

# TIME-VARYING RECONSTRUCTION OF STATIONARY PROCESSES SUBJECTED TO ANALOGUE PERIODIC SCRAMBLING

*Alban DUVERDIER and Bernard LACAZE*

National Polytechnics Institute of Toulouse  
LEN7/GAPSE, 2 rue Camichel, 31071 Toulouse Cedex 7 BP 7122  
tel: (33) 5 61 58 83 67 Fax: (33) 5 61 58 82 37  
email: duverdie@len7.enseiht.fr

## ABSTRACT

In modern telecommunications, it is often desirable to scramble the contents of the information. This paper presents a particularly efficient method of analogue signal scrambling. A stationary process is subjected to scrambling by means of a linear periodic time-varying filter. We observe then a cyclostationary process. We demonstrate that perfect reconstruction is possible. In presence of overlapping spectra, unscrambling requires a time-varying filter. We apply this method to scramble stationary binary signals. Simulations show that the system is additive noise resistant.

## 1. INTRODUCTION

These days most encryption is done digitally. Yet, analogue scramblers are known to exhibit desirable properties in numerous applications, such as speech transmission [1]. In fact, they do not need significantly bandwidth increasing. Within this framework, [2] highlighted the interest of scrambling by means of linear periodic time-varying filters. In the reconstruction phase, the authors proposed using linear invariant time filters for non-overlapped spectra. The present contribution extends the existing methods to the (in theory) perfect reconstruction of overlapped spectra. It is shown that this extension requires the application of time-varying filters in order to obtain a sufficient reconstruction quality.

## 2. LINEAR PERIODIC TIME-VARYING FILTERING

In what follows, the original signal, i.e. the process to be scrambled, will be denoted  $Z(t)$ . We let  $Z = \{Z(t), t \in \mathbb{R}\}$  be a random stationary process of zero mean and mean square continuous.  $\Theta_Z(\omega)$  is the Cramér-Loève spectral representation [3] of  $Z$  and the two are related by:

$$Z(t) = \int_{-\infty}^{+\infty} e^{i\omega t} d\Theta_Z(\omega) \quad (1)$$

The basic idea of the scrambling system is to subject the original process to a linear periodic time-varying (LPTV) filter,  $\tilde{h}$ . Let  $H_t(\omega)$  be its frequency response, periodic in  $t$  of period  $T = 2\pi/\omega_0$ . We define its Fourier development, assumed to be sufficiently regular, by:

$$H_t(\omega) = \sum_{k=-\infty}^{+\infty} \psi_k(\omega) e^{ik\omega_0 t} \quad (2)$$

with:

$$\psi_k(\omega) = \frac{1}{T} \int_0^T H_t(\omega) e^{-ik\omega_0 t} dt \quad (3)$$

Let  $X(t)$  be the response of the stationary process  $Z(t)$  through the LPTV filter  $\tilde{h}$ .  $X = \{X(t), t \in \mathbb{R}\}$  is the random process such that:

$$X(t) = \int_{-\infty}^{+\infty} e^{i\omega t} H_t(\omega) d\Theta_Z(\omega) \quad (4)$$

Equations (2) and (4) give:

$$X(t) = \sum_{k=-\infty}^{+\infty} e^{ik\omega_0 t} A_k(t) \quad (5)$$

where the  $\{A_k(t)\}_{k \in \mathbb{Z}}$  are the random harmonisable stationary process of zero mean and mean square continuous, defined by:

$$A_k(t) = \int_{-\infty}^{+\infty} e^{i\omega t} \psi_k(\omega) d\Theta_Z(\omega) \quad (6)$$

(5) corresponds to a continuous series representation of  $X$ .  $X$  is then cyclostationary [4] and its spectral representation can be written as:

$$d\Theta_X(\omega) = \sum_{k=-\infty}^{+\infty} \psi_k(\omega - \omega_0) d\Theta_Z(\omega - \omega_0) \quad (7)$$

$d\Theta_X(\omega)$  is an infinite sum of weighted shifted versions of  $d\Theta_Z(\omega)$ , centered around multiples of the period and with weights depending on  $\tilde{h}$ . The greater the support of  $d\Theta_Z(\omega)$ , the more the different versions will tend to overlap.

