IMPROVEMENT OF TDOA MEASUREMENT USING WAVELET DENOISING WITH A NOVEL THRESHOLDING TECHNIQUE

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ABSTRACT

In this paper, wavelet denoising is applied in time delay estimation between signals received at two spatially separated sensors in the presence of noise. Prior to cross correlation, each of the sensor outputs is denoised according to a novel thresholding rule in order to increase the input signal-to-noise ratio. Unlike conventional generalized cross correlators (GCCs), it does not require spectral estimation of the source signal and the corrupting noises which may introduce large delay variance. It is proved that the delay estimate provided by the proposed method is globally convergent to the true value with a high probability. Computer simulations illustrate that the technique outperforms other GCCs for different SNRs when the sampling rate is sufficiently high.

1 INTRODUCTION

Extraction of time difference of arrival (TDOA) between signals received at two spatially separated sensors has been widely used in sonar and radar to find the position and to detect the speed of a target transmitter [1]. Other applications are found in seismology, biomedical engineering, digital communications and global positioning system [2]. Mathematically, the received signals are represented by

$$x(k) = s(k) + n_1(k)$$

$$y(k) = \beta s(k - D) + n_2(k), \quad k = 0, 1, ..., L - 1 \tag{1}$$

where $s(k)$ is the source signal, $n_1(k)$ and $n_2(k)$ are the uncorrelated zero-mean Gaussian noises, $\beta \in (0, 1)$ is an attenuation factor, and L is the number of samples in each snapshot received at the two sensors. The parameter D is the TDOA to be determined. Many of the methods proposed for time delay estimation are related through a generalized cross correlation (GCC) approach [3]. In this scheme, each received signal is passed through a prefilter prior to taking cross correlation. Examples of well known prefilters include smoothed coherence transform (SCOT), maximum likelihood (ML) [3], Hassab-Boucher (HB) [4] and the Wiener processor [5]. Generally speaking, the introduction of the prefilters is to enhance the frequency bands when the signal is strong and to attenuate the bands when the noise level is high. However, the implementation of such filters usually requires spectral estimation of the transmitted source as well as corrupting noises. As a result, large delay variance may be arisen particularly for short data length. Moreover, the GCCs often assume $s(k)$ to be a Gaussian process which hinder their usage in many practical applications.

The aim of this paper is to devise an effective method for accurate TDOA estimation when the source signal is deterministic, specifically for a pure sinusoid that is commonly used in radar and certain types of underwater acoustic systems [6]. The signal is expressed as

$$s(k) = a(k)\sin[w_o k + \theta(k)] \tag{2}$$

where $a(k)$, w_o and $\theta(k)$ represent the transmitted envelope function, central radian frequency and phase function respectively. The proposed system consists of two subunits and is depicted in Figure 1. Wavelet denoising (WD) technique is first applied to each received signal to recover the corresponding source waveform by removing the contamination. The restored signals are then cross correlated and the delay estimate is given by the time argument at which the cross correlation function attains its maximum value. It can be considered as a generalized cross correlator but spectral estimation of the signal and noise spectra is avoided. Section II will describe the denoising based time delay estimation method in detail. In particular, a novel threshold is derived according to Neyman-Pearson theory of hypothesis testing [7]. The global convergence of the proposed method is proved in Section III. Finally, computer simulation for performance evaluation is presented and conclusions are drawn in Section IV.

2 THE PROPOSED METHOD

In this section, we first derive the denoising technique for removing the noisy component from $x(k)$. It consists of three steps, namely, taking wavelet transform of the received signal, thresholding the wavelet coefficients, and performing inverse wavelet transform of the modified coefficients. Without loss of generality, we choose $L = 2^{J+1}$, where J is a positive integer and denote the powers of $n_1(k)$ and $n_2(k)$ by σ^2. For notation convenience, let $X = [x(0)\ x(1)\ \cdots\ x(L-1)]^T$, $S = [s(0)\ s(1)\ \cdots\ s(L-1)]^T$, $N_1 = [n_1(0)\ n_1(1)\ \cdots\ n_1(L-1)]^T$ and let T_w be a $L \times L$ orthonormal wavelet transform matrix formed by a set of quadrature mirror filter coefficients. In vector form, the transformed output Z is related to the input vector X by $Z = T_w X$, where $Z = [z(j,i), j = -1, 0, 1, \cdots, J; i = 0, 1, \cdots, 2^j - 1]^T$. The indices j and i represent the scale level and the position localized in each scale, and $z(-1, 0)$ denotes the remaining low-pass filtered coefficient. Since T_w is a linear transform matrix, Z can be decomposed into $Z = Z_s + Z_{n_1}$, where $Z_s = [z_s(j,i)]$ and $Z_{n_1} = [z_{n_1}(j,i)]$ are the wavelet transforms of the source signal vector S and the corrupting noise vector N_1 respectively. For most radar signals in the form of (2), there are many small magnitude terms in Z_s and wavelet transform is effective in compressing this type of signals. Note that Z_n in this case is still a Gaussian vector as T_w is orthonormal. The main idea of signal restoration using wavelet denoising is to adapt each $z(j,i)$ to make its value close to $z_s(j,i)$ so that a good approximation of $s(k)$ can be obtained after taking inverse wavelet transform.

