TIME-FREQUENCY ANALYSIS OF ACOUSTIC TRANSIENTS

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ABSTRACT

We apply time-frequency analysis to various acoustic transients. Estimates of the conditional mean frequency and the conditional bandwidth exhibit characteristic trends for different transients. This information may be helpful in distinguishing between different classes of transients, and in understanding the underlying mechanisms generating the transient.

1. INTRODUCTION

One of the primary means for locating and identifying submerged submarines is via the detection and analysis of acoustic emissions from the submarine. Spectral analysis has been particularly effective for identifying so called steady-state acoustic sounds, such as those associated with propeller noise. This analysis has been beneficial to the design of "quiet" submarines: silencing efforts, such as hull isolation and damping, have resulted in significant reduction in the radiation of steady-state acoustic energy from modern submarines. However, as the level of radiated steady-state or stationary energy has been reduced, the level of radiated energy associated with transient events, such as hatch slams and control surface movements, has not been significantly reduced. Consequently, transients have become a focus for detecting submerged submarines.

Because transients are potentially a primary source for detecting (and possibly identifying) submarines, it is becoming increasingly important to identify the threat they pose to detection by enemy ships. If US submarines are to acquire the same level of "quiet character" for transients as they currently have for steady-state acoustic emissions, a thorough understanding of the spectral structure and character of transients is essential. Unlike steady-state emissions, transients can not be fully characterized by the spectrum because, by their very nature, transients are time-varying, or nonstationary, signals. Accordingly, in this paper, we apply time-frequency analysis to study the changing spectral characteristics of a variety of transients recorded from several resonant structures. This analysis clearly reveals the time-varying spectral characteristics of transients.

While the spectral character of transients is often labeled "broadband," we show that the time-varying spectral characteristics of transients are informative and discriminative between different classes of transients. In particular, the conditional mean frequency and the conditional bandwidth exhibit characteristic trends for different transients. Furthermore, this analysis provides insight into the build-up and dissipation of energy in various resonant modes of the structure from which the transient emanated. This information may be helpful in distinguishing between different classes of transients, and in understanding the underlying mechanisms generating the transient.

2. BACKGROUND: TIME-FREQUENCY ANALYSIS

Conventional Fourier spectral analysis is a powerful technique that reveals the intensity of the frequencies in a wave. However, it does not indicate when these frequencies occurred, and thus the spectrum does not completely characterize processes for which frequencies change over time (e.g., transients, FM waves, music, speech and many biomedical signals). Time-frequency distributions (TFDs), $P(t,f)$, on the other hand, reveal the evolution of the intensity of the frequencies in a wave over time, and thus we learn not only what frequencies occurred and their respective intensity, but when they occurred as well.[1]

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As a joint energy density, it is required that [1]

$$P(t,f) \geq 0 \tag{1}$$

$$\int P(t,f)\,df = |s(t)|^2 \quad \text{and} \quad \int P(t,f)\,dt = |S(f)|^2 \tag{2}$$

where $|s(t)|^2$ and $|S(f)|^2$ are the temporal and spectral densities, respectively $\left(S(f) = \int s(t)\, e^{-j2\pi ft} dt \right)$, which are the marginal densities of $P(t,f)$. We note that the spectrogram, which is the most commonly used method of time-frequency analysis, satisfies the positivity condition above, but not the marginal conditions (the marginals of the spectrogram are $|s(t)|^2 * |h(t)|^2$ and $|S(f)|^2 * |H(f)|^2$, where $*$ denotes convolution, $h(t)$ is the window used in computing the spectrogram and $H(f) = \int h(t)\, e^{-j2\pi ft} dt$) [1]. Cohen and Posch [3] developed the theory and formulation for distributions satisfying eqs. (1) & (2), and Loughlin et al. [5] developed a method of implementation. A fast algorithm for this method was recently developed by Groutage [9], which allows the computation of these distributions for signals consisting of tens of thousands of samples.

