximize the bound, the values of $\mathbf{\Theta}_k$ should be chosen near values of $\mathbf{\Theta}$ corresponding to location ambiguities. They can be pre-determined, or they can be found by maximizing the right hand side of (2). The elements of the matrix $\mathbf{B}$ are given by

$$B_{ij}(\mathbf{\Theta}) \triangleq E\left\{L(\mathbf{y}, \mathbf{\Theta}_i, \mathbf{\Theta}) L(\mathbf{y}, \mathbf{\Theta}_j, \mathbf{\Theta})\right\} \ , \quad i, j = 1, \cdots, K \ , \tag{4}$$

where $L(\mathbf{y}, \mathbf{\Theta}_i, \mathbf{\Theta})$ is the likelihood ratio:

$$L(\mathbf{y}, \mathbf{\Theta}_i, \mathbf{\Theta}) \triangleq \frac{f(\mathbf{y}|\mathbf{\Theta}_i)}{f(\mathbf{y}|\mathbf{\Theta})}. \tag{5}$$

For our problem, consider the following data model:

$$\mathbf{y} = \mathbf{a}(\mathbf{\Theta}) + \mathbf{n} \ , \tag{6}$$

where $\mathbf{a}(\cdot)$ is a known vector function representing the channel spatial transfer function, and $\mathbf{n}$ is an additive, zero-mean, Gaussian noise with spatial covariance matrix $\mathbf{R_n}$. Therefore, the conditional probability distribution of $\mathbf{y}$, given the parameter vector $\mathbf{\Theta}$ is:

$$f(\mathbf{y}|\mathbf{\Theta}_i) = \frac{1}{det(\pi \mathbf{R_n})} exp\left[-(\mathbf{y} - \mathbf{a}(\mathbf{\Theta}))^H \mathbf{R_n}^{-1} (\mathbf{y} - \mathbf{a}(\mathbf{\Theta}))\right] \ . \tag{7}$$

Substitution of (7) into (5) for evaluation of (4) yields:

$$B_{ij}(\mathbf{\Theta}) = \frac{1}{det(\pi \mathbf{R_n})} \int_{-\infty}^{\infty} exp\left(C_{ij}(\mathbf{\Theta})\right) d\mathbf{y} \ , \tag{8}$$

where $C_{ij}(\mathbf{\Theta})$ is defined as:

$$\begin{aligned} C_{ij}(\mathbf{\Theta}) \ &= (\mathbf{y} - \mathbf{a}(\mathbf{\Theta}_i))^H \mathbf{R_n}^{-1} (\mathbf{y} - \mathbf{a}(\mathbf{\Theta}_i)) \\ &+ (\mathbf{y} - \mathbf{a}(\mathbf{\Theta}_j))^H \mathbf{R_n}^{-1} (\mathbf{y} - \mathbf{a}(\mathbf{\Theta}_j)) \\ &- (\mathbf{y} - \mathbf{a}(\mathbf{\Theta}))^H \mathbf{R_n}^{-1} (\mathbf{y} - \mathbf{a}(\mathbf{\Theta})) \ . \end{aligned} \tag{9}$$

Plugging (9) into (8), and after a few lines of algebra, one obtains:

$$\begin{aligned} B_{ij}(\mathbf{\Theta}) = G exp\Big\{2Re\Big[&\mathbf{a}^H(\mathbf{\Theta}_i)\mathbf{R_n}^{-1}\mathbf{a}(\mathbf{\Theta}_j) - \mathbf{a}^H(\mathbf{\Theta})\mathbf{R_n}^{-1}\mathbf{a}(\mathbf{\Theta}_i) \\ &- \mathbf{a}^H(\mathbf{\Theta})\mathbf{R_n}^{-1}\mathbf{a}(\mathbf{\Theta}_j) + \mathbf{a}^H(\mathbf{\Theta})\mathbf{R_n}^{-1}\mathbf{a}(\mathbf{\Theta})\Big]\Big\} \end{aligned} \tag{10}$$