### 3. LINEAR RECONSTRUCTION SCHEME AFTER AN LPTV FILTERING

The scrambling system that we have introduced is depicted by figure 1.

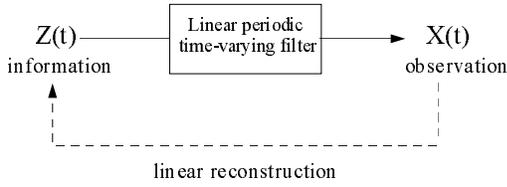


Figure 1, Scrambling method

In order to have an efficient scrambling system, we need a method of perfect information reconstruction. If we suppose known  $d\Theta_X(\omega)$  and the functions  $\{\psi_k(\omega)\}_{k \in \mathbb{Z}}$ , the inversion of equation (7) allows the identification of  $d\Theta_Z(\omega)$ . In practice, it is possible when the information is supposed to be bandlimited. In this case, we assume that the support of  $d\Theta_Z(\omega)$  is contained in  $[-\omega_0, \omega_0]$ . Equation (7) shows that the overlap is then limited to adjacent components. The modulus of  $d\Theta_X(\omega)$  is represented by figure 2.

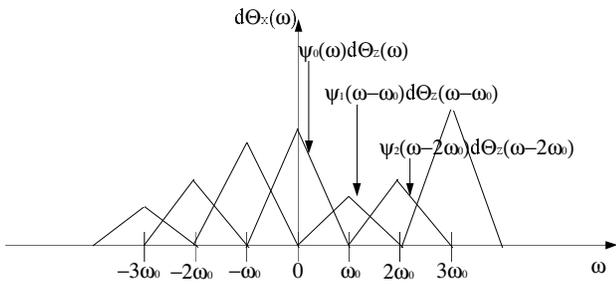


Figure 2, Modulus of the observed signal spectra

Equation (7) gives:

$$\begin{aligned} \forall l \in \mathbb{Z}, \quad \forall \omega \in [0, \omega_0[, \\ \psi_{l+1}(\omega - \omega_0) d\Theta_Z(\omega - \omega_0) \\ + \psi_l(\omega) d\Theta_Z(\omega) = d\Theta_X(\omega + l\omega_0) \end{aligned} \quad (8)$$

(8) corresponds to an infinity of linear redundant combinations of a finite number of variables. To identify these variables, we just have to choose two equations of (8) for two different values of  $l$ . For example, if we work with  $l$  and  $l+1$ , we obtain:

$$\begin{cases} \forall \omega \in [0, \omega_0[, \\ \psi_{l+1}(\omega - \omega_0) d\Theta_Z(\omega - \omega_0) \\ + \psi_l(\omega) d\Theta_Z(\omega) = d\Theta_X(\omega + l\omega_0) \\ \\ \psi_l(\omega - \omega_0) d\Theta_Z(\omega - \omega_0) \\ + \psi_{l-1}(\omega) d\Theta_Z(\omega) = d\Theta_X(\omega + (l-1)\omega_0) \end{cases} \quad (9)$$

The perfect reconstruction of  $Z(t)$  is then possible if there exists an  $l$  such that:

$$\begin{aligned} \forall \omega \in [0, \omega_0[, \\ \psi_l(\omega) \psi_l(\omega - \omega_0) - \psi_{l-1}(\omega) \psi_{l+1}(\omega - \omega_0) \neq 0 \end{aligned} \quad (10)$$