Based on the well known Neyman-Pearson lemma, an effective thresholding rule for adjusting the wavelet coefficients is designed. This lemma is stated as follows. Let z be a Gaussian random variable with known variance σ^2 and a test is conducted upon the hypotheses $H_0 : E\{z\} = \mu_o$ versus $H_1 : E\{z\} \neq \mu_o$. Denote the decision of hypotheses H_0 and H_1 as D_0 and D_1 respectively. The type II error $P(D_0 \mid H_1)$ will be minimized for a given type I error $\alpha = P(D_1 \mid H_0)$ if H_0 is assured when z falls within the interval

$$|z - \mu_o| \leq \sqrt{2}\sigma \operatorname{erf}^{-1}(1 - \alpha) \tag{3}$$

where

$$\operatorname{erf}(v) \triangleq \frac{2}{\sqrt{\pi}} \int_0^v e^{-t^2} dt \tag{4}$$

The denoising method is to discard individual element in Z_s that is predicted to be small in magnitude. For each $z(j,i)$, a decision is made between the hypotheses,

$$H_0 : E\{z(j,i)\} = 0 \quad \text{versus} \quad H_1 : E\{z(j,i)\} \neq 0 \tag{5}$$

Since $E\{z(j,i)\} = z_s(j,i)$, the above test is simply to check on each unknown wavelet coefficient $z_s(j,i)$ to see whether it is null or not.

Applying the Neyman-Pearson lemma, the wavelet coefficient $z(j,i)$ is regarded as totally due to noise if

$$|z(j,i)| \leq \lambda \triangleq \sqrt{2}\sigma \operatorname{erf}^{-1}(1 - \alpha) \tag{6}$$

As a result, $z_s(j,i)$ will be estimated as $\hat{z}_s(j,i)$ which is given by

$$\hat{z}_s(j,i) = \begin{cases} z(j,i) & , \quad |z(j,i)| \geq \lambda \\ 0 & , \quad \text{otherwise} \end{cases} \tag{7}$$

Similarly, the received signal $y(k)$ is denoised. The restored signals of $s(k)$ and $\beta s(k-D)$, denoted by $\hat{s}(k)$ and $\beta \hat{s}(k-D)$, respectively, are then constructed by inverse transform of the modified wavelet coefficients. Finally, the delay estimate $\hat{D}$ is given by the peak of the cross correlation function of $\hat{s}(k)$ and $\beta \hat{s}(k-D)$, that is,

$$\hat{D} = \arg \max_\tau \{\hat{f}(\tau)\} \tag{8}$$

where

$$\hat{f}(\tau) \triangleq \frac{\beta}{L} \sum_{k=0}^{L-\tau-1} \hat{s}(k)\hat{s}(k - D + \tau) \tag{9}$$

3 PERFORMANCE ANALYSIS

Denote the ideal cross correlation function by $f(\tau) = \frac{\beta}{L} \sum_{k=0}^{L-\tau-1} s(k) s(k-D+\tau)$. For simplicity, the functions $\hat{f}(\tau)$ and $f(\tau)$ can be expressed as $\hat{f}(\tau) = \frac{\beta}{L} \hat{s}_{1\tau}^T \hat{s}_{2\tau}$ and $f(\tau) = \frac{\beta}{L} s_{1\tau}^T s_{2\tau}$, where $\hat{s}_{1\tau}$, $\hat{s}_{2\tau}$, $s_{1\tau}$ and $s_{2\tau}$ are the corresponding data blocks of length $L - |\tau|$. Their mean square difference is bounded by