where G is defined by

$$\begin{aligned} G \triangleq \ & \tfrac{1}{det(\pi \mathbf{R_n})} \\ & \textstyle\int_{-\infty}^{\infty} exp\left[-(\mathbf{y} - \mathbf{a}(\mathbf{\Theta}_i) - \mathbf{a}(\mathbf{\Theta}_j) + \mathbf{a}(\mathbf{\Theta}))^H \mathbf{R_n}^{-1}\right. \\ & \left.(\mathbf{y} - \mathbf{a}(\mathbf{\Theta}_i) - \mathbf{a}(\mathbf{\Theta}_j) + \mathbf{a}(\mathbf{\Theta}))\right] d\mathbf{y} \ . \end{aligned} \tag{11}$$

G is an integral from $-\infty$ to ∞ of a Gaussian probability density function with covariance matrix $\mathbf{R_n}$ and mean $\mathbf{a}(\mathbf{\Theta}_i) + \mathbf{a}(\mathbf{\Theta}_j) - \mathbf{a}(\mathbf{\Theta})$. Therefore $G = 1$, and $B_{ij}(\mathbf{\Theta})$ from (10) becomes:

$$B_{ij}(\mathbf{\Theta}) = exp\left[(\mathbf{a}(\mathbf{\Theta}) - \mathbf{a}(\mathbf{\Theta}_i)])^H \mathbf{R_n}^{-1} \left(\mathbf{a}(\mathbf{\Theta}) - \mathbf{a}(\mathbf{\Theta}_j)\right)\right] . \tag{12}$$

Now the bound can be evaluated by using expression (12) into (2). The key point for achieving a tight bound is proper selection of the test-points. In particular, in an ocean environment the number of unknown parameters is large. Therefore, calculation of inverse of the matrix $\mathbf{B}(\mathbf{\Theta})$ evaluated according to a multidimensional grid of test points is required. In the next section, reduction in the number of computations and selection of the test-points will be discussed.

4. EVALUATION OF THE BOUND FOR SHALLOW WATER SOURCE LOCALIZATION PROBLEM

Environmental uncertainties cause errors in the modal horizontal wavenumber and eigenfunctions. In a shallow water waveguide, where the source range is much larger than the channel depth, the effect of errors in the modal horizontal wavenumbers is very significant, since the modal phases are the product of the horizontal wavenumbers and the source range, resulting in large modal phase perturbations. Here, we assume that the environmental uncertainties cause errors only in these modal phases. In other words, we represent the environmental uncertainties by uncertainties in the modal horizontal wavenumbers (see [7] for validity of this assumption).

In order to further reduce the number of unknown parameters, the mean and covariance matrix of the modal horizontal wavenumber vector, $\bar{\kappa}$ and $\mathbf{R}_\kappa$, are estimated using a Monte-Carlo method:

$$\bar{\kappa} = \frac{1}{L}\sum_{l=1}^{L} \kappa(l) \ , \tag{13}$$

$$\hat{\mathbf{R}}_\kappa = \frac{1}{L}\sum_{l=1}^{L} \kappa(l)\kappa(l)^H \ , \tag{14}$$

where $\{\kappa(l)\}_{l=1}^{L}$ are L realizations of modal horizontal wavenumber vectors computed assuming a uniform distribution of environmental parameters over the intervals indicated in Fig. 1. For calculation of these wavenumbers, KRAKEN [4], a normal mode propagation model program, was used to generate a database containing scenarios with independent perturbations of the environmental uncertain parameters.

The horizontal wavenumbers can be efficiently expressed by:

$$\kappa = \bar{\kappa} + \mathbf{U}\mathbf{g} \ . \tag{15}$$

where $\mathbf{U}$ is the matrix of eigenvectors of $\hat{\mathbf{R}}_\kappa$, and the diagonal matrix $\Lambda_{\mathbf{g}}$ is the covariance of the vector $\mathbf{g}$:

$$\hat{\mathbf{R}}_\kappa = \mathbf{U}\Lambda_{\mathbf{g}}\mathbf{U}^H \ . \tag{16}$$