From the density, one can extract information about the process in the form of conditional moments, such as the average frequency at a given time and the spread (standard deviation) about that quantity,

$$\langle f \rangle_t = \int f P(f|t)\, df \tag{3}$$

$$\sigma_{f|t} = \sqrt{\int (f - \langle f \rangle_t)^2 P(f|t)\, df} \quad . \tag{4}$$

We refer to these quantities as the conditional mean frequency and the conditional bandwidth, respectively. While it is often stated that the conditional mean frequency should equal the instantaneous frequency of the signal, that requirement is incompatible with eqs. (1) & (2) [1]. It is quite simple to give many cases where the instantaneous frequency can not be interpreted as the average frequency at each time; accordingly, it is our view that these two quantities are generally different time-varying frequencies (just as the mean, median and mode frequencies at each time are generally different for many signals) [6], [7], [8].

Cohen and Lee have shown that accurate estimates of the instantaneous frequency of a signal can be obtained from a (very) wideband spectrogram (for which the frequency marginal is grossly inaccurate) [2]. Conversely, it has been empirically demonstrated that a narrowband spectrogram can be used to estimate the time-varying average frequency (i.e., the conditional mean frequency) [4]. Examples of these two different estimates are shown in Figure 1. We used this latter approach to estimate the conditional mean frequency and conditional bandwidth of the transients investigated here.

3. Methods and Results

Ten occurrences of two different classes of acoustic transients were recorded from a resonant, thin-walled fluid-filled canister. The transients were generated by striking the canister with an impact force, and the response was observed using an accelerometer and recorded onto a hand-held DAT recorder. The class of the transients was pre-determined by the mechanism that generated the transient (i.e., two different impact forces). Time-frequency distributions and short-time (narrowband) estimates of the conditional mean frequency $\langle f \rangle_t$ and the conditional bandwidth $\sigma_{f|t}$, were computed.

Figure 2 illustrates a representative time series and spectral density of a transient from the two different classes. In both classes, the impact force was very brief, but more impulsive-like in the second class (bottom in Fig. 2) than in the first class (top). While the spectra of these two transients are similar (coincident spectral peaks), resonances above 2 kHz are significantly attenuated in class one compared to class two. Evidently, the more impulsive-like impact in the second class imparted greater energy into these higher resonances of the structure.

The differences between these two classes are even more dramatically revealed by time-frequency analysis. Figure 3 shows conditional mean frequencies and bandwidths for each of ten transients in class one (top) and class two (bottom). Apart from the expected difference in the magnitudes of the conditional mean frequencies between the two classes (with class two having much higher average frequency, as anticipated from the spectra in Fig. 2), the time-varying trend of the mean frequency is very different. In the first class, conditional mean frequency decays rapidly from approximately 400 Hz to 100 Hz

1. Actually, it is the conditional density $P(f|t)$ that reveals the changing spectral intensity over time. The joint density, $P(t,f)$, simultaneously indicates the temporal and spectral intensities of the signal and any interactions between the two. Thus, when the signal intensity is low (high) at certain times, the joint density is also low (high) at those times. Likewise, when the spectral intensity is low (high) at certain frequencies, the joint density is low (high) at those frequencies [5], [6].

in the first 60 msec. The second class, however, exhibits an initial *rise* in conditional mean frequency from approximately 3 kHz to 6 kHz in about 20 msec, followed by a rapid decay from 6 kHz to 2 kHz in about 60 msec. The differences in conditional bandwidth are equally striking. In the first class, there is no apparent trend in the time-varying bandwidth; on average, it is approximately constant. The second class, however, while exhibiting a much higher bandwidth as expected, shows a decreasing trend over time, with the time-varying bandwidth narrowing from approximately 11 kHz to 3 kHz in the first 100 msec of the transient.

A possible cause for the differences observed between these two classes is that the energy in the higher frequency modes of this structure, which are strongly excited in the second class, builds up slower than that in the lower frequency modes (which would account for the initial rise in conditional mean frequency in class two). This energy then dissipates more rapidly than the energy in the lower modes (which would explain the drop in conditional mean frequency after 20 msec and conditional bandwidth in class two.) The apparent absence of a change in conditional bandwidth of class one is perplexing, as the lower frequency modes obviously die away over time. It may be that the conditional bandwidth does indeed decrease, but the variance of the estimates (particularly at low signal levels) coupled with the small number of transients plotted obscures such a decrease (which would naturally be small compared to that of class two).

4. Conclusion

Time-frequency analysis provides complementary information to that obtained from the spectral density, which is of particular value in the study of time-varying signals, such as transients. In our analysis of two classes of transients, the conditional mean frequency and the conditional bandwidth exhibited characteristic trends that are dramatically different between the two classes. This information may be helpful in not only categorizing different classes of transients, but also in understanding the underlying mechanisms that generated the transient.

