THE LMMSE ESTIMATE-BASED MULTIUSER DETECTOR: PERFORMANCE ANALYSES AND ADAPTIVE IMPLEMENTATION

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ABSTRACT

Presented in this work are analytical expressions of the performance measure on the LMMSE estimate-based multiuser detector, including error probability expression and its computationally and notationally efficient approximations, signal to interference-plus-noise ratio, and asymptotic efficiency. Also included in this work are adaptive implementation schemes of the LMMSE detector and the equivalent relation between them under appropriate assumptions. Simulations are included to show the tightness of approximate results over a wide range of near-far ratio and various combinations of SNRs of interfering multiple-access users.

1. INTRODUCTION

Multiuser separation and interference suppression is an active research topic in CDMA communications. The major driving force for continued research in multiuser detection is the combination of a better understanding on the statistics of multiple-access interference (MAI) and the promising feature of near-far resistance offered by multiuser detector that eliminates the need for stringent power control [1]. The optimum solution to the problem under ideal Gaussian MA channel was proposed in [2]. Due to the computational complexity (*exponential* to the number of users) of the optimum solution, sub-optimum solutions $[3-8]$ become more attractive for practical applications. Among various sub-optimum detectors proposed recently, the linear decorrelating detector $[3-4]$ has received most attention and is well-cited due to its computational simplicity (linear in the number of users) and near-far resistant properties. A drawback associated with this detector is the effect of noise enhancement (analogous to that in the zero-forcing equalizer), which limits its performance in situation where noise level is dominant over or comparable to MAI. The linear minimum mean squared error (LMMSE) estimate-based multiuser detector $[7-8]$ overcomes this performance limit, at the same time maintains computational simplicity and near-far resistance. The LMMSE detector is of most practical importance among all *linear* estimate-based multiuser detectors. Since the LMMSE detector belongs to *linear* Bayesian-based multiuser detector, it offers the best (among all *linear* detectors) trade-off between bias (MAI residue) and noise variance in terms of mean squared error (MSE) on the estimate, on which a decision is based. This finally provides improved performance (see details in sections 2-3 of this work). Other important features of the LMMSE detector are the possibility of adaptive implementation and performance robustness in changing environment [8].

This work focuses on performance analyses of the LMMSE multiuser detector and relationship between different performance measure, such as error probability and its approximations, signal to interference-plus-noise ratio, and asymptotic efficiency.

2. NOTATION AND PROBLEM FORMULATION

In code-division multiple-access (CDMA) systems, all MA users share the same wideband channel simultaneously (joint time-frequency sharing), while each user is assigned a distinct spreading signature waveform. Therefore the baseband data $r(t)$ at a receiver is actually a summation of multiuser signals embedded in additive noise, or,

$$r(t) = \sum_i \sum_{k=1}^{K} \sqrt{a_k}\, b_k(i)\, s_k(t - iT - \tau_k) + n(t)\,, \quad (1)$$

where K is the number of users; i is the symbol index; a_k, $b_k(i)$, $s_k(t)$, and τ_k are the bit energy, information bit, signature waveform (of duration T), and propagation delay of the kth user, respectively; $n(t)$ is a white Gaussian process, with two-sided power spectral density of σ^2.

For synchronous channel, all the delays τ_k's are equal, we can then treat $\tau_k = 0$ $(k = 1, 2, \ldots, K)$ without

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loss of generality. In this work, we only consider the synchronous channel for the purpose of national simplicity. Note that once the channel is synchronized, all the information bits of the multiusers in the ith symbol interval are completely contained in the data $r(t)$ within the ith symbol interval. Therefore, we can concentrate on solving multiuser separation problem within a specific symbol interval. For a specific bit interval, say $i = 0$ to ignore the symbol index i, we can rewrite (1) as,

$$\begin{aligned} r(t) &= \left[\ s_1(t)\ \ s_2(t)\ \ \cdots\ \ s_K(t)\ \right] \cdot \mathbf{A} \cdot \mathbf{b} + n(t)\ , \\ &= \mathbf{S}(t) \cdot \mathbf{A} \cdot \mathbf{b} + n(t)\ , \qquad (\, 0 \le t \le T\,)\ , \end{aligned} \tag{2}$$

with $\mathbf{A} = \mathrm{diag}\{\sqrt{a_1},\, \sqrt{a_2},\, \cdots,\, \sqrt{a_K}\}$ being a positive diagonal matrix of amplitudes of MA users, and $\mathbf{b} = [\, b_1\ b_2\ \cdots\ b_K\,]^T$ being a vector of binary information bit of MA users ($b_k \in \{-1, +1\}$ with equal probability, and b_k's are i.i.d.). Columns of $\mathbf{S}(t)$ are signature waveforms of MA users (assuming $s_k(t)$ is of duration T and normalized $\|\, s_k(t)\,\| = 1$).