Now, the random environmental parameters given by the vector $\mathbf{g}$ cause wavenumber vector perturbations described by the column space of $\hat{\mathbf{R}}_\kappa$. Furthermore, the first element of $\mathbf{g}$ associated with the highest eigenvalue of $\hat{\mathbf{R}}_\kappa$ captures the largest component of the environmental uncertainty. In the following, all the elements of $\mathbf{g}$, except the first one, are assumed to be known. By this assumption the bound will be lower than assuming all the elements in $\mathbf{g}$ are unknown, but it will enable us to reduce the amount of computations for evaluation of the bound. Thus the modal horizontal wavenumbers can be approximated by

$$\kappa_m = \bar{\kappa}_m + U_{m1}g_1 \quad m = 1, \cdots, M. \tag{17}$$

The importance of the last step is that the environmental uncertainties are expressed by a single parameter, g_1. However, for the single source case, there are five unknown parameters: source range and depth, the environmental parameter and the real and imaginary parts of the complex signal amplitude. Taking a grid in each dimension of this parameter space would result in a huge number of test points, precluding alculation of the bound which involves a matrix inversion whose size is determined by the number of the test-points. In order to avoid inversion of a matrix with such a large size, the most test-points making the greatest contribution will be identified. The most contributing test-points are the K lowest values of the weighted norm $\|\mathbf{a}(\Theta) - \mathbf{a}(\Theta_j)\|^2_{\mathbf{R}_{\mathbf{n}}^{-1}}$ for $j = 1, \cdots, J$ where J is the number of candidate test-points. The field is evaluated for the J candidate test-points on the grid of the parameters, r_o, z_o and g_1. Because of the linear dependence of the field on the signal amplitude, b, one is able to choose test-points on b which minimize the weighted norm of the difference between the true value of the field and the test-points in order to increase the bound. The test-points which have been declared as the most contributing test-points are selected in order to construct the matrices $\mathbf{B}$ and $\mathbf{T}$. By this method, evaluation of the field is required only on a three dimensional parameter space. In order to avoid inversion of a large matrix, among these candidate test points only the most contributing ones are selected for calculation of the bound.

5. SIMULATION RESULTS

In this section, numerical evaluation of the Barankin bound for depth and range estimation for the channel of Fig. 1. is presented. Two scenarios are considered: (1) the "genlmis" scenario presented at the 1993 NRL workshop (Fig. 1), and (2) unknown channel depth scenario with known signal amplitude. In the first scenario the environmental uncertainties are assumed to affect the modal horizontal wavenumbers only while in the second scenario the uncertainty in the channel depth affect both the modal horizontal wavenumbers and the modal eigenfunctions. Source localization performance in these scenarios is compared to the case of known environment. For evaluating the bounds many test points on a grid in the parameter space were used. We consider a single source which radiates a monochromatic signal at 250Hz, located at depth $z_0 = 50$m in a channel of depth 102.5m. The acoustic field is sampled by a vertical array of 20 sensors, equally spaced 5m apart, located at range $r_0 = 7.5$km from the source. In all cases, $K = 200$ contributing test-points were selected to construct the matrices $\mathbf{B}$ and $\mathbf{T}$.

In the first scenario, the modified model of (17) was considered while in the second scenario in which only the channel depth is unknown, the exact propagation model was considered. The bounds on range and depth estimation error as a function of SNR are shown in Figs. 2,3 for the first scenario and in Figs. 4,5 for the second. For comparison, the CRLB is also plotted in these figures. The threshold phenomenon is clearly identified. Note, that the threshold SNR of the range estimation is more sensitive to the knowledge of the channel than that of the depth estimation indicating that the uncertainties in the modal horizontal wavenumber affect primarily range estimation error. Figs. 2 and 4 show that the uncertainties cause the threshold SNR for range estimation to increase by $\sim$ 3dB. Further Barankin bound results for different scenarios, including study of the threshold channel uncertainty level, rather than threshold SNR are presented in [8].

