AN EFFICIENT FRACTIONAL SAMPLE DELAYER FOR DIGITAL BEAM STEERING

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ABSTRACT

In this paper we propose to use digital fractional sample delayers to perform high precision beam steering at the baseband sampling frequency. The major advantages of the proposed technique are that the fractional sample delayer (FSD) used has a very flat magnitude response within the baseband width allowing greater than 20-bit resolution for a 35-tap Finite Impulse Response filter. It also has a delay which is continuously variable providing resolutions greater than 220,000ths of the baseband sampling time. Owing to the signal delay being performed at the baseband rate, elements with different delays may be placed in parallel, allowing for the formation of multiple beams (e.g. tracking and surveillance capability simultaneously).

1. INTRODUCTION

Phased-array antennas find uses in many application areas such as radar, sonar, teleconferencing and medical ultrasound [1]-[4]. One operation that is normally required is the ability to electronically steer the array so that signals arriving from angles off bore-sight can be received with the maximum power. This is achieved by associating a delay-line with each antenna element, so that the far-field wave-front from the desired direction is realigned to broadside on the array. The classical approach towards performing this task is based on hardware phase shifters. These include: digitally switched phase shifters, diode phase shifters, ferromagnetic phase shifters, Reggia-Specer phase shifter and many others [1].

Figure 1 shows the structure for the proposed signal processing method, which includes a multirate version of the filter presented in [5], for a single antenna element. The fractional sample delay (FSD) filter in this configuration is designed to provide the required allpass characteristic for half-bandwidth only as the signal is bandlimited to approximately half-bandwidth by the preceding decimation filter. Using the Noble identity [6], the FSD is placed downstream from the down-

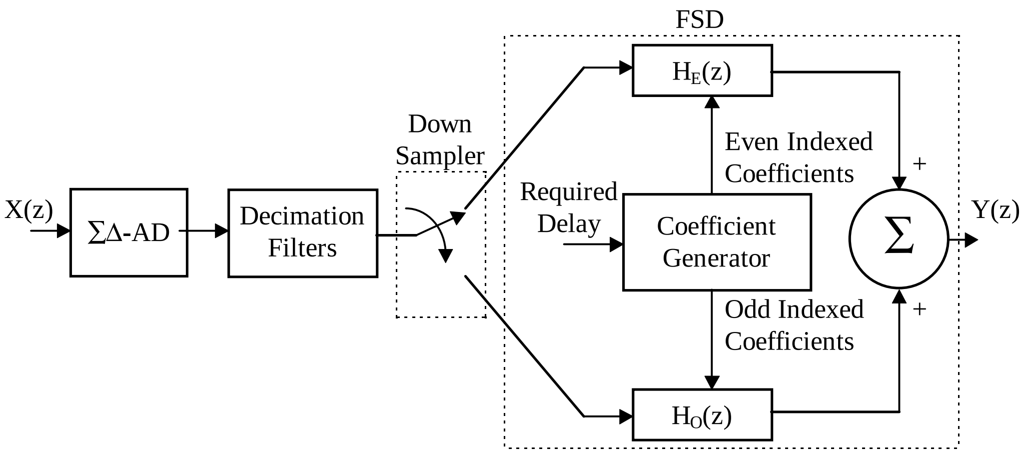

Figure 1: The signal delay scheme

sampler thus allowing it to be calculated at the baseband sampling rate, with its even and odd indexed coefficients being split between the upper and lower branches respectively. The delay is specified in terms of sample periods and, as the delay element is in effect operating at twice the baseband frequency, the required baseband fractional delay must be doubled.

Owing to the delay element being a finite impulse response (FIR) filter, the delay can be continuously changed on every output sample clock, without any transients. Integer delays are simply catered for by allowing the fractional delay filter to slide across the input buffers whose length is made longer than the number of branch taps of the filter to account for this.

The output from the decimation filter in Figure 1 is clocked at twice the baseband sampling frequency. This clock rate can be increased if the filter's cutoff frequency is altered proportionally (e.g. the clock rate is increased by two and the filter is given a quarter band specification). The advantage of this is that the proceeding fractional delay filter need only be designed for quarter band usage, with a corresponding reduction in its length. Again, using the Noble identity the FSD is split between four branches, allowing it to be operated at the baseband sampling rate. Of course, the disadvantage is that the decimation filter complexity will increase in order to maintain the baseband width specification. For example, to achieve a usable bandwidth of 96% with a linear-phase half band filter designed for 20-bit resolution, 413-taps are required. This increases to 859-taps when a quarter band filter is used.

2. DELAY SPECIFICATION

The main requirements for a beamsteering system are that it can steer the beam with a particular accuracy and not introduce magnitude distortion into the signals. Thus, to produce no magnitude distortion the passband ripple of the delay filter must be less than half the least significant bit of the required resolution within the design bandwidth of the final decimation filter. In [7] it was shown that of the commonly used interpolation techniques, Lagrange filters give excellent fractional delay characteristics. They can also be computed in closed form which makes them ideal for real-time applications.