If we filter $r(t)$ with a bank of matched filters (MFs), whose impulse responses are given by, $h_k(t) = s_k(T - t)$, $(k = 1, 2, \ldots, K)$ and columnize the sampled outputs of the bank of matched filters at $t = T$, we get the following matrix notation,

$$\mathbf{x} = \mathbf{P} \cdot \underbrace{\mathbf{A} \cdot \mathbf{b}}_{\boldsymbol{\theta}} + \mathbf{n}\ , \qquad \text{— linear model} \tag{3}$$

where $\mathbf{x} = \left[\ x_1(T)\ \ x_2(T)\ \ \cdots\ \ x_K(T)\ \right]^T$, with $x_k(T) = r(t) * h_k(t)\,|_{t=T}$ being the kth matched filter output sampled at $t = T$; and $\mathbf{n} \sim \mathcal{N}(\mathbf{0},\, \sigma^2\, \mathbf{P})$ being *colored* Gaussian noise due to matched filtering.

Note that the *linear model* in (3) is also valid for asynchronous channel, except for a larger dimension. Since for asynchronous channel, by introducing the partitioned signature waveforms $s_1(t)$, $s_2^L(t)$, $s_2^R(t)$, $\cdots$, $s_K^L(t)$, and $s_K^R(t)$, the resultant matrix $\mathbf{P}$ in (3) is of a dimension $(2K - 1) \times (2K - 1)$. For synchronous channel, the matrix $\mathbf{P}$ in (3) is simply a nonsingular, positive definite, symmetric, cross-correlation matrix of the signature waveforms. Its elements are given by,

$$\begin{aligned} P[i,\, j] \;\; &= \int_0^T s_i(t)\ s_j(t)\ dt \stackrel{\Delta}{=} \rho_{i\,j}\ , \\ & \qquad i = 1, 2, \ldots, K;\ j = 1, 2, \ldots, K. \end{aligned}$$

In practice, due to the finite bandwidth constraint and the existence of a large number of users, the signature waveforms are not ideally orthonormal. The non-diagonal nature of the $\mathbf{P}$ matrix will cause the MAI, which is the cause of the near-far problem in conventional matched filter receiver. In order to combat the near-far problem, various detectors have been proposed [1−8]. In this work, we mainly concentrated on the performance measure and implementation issues of LMMSE multiuser detectors [7 − 9].

3. THE LMMSE DETECTOR: PERFORMANCE MEASURE & APPROXIMATIONS

The LMMSE detector [6 − 9] was proposed and analyzed in comparison with the decorrelating detector from various aspects, mainly through simulations. The most commonly known form of it is the following one,

$$\begin{aligned} &\text{Decision rule:} \quad \hat{\mathbf{b}} = \mathrm{sgn}\left\{\mathbf{W}\,\mathbf{x}\right\}\ , \\ &\text{with} \quad\ \ \mathbf{W} = (\mathbf{P} + \sigma^2\, \mathbf{A}^{-2})^{-1}\ . \end{aligned} \tag{4}$$

where $\mathbf{x}$ is a vector of MF output defined in (3).