ACKNOWLEGMENT

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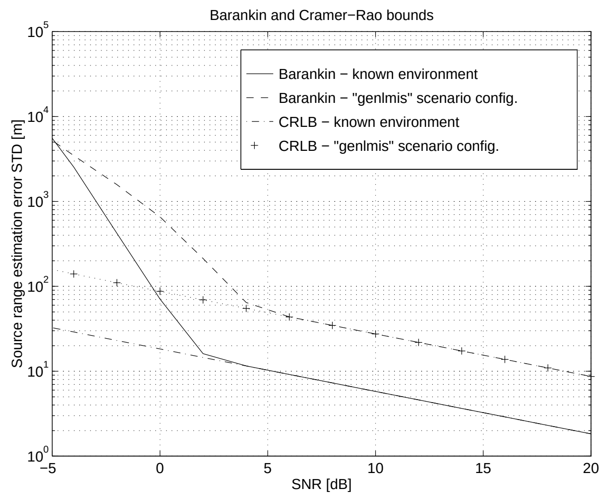

Figure 2. The bounds on range estimation error, "genlmis" scenario configuration

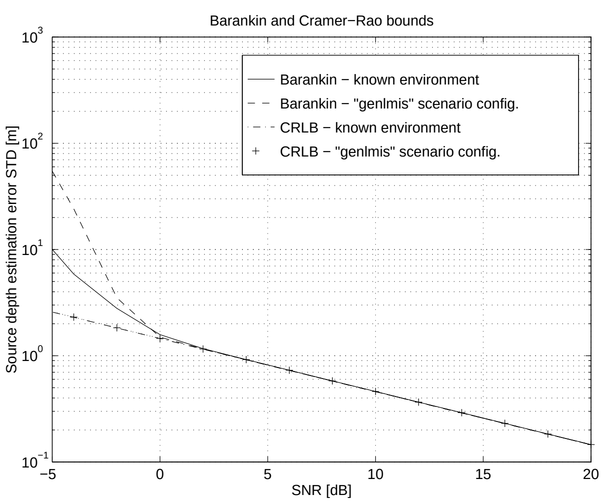

Figure 3. The bounds on depth estimation error, "genlmis" scenario configuration

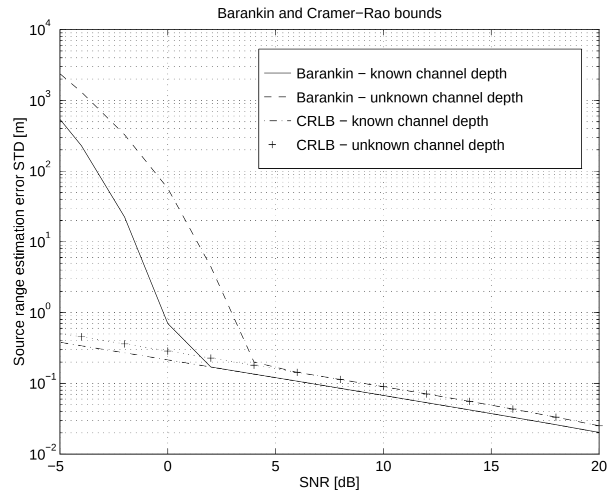

Figure 4. The bounds on range estimation error, unknown channel depth

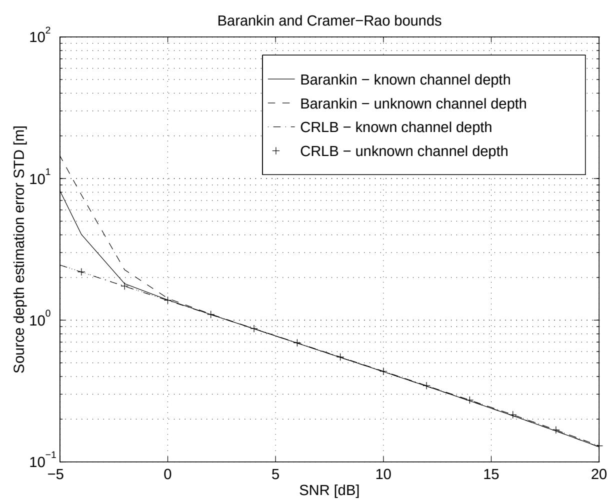

Figure 5. The bounds on depth estimation error, unknown channel depth