Figure 2 shows the effects of Lagrange filter length on the passband magnitude peak error for usable bandwidths between 80% and 96% of the baseband

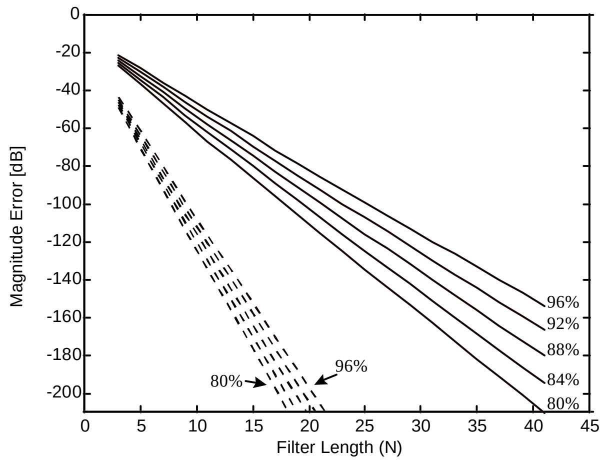

Figure 2: Dependance between the magnitude's peak error and filter length for various usable percentage band-widths. . Half-band filter (solid line) and quarter-band filter (dashed line).

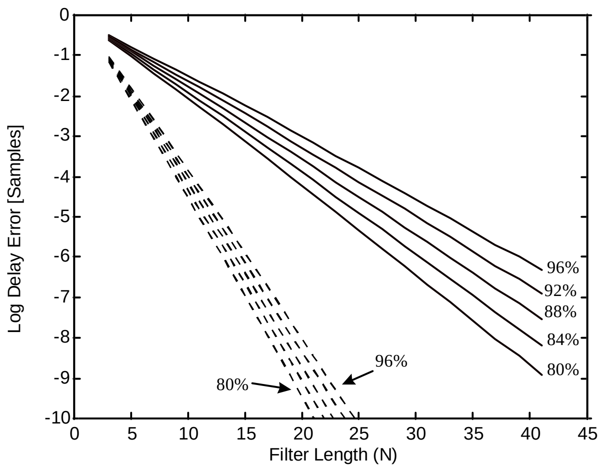

Figure 3: Delay's peak error for different filter lengths when various percentage band-widths are used. Half-band filter (solid line) and quarter-band filter (dashed line).

Nyquist frequency. In measuring the frequency response error, the required delay is set to a half sample as this provides the worst case for an odd length filter. The peak error occurs at the design band-edge owing to the interpolator's monotonic frequency response. Results are shown for both a two-branch structure (solid lines) and a four-branch structure (dashed lines). As expected the four-branch structure provides much better accuracy owing to the design bandwidth only having to approach quarter band. The actual design bandwidth required is determined by the quality of the preceding decimation filter (i.e. many commercial analogue-to-digital converters only provide a usable bandwidth of 80% to 90%.).

Figure 3 indicates the effects of the filter length on the delay resolution for the same conditions as used in Figure 2. It can be seen that for a 35-tap Lagrange filter it is possible to achieve 20-bit magnitude accuracy and 220,000ths of a sample delay quantization. This quantization level can be related to the steering angle step size via:

$$\theta_\Delta(m) = \sin^{-1}\left(\frac{mC}{f_s\delta d}\right) - \sin^{-1}\left(\frac{(m+1)C}{f_s\delta d}\right) \qquad (1)$$

where, f_s = baseband sampling frequency,
C = wave propogation velocity,
δ = delay quantization size,
d = array element spacing,
m = quantization level index

Naturally, as the steering angle is increased (i.e. m is raised) the angular resolution, θ_Δ decreases. Figures 2 and 3 also show that the delay resolution is directly related to the magnitude accuracy and, therefore, more magnitude accuracy than required may be provided in order to achieve a desired angular step size accuracy around a particular steering angle.

3. COEFFICIENT CALCULATION

The fractional sample delay filter is designed as a Lagrange interpolator with a novel scheme for the coefficient calculation which allows higher order filters to be recalculated in real-time [8]. The coefficients, h_k, can be computed from the Lagrange Interpolation formula:

$$h_k(\beta) = \prod_{\substack{i=0 \\ i\neq k}}^{N-1} \frac{\beta - i}{k - i} = \frac{n_k}{d_k} \qquad (2)$$

where β is the combined mid-array delay of the interpolator and required fractional delay. This requires a number of multiplications proportional to N^2, where N is the number of coefficients, when using either direct calculation methods or the Farrow structure [9],[10]. The scheme used in our filter design requires only 4N-8 multiplications giving almost an 88% saving in the number of multiplications required for a 35-tap filter. It is also applicable to both odd and even length filters. As with the former coefficient calculation schemes the reciprocals of the denominator factors, d_k, of (2) are pre-stored The numerator factors, n_k, are computed by firstly calculating the partial products (3):