The results in (4) give an impression that the knowledge of signature waveforms of MA users (used to form MFs and get $\mathbf{P}$ matrix) and SNR matrix of MA users are needed in order to implement this detector. In this work, we prove the equivalence of the LMMSE detector in (4) and the following one,

$$\text{Decision rule:} \quad \hat{\mathbf{b}} = \mathrm{sgn}\left\{\mathbf{S}^T\, \Sigma_{\mathbf{r}\,\mathbf{r}}^{-1}\, \mathbf{r}\right\}\ , \tag{5}$$

where $\mathbf{r}$ is a vector of chip-rate sampled data in (2), and $\Sigma_{\mathbf{r}\,\mathbf{r}}$ is the covariance matrix of data $\mathbf{r}$. Or, specifically, $\Sigma_{\mathbf{r}\,\mathbf{r}} = \mathbf{S}\,\mathbf{A}^2\,\mathbf{S}^T + \sigma^2\,\mathbf{I}$. In deriving the equivalence of detectors in (4) and (5), we first notice the fact that the sgn$(\cdot)$ operator on any vector is invariant to any positive definite diagonal matrix operation, i.e. $\mathrm{sgn}(\,\mathbf{A}\,\boldsymbol{\theta}\,) = \mathrm{sgn}(\boldsymbol{\theta})$ in (5). Based on the above observation, we then have,

$$\begin{aligned} \mathrm{sgn}\left\{\mathbf{S}^T\, \Sigma_{\mathbf{r}\,\mathbf{r}}^{-1}\, \mathbf{r}\right\} &= \mathrm{sgn}\left\{\mathbf{A}^2\, \mathbf{S}^T\, \Sigma_{\mathbf{r}\,\mathbf{r}}^{-1}\, \mathbf{r}\right\}, \\ &= \mathrm{sgn}\left\{\mathbf{A}^2\, \mathbf{S}^T\, (\,\mathbf{S}\,\mathbf{A}^2\,\mathbf{S}^T + \sigma^2\,\mathbf{I}\,)^{-1}\, \mathbf{r}\right\}, \\ &= \mathrm{sgn}\left\{\sigma^{-2}\,\mathbf{A}^2\,(\,\mathbf{I} - \mathbf{P}\,\mathbf{W}\,)\,\mathbf{S}^T\,\mathbf{r}\right\}, \\ &= \mathrm{sgn}\left\{(\,\mathbf{P} + \sigma^2\,\mathbf{A}^{-2}\,)^{-1}\,\mathbf{x}\right\}\ . \end{aligned} \tag{6}$$

In getting the last two equations in (6), matrix inversion lemma in combination with the facts $\mathbf{P} = \mathbf{S}^T\,\mathbf{S}$, and $\mathbf{x} = \mathbf{S}^T\,\mathbf{r}$ is used. As a matter of fact, the LMMSE detector of (5) can be derived by applying the LMMSE criterion and the invariance property on the chip-rate sampled data $\mathbf{r} = \mathbf{S}\,\mathbf{A}\,\mathbf{b} + \mathbf{n}_0$ with $\mathbf{n}_0$ being white noise (covariance matrix $\sigma^2\,\mathbf{I}$) independent of $\mathbf{b}$. Since,

$$\begin{aligned} \hat{\mathbf{b}} \;\; &= \mathrm{sgn}\left\{\hat{\theta}_{LMMSE}\right\} \\ &= \mathrm{sgn}\left\{\Sigma_{\boldsymbol{\theta}\,\mathbf{r}}\, \Sigma_{\mathbf{r}\,\mathbf{r}}^{-1}\, \mathbf{r}\right\} = \mathrm{sgn}\left\{\mathbf{A}^2\, \mathbf{S}^T\, \Sigma_{\mathbf{r}\,\mathbf{r}}^{-1}\, \mathbf{r}\right\} \\ &= \mathrm{sgn}\left\{\mathbf{S}^T\, \Sigma_{\mathbf{r}\,\mathbf{r}}^{-1}\, \mathbf{r}\right\}\ . \end{aligned}$$

What makes the second form of LMMSE detector in (5) attractive is the fact that only the signature waveform of the *desired* user, say the kth user, is need in decoding its information bit. That is,

$$\hat{b}_k = \mathrm{sgn}\left\{\mathbf{s}_k^T\, \Sigma_{\mathbf{r}\,\mathbf{r}}^{-1}\,\mathbf{r}\right\}\,, \quad \text{— generalized MF}\,, \qquad (7)$$

where $\mathbf{s}_k^T$ is the kth row of $\mathbf{S}^T$ (the kth user's signature waveform sampled at chip-rate), and $\Sigma_{\mathbf{r}\,\mathbf{r}}^{-1}$ can be sequentially estimated from data sequence $\{\mathbf{r}(i)\}$ after proper initialization (see section 4 for a data-driven adaptive implementation scheme).