It follows that the identification of the spectral representation of  $Z$  may be obtained by:

$$\begin{cases} \forall \omega \in [-\omega_0, 0[, \quad d\Theta_Z(\omega) = \\ \frac{\psi_l(\omega + \omega_0) d\Theta_X(\omega + l\omega_0) - \psi_{l-1}(\omega + \omega_0) d\Theta_X(\omega + (l+1)\omega_0)}{\psi_l(\omega) \psi_l(\omega + \omega_0) - \psi_{l-1}(\omega + \omega_0) \psi_{l+1}(\omega)} \\ \\ \forall \omega \in [0, \omega_0[, \quad d\Theta_Z(\omega) = \\ \frac{\psi_l(\omega - \omega_0) d\Theta_X(\omega + l\omega_0) - \psi_{l+1}(\omega - \omega_0) d\Theta_X(\omega + (l-1)\omega_0)}{\psi_l(\omega) \psi_l(\omega - \omega_0) - \psi_{l-1}(\omega) \psi_{l+1}(\omega - \omega_0)} \end{cases} \quad (11)$$

$Z(t)$  is then given by the response of  $X(t)$  through a periodic time-varying filter  $\tilde{g}$ , such that its frequency response  $G_t(\omega)$  is defined by:

$$\begin{aligned} \forall \omega \in [0, \omega_0[, \\ \begin{cases} \forall k \notin \{l-1, l\}, \quad G_t(\omega + k\omega_0) = 0 \\ G_t(\omega + (l-1)\omega_0) = \frac{\psi_l(\omega) e^{-i l \omega_0 t} - \psi_{l+1}(\omega - \omega_0) e^{-i (l-1) \omega_0 t}}{\psi_l(\omega) \psi_l(\omega - \omega_0) - \psi_{l-1}(\omega) \psi_{l+1}(\omega - \omega_0)} \\ G_t(\omega + l\omega_0) = \frac{\psi_l(\omega - \omega_0) e^{-i l \omega_0 t} - \psi_{l-1}(\omega) e^{-i (l+1) \omega_0 t}}{\psi_l(\omega) \psi_l(\omega - \omega_0) - \psi_{l-1}(\omega) \psi_{l+1}(\omega - \omega_0)} \end{cases} \end{aligned} \quad (12)$$

In general, there exists several  $l$  that verify (10). We can then obtain several reconstructions of  $Z$ . This also allows to introduce error correction in the presence of interference or frequency selective noise.

### 4. EXAMPLE

Let  $Z(t)$  be an N.R.Z. signal [5], approximately bandlimited on  $[-\omega_0, \omega_0]$ , and  $\tilde{h}$  a sum of periodic clock changes. The latter is a special case of linear periodic time-varying filters [6].  $\tilde{h}$  is defined then by:

$$H_t(\omega) = e^{-i\omega\alpha \sin(2\omega_0 t)} + e^{i\omega_0 t} e^{-i\omega\beta \sin(\omega_0 t)} \quad (13)$$



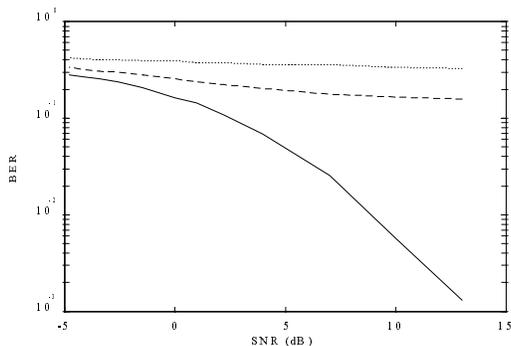


Figure 7, BER of reconstruction schemes

## 5. CONCLUSION

In this article we have presented a method of perfect linear reconstruction of a stationary process subjected to a linear periodic time-varying filter. We showed, in particular, that the reconstruction was possible even with overlapping spectral components. Finally, this was verified by a simulation example and we indicated the resistance of this system to the presence of additive noise.

## 6. REFERENCES

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