5. References

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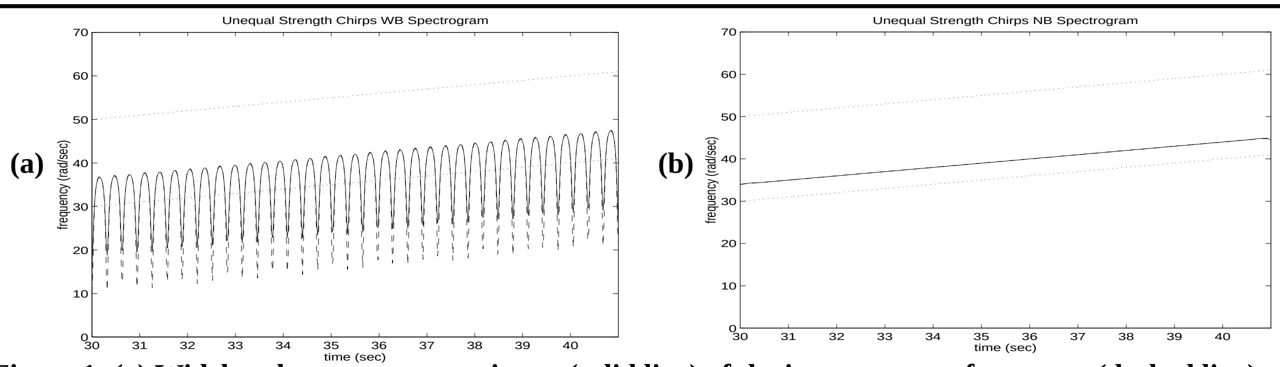

Figure 1: (a) Wideband spectrogram estimate (solid line) of the instantaneous frequency (dashed line) of a signal consisting of the sum of two chirps (dotted lines show individual chirp rates) of unequal strength amplitudes. (b) Narrowband spectrogram estimate (solid line) of the average frequency at each time (dashed line, obscured by estimate) of the same signal. Note that the average frequency at each time, which is a weighted sum of the individual instantaneous frequencies of the two chirps (dotted lines), is generally different from the instantaneous frequency. Interestingly, when the components are of equal strength, these two time-varying frequencies coincide. Similar results hold for the sum of two tones and other signals, as well. See [4], [6], [7], [8] for further details and discussion.