In this work, we derive some other performance measure of the LMMSE detector. One of the performance measure commonly used in practice is the signal to interference-plus-noise ratio (SINR). For the LMMSE detector, both bias (MAI residue) and noise exist in the decision statistics/estimator [10]. We calculated the kth user's SNIR as,

$$SNIR_k = \frac{a_k\,\left(1 - \underline{\boldsymbol{\rho}}_k^T\,(\mathbf{P}_k + \sigma^2\,\mathbf{A}_k^{-2})^{-1}\,\underline{\boldsymbol{\rho}}_k\right)}{\sigma^2}\,, \qquad (8)$$

where matrices $\mathbf{P}_k$, $\mathbf{A}_k$ and vector $\underline{\boldsymbol{\rho}}_k$ are constructed from $\mathbf{P}$, $\mathbf{A}$, and $\boldsymbol{\rho}_k$, respectively, by removing the contribution (the kth row and column for matrix, the kth element for vector) of the kth user.
In addition, we calculated the minimum mean squared error (MMSE) of the kth user,

$$\begin{aligned} MMSE_k &= \mathcal{E}\left\{(\theta_k - \hat{\theta}_k\,)^2\right\}\,, \quad \text{with } \theta_k = \sqrt{a_k}\,b_k\,, \\ &= \sigma^2\,\mathbf{w}_k(k)\,, \\ &= \frac{\sigma^2}{1 + \sigma^2/a_k - \underline{\boldsymbol{\rho}}_k^T\,\left(\mathbf{P}_k + \sigma^2\,\mathbf{A}_k^{-2}\right)^{-1}\,\underline{\boldsymbol{\rho}}_k}\,. \end{aligned} \qquad (9)$$

The relation between $SINR_k$ in (8) and $MMSE_k$ in (9) can then be established as,

$$SINR_k = \frac{a_k}{MMSE_k} - 1\,. \qquad (10)$$

Formula (10) reveals the equivalence between minimizing MSE and maximizing SINR under *linear* constraint. In [9], we derived the following analytical expression of error probability (kth user) of the LMMSE detector,

$$P_e(k) = \frac{1}{2^K}\sum_{\mathbf{b}} Q\left(\frac{\sqrt{a_k} - b_k\,\sigma^2\,\mathbf{w}_k^T\,\mathbf{A}^{-1}\,\mathbf{b}}{\sigma\,\sqrt{\mathbf{w}_k^T\,\mathbf{P}\,\mathbf{w}_k}}\right)\,, \qquad (11)$$

where $\mathbf{w}_k$ is the kth column of $\mathbf{W}$ matrix in (4) (see [9] for precise expression of $\mathbf{w}_k$). $Q(\xi) = \int_\xi^\infty \frac{1}{\sqrt{2\,\pi}}\,e^{-\frac{x^2}{2}}\,dx$. Based on the assumption on the statistics of $\mathbf{b}$ in section 2, we can write $P_e(k)$ in (11) as,

$$\begin{aligned} &P_e(k) = \mathcal{E}\,\{\,Q(\xi_k)\,\}\,, \\ &\text{with } Q(\xi_k) = Q\left(\frac{\sqrt{a_k} - b_k\,\sigma^2\,\mathbf{w}_k^T\,\mathbf{A}^{-1}\,\mathbf{b}}{\sigma\,\sqrt{\mathbf{w}_k^T\,\mathbf{P}\,\mathbf{w}_k}}\right) \end{aligned} \qquad (12)$$

where the expectation $\mathcal{E}(\cdot)$ is with respect to $\mathbf{b}$, the random information bit of MA users.
In order to get computationally and notationally more efficient approximations to $P_e(k)$ in (11) and (12), we first expand the above $Q(\xi_k)$ function (it is a 1-D function of random vector $\mathbf{b}$) at a non-random point specified by the square root of $SINR_k$ in (8),

$$\begin{aligned} Q(\xi_k) &= Q(\sqrt{SINR_k}) \\ &+ \sum_{n=1}^{\infty} \frac{\left(\xi_k - \sqrt{SINR_k}\,\right)^n}{n!}\,Q^{(n)}\left(\sqrt{SINR_k}\right)\,. \end{aligned} \qquad (13)$$