$$ppf_0 = \beta \qquad (3a)$$

$$ppf_i = ppf_{i-1} \times (\beta - i)\,; \quad i = 1,2,\cdots,N-2 \qquad (3b)$$

$$ppr_0 = (\beta - N + 1) \qquad (3c)$$

$$ppr_i = ppr_{i-1} \times (\beta - N + 1 + i)\,; \quad i = 1,2,\cdots,N-2 \qquad (3d)$$

where $ppf_{0\ldots N-2}$ and $ppr_{0\ldots N-2}$ represent the stored forward and reverse partial products. These are then combined using (4) to give the numerator terms of (2):

$$n_0 = ppr_{N-2} \qquad (4a)$$

$$n_{N-1} = ppf_{N-2} \qquad (4b)$$

$$n_1 = n_0 + ppr_{N-3} \qquad (4c)$$

$$n_{N-2} = n_{N-1} - ppf_{N-3} \qquad (4d)$$

$$n_i = ppf_{i-1} \times ppr_{N-2-i}\,; \quad i = 2,\cdots,N-3 \qquad (4e)$$

In an architectural design to implement this scheme, savings in controller complexity can be achieved by removing Equations (4c) and (4d) and allowing the index, i to start from one in (4e), at the cost of two extra multiplications.

Figure 4 shows that further reductions in computational costs can be made when the coefficients are made to be fixed-point. In doing this some of the outlying coefficients become zero resulting in the number of multiplications, to produce an output, being reduced to:

$$Mults = N + 5M - 6 \qquad (5)$$

where M is the reduced length of the filter.

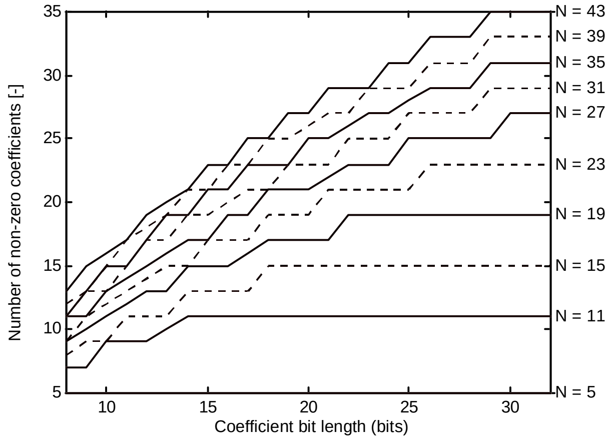

Figure 4: Comparison of filter length reductions owing to coefficient wordlength quantization.

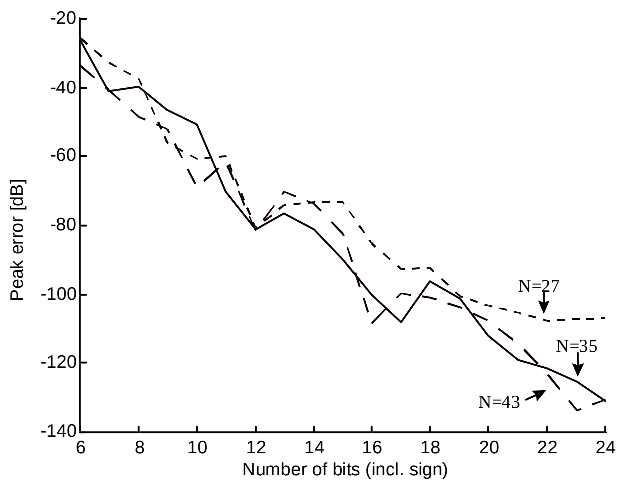

Figure 5: Effect of coefficient wordlength on peak magnitude error for different filter lengths.

Figure 5 shows the variation of the peak error with respect to coefficient wordlength for a family of filters with different filter lengths. These results are for a required delay of 0.5 samples and 96% base-bandwidth and show that a 35-tap FSD with 23-bit signed-coefficients is able to achieve greater than 20-bit resolution. It has been confirmed that this graph provides the worst case peak error by performing the experiment with delay values other than 0.5.

4. CONCLUDING REMARKS

This paper has presented an efficient method for performing beam steering at the baseband sampling rate. The proposed digital signal processing scheme utilizes the concept of the variable fractional sample delay filter which was presented in our earlier publications. Here we provide more detailed indications on how to select the principal parameters of the filter in order to achieve required properties (i.e. bandwidth usage and bit-resolution) of the overall system. Our early investigations suggest that the technique has many benefits over the existing solutions. The most important advantage is better angular resolution of the beam steering.

Owing to the simplicity of the filter design method the scheme does not only have to be used for linear arrays, but can also be used with non-uniformly spaced arrays. Wavefront curvature can also be easily handled as the delay elements associated with each receiver element may be controlled independently to a very high accuracy.

The implementation of the scheme on a semi-custom VLSI chip is currently under investigation.

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