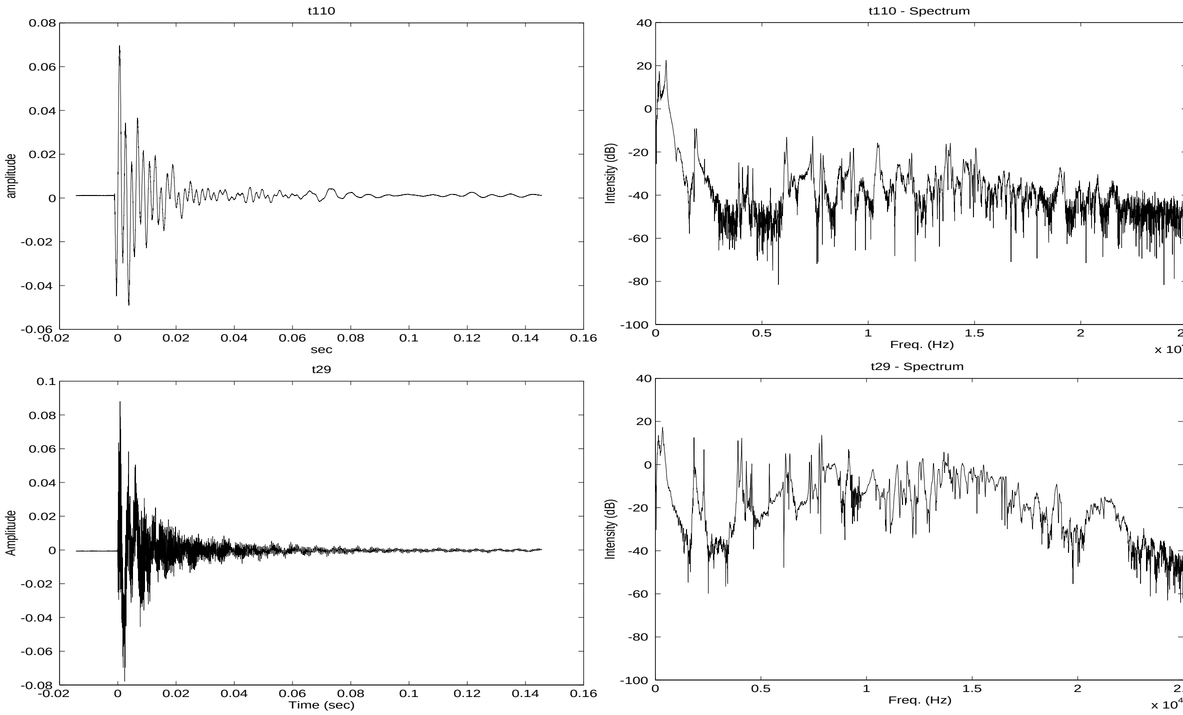

Figure 2: Representative transient time series (left) and spectra (right) from class one (top) and two (bottom).

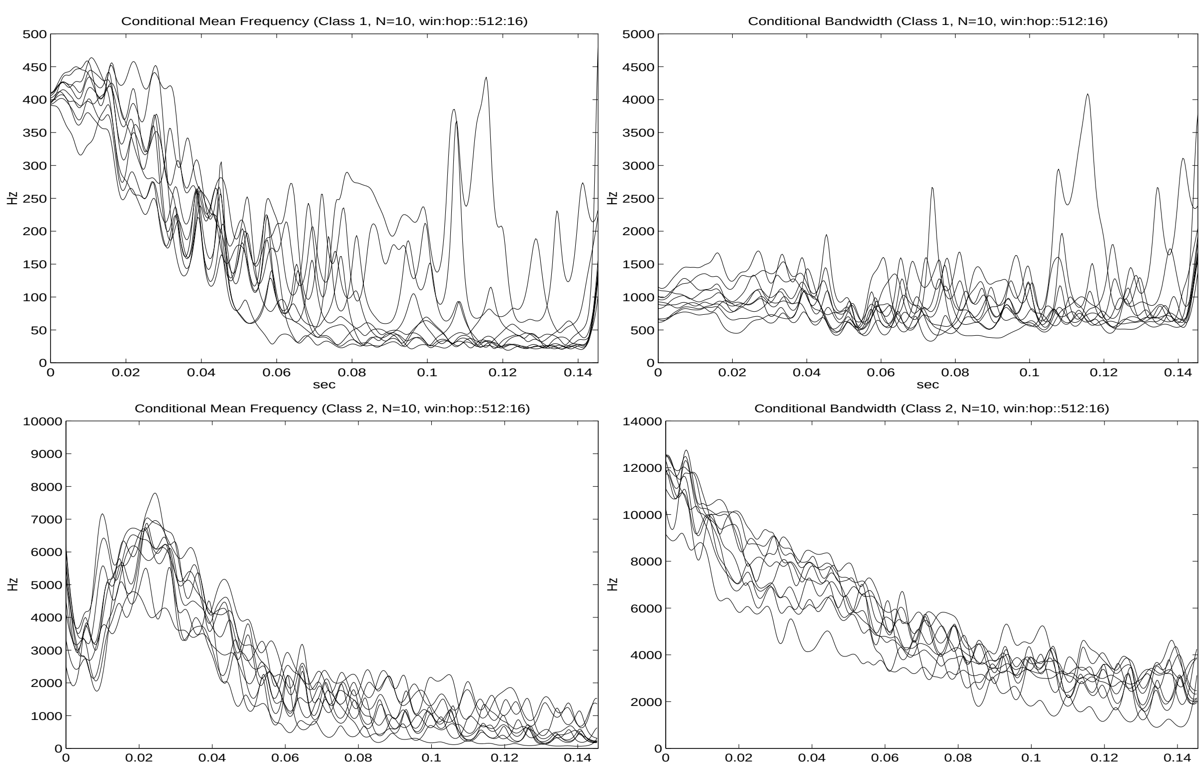

Figure 3: Conditional mean frequencies (left) and conditional bandwidths (right) for each of the ten transients in class one (top) and class two (bottom).