Taking the $\mathcal{E}(\cdot)$ operation on both sides of (13) and ignoring the contribution from the second term (infinite sum), we then obtain the *simplest* approximation of $P_e(k)$ of (11) as follows,

$$P_e(k) \approx Q\left(\sqrt{\frac{a_k\,\left(1 - \underline{\boldsymbol{\rho}}_k^T\,(\mathbf{P}_k + \sigma^2\,\mathbf{A}_k^{-2})^{-1}\,\underline{\boldsymbol{\rho}}_k\right)}{\sigma^2}}\right)\,, \qquad (14)$$

The reason of ignoring the remaining terms in (13) in getting (14) is that the *combined* contribution from the *expectation* of remaining terms (grouping every two successive terms) of infinite sum in (13) is very small compared to the first dominant term over a wide range of SNRs. We also verify this fact through simulations in section 4. As a by-product, we also expand the $Q(\xi_k)$ at ponit $\overline{\xi}_k$, the mean value of ξ_k, and obtaine a less efficient approximation (it contains more terms),

$$P_e(k) \approx Q\left(\,\overline{\xi}_k\,\right) + \frac{\sigma^2\,\underline{\mathbf{w}}_k^T\,\mathbf{A}_k^{-2}\,\underline{\mathbf{w}}_k}{2\,\mathbf{w}_k^T\,\mathbf{P}\,\mathbf{w}_k}\,\frac{\overline{\xi}_k}{\sqrt{2\,\pi}}\,e^{-\frac{\overline{\xi}_k^2}{2}}\,, \qquad (15)$$

with $\overline{\xi}_k = \dfrac{\sqrt{a_k} - \sigma^2\,\mathbf{w}_k(k)\,\frac{1}{\sqrt{a_k}}}{\sigma\,\sqrt{\mathbf{w}_k^T\,\mathbf{P}\,\mathbf{w}_k}}$, and $\underline{\mathbf{w}}_k$ is formed from $\mathbf{w}_k$ by eliminating its kth element.

Based on (14), a good approximate expression for the asymptotic efficiency of the LMMSE detector can be obtained. Or,

$$\gamma_k \approx 1 - \underline{\boldsymbol{\rho}}_k^T\,(\mathbf{P}_k + \sigma^2\,\mathbf{A}_k^{-2})^{-1}\,\underline{\boldsymbol{\rho}}_k\,. \qquad (16)$$

In section 4, we numerically evaluate all these analytical results along with computer simulations to show the tightness of approximations in (14) and (15) over a wide range of different combinations of SNRs of MA users. Formula (14) also reveals an important fact that there exists an approximate equivalence between minimizing MSE (maximizing the SINR) and minimizing the error probability under *linear* contsraint and Gaussian noise. In order to make a comparision with the well-known decorrelating detector ($\hat{\mathbf{b}} = \mathrm{sgn}\{\,(\mathbf{P}^{-1}\,\mathbf{x}\,\})$, we also derived the error probability and asymptotic efficiency of of it as follows, bbb

$$P_e^{DEC}(k) = Q\left(\sqrt{\frac{a_k\,(1-\underline{\boldsymbol{\rho}}_k^T\,\mathbf{P}_k^{-1}\,\underline{\boldsymbol{\rho}}_k)}{\sigma^2}}\right) \le Q\left(\sqrt{\frac{a_k}{\sigma^2}}\right) ,$$

Asymptotic efficiency: $\gamma_k^{DEC} = 1 - \underline{\boldsymbol{\rho}}_k^T\,\mathbf{P}_k^{-1}\,\underline{\boldsymbol{\rho}}_k$,

SINR: $SNIR_k^{DEC} = \dfrac{a_k\,(1-\underline{\boldsymbol{\rho}}_k^T\,\mathbf{P}_k^{-1}\,\underline{\boldsymbol{\rho}}_k)}{\sigma^2}$.

From the above analytical expressions, it can be easily seen that the asymptotic efficiency of the LMMSE detector γ_k^{LMMSE} is always lower bounded by that of the decorrelating detector γ_k^{DEC}. Similarly, the error probability of the LMMSE detector $P_e^{LMMSE}(k)$ is always upper bounded by that of the decorrelating detector $P_e^{DEC}(k)$. Therefore, the LMMSE detector always provides better detection performance than decorrelating detector. Of most importance, the proposed form of LMMSE in (7) needs *less* knowledge (only the signature of the desired user is needed) about MA users' signatures than the decorrelating detector. A practical data-driven version of the LMMSE detector is proposed in section 4.