$$
\begin{aligned}
&E\{|\hat{f}(\tau) - f(\tau)|\}^2 \\
&= \frac{\beta^2 E\{|\hat{s}_{1\tau}^T \hat{s}_{2\tau} - s_{1\tau}^T s_{2\tau}|^2\}}{L^2} \\
&= \frac{\beta^2 E\{|(\hat{s}_{1\tau} - s_{1\tau})^T \hat{s}_{2\tau} + s_{1\tau}^T(\hat{s}_{2\tau} - s_{2\tau})|^2\}}{L^2} \\
&\leq \frac{2\beta^2 E\{\|\hat{s}_{1\tau} - s_{1\tau}\|^2\|\hat{s}_{2\tau}\|^2 + \|s_{1\tau}\|^2\|\hat{s}_{2\tau} - s_{2\tau}\|^2\}}{L^2} \\
&\leq \frac{4}{L}\rho E_s \tag{10}
\end{aligned}
$$

where

$$E_s \triangleq \int_{t_1}^{t_2} |s(t)|^2 dt \tag{11}$$

$$\rho \triangleq (2 \log(L) + 1)[\sigma^2 + \sum_{j,i} \min\{\sigma^2, \ z_s^2(j,i)\}] \quad (12)$$

The signal $s(t)$ represents the continuous version of $s(k)$ which vanishes for $t \notin [t_1, t_2]$. Under more restrictive conditions, say $s(t)$ and the wavelet function are both smooth enough, it can be derived further that $\rho = O(\log^2(L))$ [8]. Therefore, when L tends to infinity, $E\{|\hat{f}(\tau) - f(\tau)|^2\}$ will tend to 0 which implies that the estimator is consistent. Moreover, as $L \to \infty$ and for any given small tolerance $\epsilon > 0$, using the Chebyshev's inequality we have

$$P\{|\hat{f}(\tau) - f(\tau)| > \epsilon\} \leq \frac{E\{|\hat{f}(\tau) - f(\tau)|^2\}}{\epsilon} \to 0 \quad (13)$$

This verifies that $\hat{f}(\tau)$ uniformly converges to $f(\tau)$ with high probability. The convergence, however, is not in the mean square sense but uniformly. This strong convergence property enables us to prove that if the discrete peak τ^* of $\{f(\tau)\}$ is close to the true peak u^* of $f(u)$ (it is true when the sampling interval is small enough), then the peak $\hat{\tau}$ of $\{\hat{f}(\tau)\}$ will also be close to u^* with probability one. In fact, if it is not true, there exists a positive constant δ such that $|\hat{\tau} - u^*| \geq \delta$ holds with a high probability. Yet another positive constant, say γ, can be found such that $f(u^*) - f(\hat{\tau}) \geq \gamma$, since $f(u)$ is continuous and u^* is the unique peak. On the other hand, when the sampling interval is sufficiently small, or equivalently when L is large, we have

$$0 \leq f(u^*) - f(\tau^*) < \frac{\gamma}{4} \quad (14)$$

From the inequality (13) we may conclude that

$$-\frac{\gamma}{4} \leq \hat{f}(\tau) - f(\tau) \leq \frac{\gamma}{4} \quad (15)$$

hold for each τ with a high probability when L is large, which also implies that

$$-\frac{\gamma}{4} \leq \hat{f}(\hat{\tau}) - f(\tau^*) \leq \frac{\gamma}{4} \quad (16)$$

Hence, we may deduce that

$$\begin{aligned} \gamma \ &\leq \ f(u^*) - f(\hat{\tau}) \\ &< \ f(\tau^*) + \frac{\gamma}{4} - f(\hat{\tau}) \\ &\leq \ f(\tau^*) + \frac{\gamma}{4} - (\hat{f}(\hat{\tau}) - \frac{\gamma}{4}) \\ &\leq \ \frac{3\gamma}{4} \end{aligned} \quad (17)$$

which is a contradiction. Therefore, the assertion that the peak $\hat{\tau}$ of $\{\hat{f}(\tau)\}$ will be close to u^* with probability one is validated.

4 EXPERIMENTATION & CONCLUSIONS

Extensive simulation tests had been carried out to evaluate the performance of the proposed method for time delay estimation. Comparison with the direct cross correlator (CC), generalized cross correlator with maximum likelihood prefilters (GCC-ML) and SCOT prefilters (GCC-SCOT) were made. Cross correlation with wavelet denoising using Donoho's soft threshold [9] were also tried in order to contrast the two decision rules. In our experiments, the source signal was given by (2) with $w_o = 160\pi$ and $\theta(k) = 0$. The envelope $a(k)$ had a value of 10 for time interval between 0 and 0.2, and 0 otherwise. The corrupting noise sequences $n_1(k)$ and $n_2(k)$ were white Gaussian processes produced from a random number generator. The energy of $s(k)$ was 10 and different signal-to-noise ratios (SNRs) were obtained by proper scaling of the noises. The actual delay was assigned to 0.29296875. Prior to wavelet transform of the received signals, each sample was averaged with its adjacent samples in order to reduce noise power. In the wavelet denoising procedure, 4-tap Daubechies wavelet coefficients were used and the type I error α was set to 0.05. All results provided were averages of 200 independent runs.