4. DATA-DRIVEN ADAPTIVE IMPLEMENTATION

As mentioned above, the new form of LMMSE detector in (5) and (7) provides the possibility of implementing it in a data adaptive way, so that only the signature of the *desired kth user* is needed in order to decode its information bits $\{\,b_k(i)\,\}$. In this proposed *computationally efficient* data-adaptive scheme, we use the *properly initialized* and data-driven updated sample covariance estimate $\hat{\Sigma}_{\mathbf{rr}}$ to replace the true $\Sigma_{\mathbf{rr}}$ without actually conducting the matrix inversion. The algorithm is summarized as follows,

- Initialization stage (i=1):

  $$\hat{\Sigma}_{\mathbf{r}\,\mathbf{r}}^{-1}(i) = \mathbf{I}_L\ ,\ \ \hat{b}_k(i) = \mathrm{sign}\left\{\,\mathbf{s}_k^T\,\hat{\Sigma}_{\mathbf{r}\,\mathbf{r}}^{-1}(i)\,\mathbf{r}(i)\,\right\}\ .$$

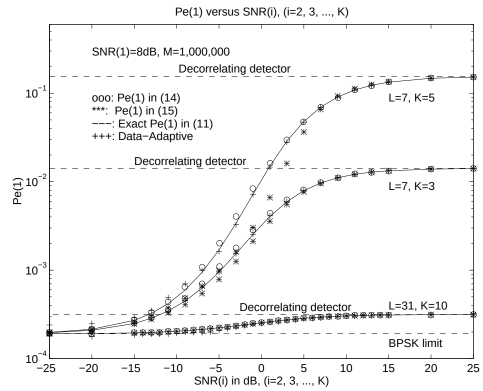
- Update stage (for $i = 2, 3, \cdots$):

  $$\begin{aligned}\hat{\Sigma}_{\mathbf{r}\,\mathbf{r}}^{-1}(i) \;&= \frac{1}{\beta_i}\,\hat{\Sigma}_{\mathbf{r}\,\mathbf{r}}^{-1}(i-1)\\ &-\frac{\alpha_i}{\beta_i^2}\,\frac{\hat{\Sigma}_{\mathbf{r}\,\mathbf{r}}^{-1}(i-1)\,\mathbf{r}(i)\,\mathbf{r}^T(i)\,\hat{\Sigma}_{\mathbf{r}\,\mathbf{r}}^{-1}(i-1)}{1+\dfrac{\alpha_i}{\beta_i}\,\mathbf{r}^T(i)\,\hat{\Sigma}_{\mathbf{r}\,\mathbf{r}}^{-1}(i-1)\,\mathbf{r}(i)},\end{aligned}$$

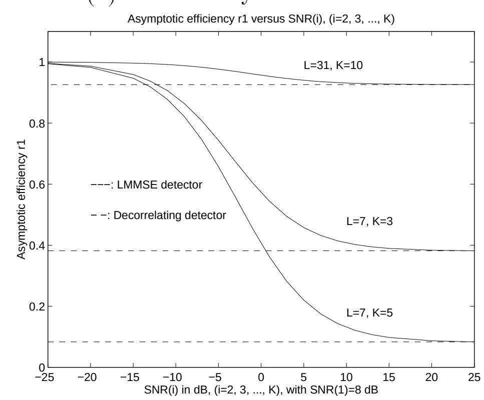
  $$\hat{b}_k(i) = \mathrm{sign}\left\{\,\mathbf{s}_k^T\,\hat{\Sigma}_{\mathbf{r}\,\mathbf{r}}^{-1}(i)\,\mathbf{r}(i)\,\right\}\ .$$

  where $\alpha_i = 1/i$, $\beta_i = 1 - \alpha_i$.

Numerical evaluations and computer simulations shown in the following figures verify our analyses. Gode codes of various length (L) were used as spreading signatures in our simulations.



(a). Probability of error.



(b). Asymptotic efficiency.

Figure 1: Performance comparison of proposed data-adaptive LMMSE detectors and analysis results.

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