Figure 2 plots the mean square delay error versus the sampling interval for different methods at SNR $= -10$ dB. In general, the accuracy of all five methods increased as the sampling frequency increased. Although it has been proved [9] that Donoho's soft-thresholding method gives the best spatial adaptation and optimum theoretical risk performance, the delay estimates obtained using this threshold were less accurate than the other four methods for the entire range of different sampling intervals. One reason might be due to the fact that the constant threshold used, viz $\sigma\sqrt{2\log(L)}$, was too large to keep all the pertinent signal information. It can also be seen that the two generalized cross correlators performed even worse than the direct cross correlation method. It is because (i) the GCCs were not optimum time delay estimators for deterministic signals and (ii) the prefilter coefficients computed from the spectral estimates of the received signals had fairly large variances. When the sampling interval was smaller than 0.001, WD-CC with the new threshold achieved the minimum delay error whereas CC was the best delay estimator for other cases.

Figure 3 plots the variances of the five methods versus SNR with the sampling rate fixed at 2^{-11}. As expected, the error of all techniques decreased as the SNR increased. The WD-CC method with the new threshold attained the best performance for all conditions. The GCCs were less satisfactory than the CC method

whilst the WD-CC with Donoho's threshold was the poorest delay estimator among all the others.

In conclusion, an efficient method based on wavelet denoising is proposed for time delay estimation. Similar to the generalized cross correlation approach, the system consists of a pair of wavelet denoising units for recovering the source signals, followed by a cross correlator. A new thresholding rule for denoising is also designed. It is demonstrated that the proposed method generally gives a better delay estimation performance over other correlation based techniques.

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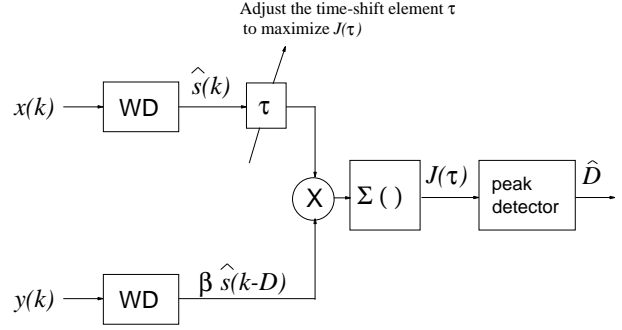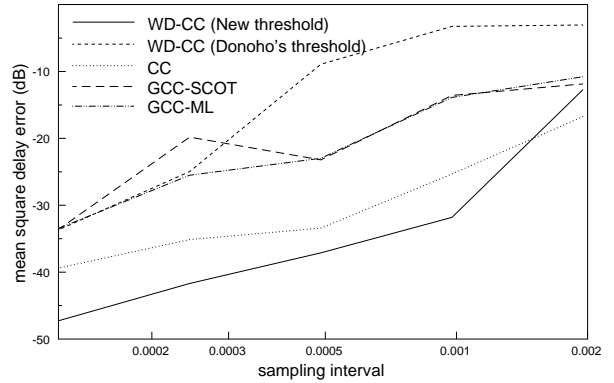
Figure 1: System block diagram of WD-CC
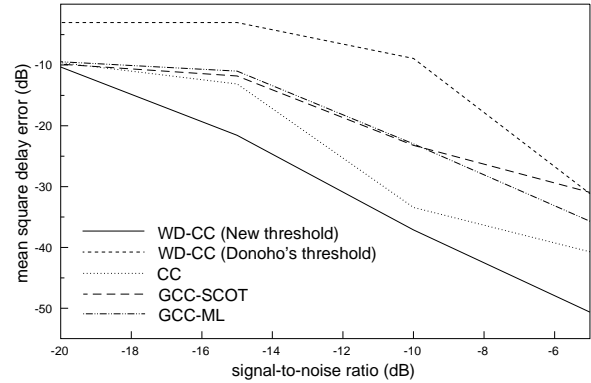

Figure 2: Mean square delay error vs sampling interval

Figure 3: Mean square delay error vs SNR