where $\frac{1}{E\{|a_i|^2\}^2}$ and $\frac{1}{E\{|a_i|^4\}-3E\{|a_i|^2\}^2}$ are *a priori* known input dependent constants which we will call α and β respectively. We now differentiate the above cost function with respect to $\bar{\mathbf{w}}$ where $\bar{\mathbf{w}} = [w_N, \cdots, w_1, w_{-1}, \cdots, w_{-N}]^T$. The "barred" $\mathbf{w}$ is used because the middle component of $\mathbf{w}$ has to be fixed at 1 to avoid $\mathbf{s} = \mathbf{0}$. This gives the newly proposed algorithm

$$\bar{\mathbf{w}}(i+1) = \bar{\mathbf{w}}(i) - \mu\left[(\alpha+3\beta)E\{|z_i|^2\} - \beta|z_i|^2\right] \cdot \cdot(z_i^*\bar{\mathbf{y}}_i + z_i\bar{\mathbf{y}}_i^*) \quad (15)$$

where $\bar{\mathbf{y}}_i = [y_{i+N}, \cdots, y_{i+1}, y_{i-1}, \cdots, y_{i-N}]^T$, * denotes complex conjugation and 2N is the length of the equalizer adaptive parameter vector $\bar{\mathbf{w}}$. In the actual implementation of this algorithm the expectation $E\{|z_i|^2\}$ is approximated on-line in the following manner: $E\{|z_{i+1}|^2\} = E\{|z_i|^2\} + \frac{1}{i}(|z_i|^2 - E\{|z_i|^2\})$.

Other cost functions from the above class can be implemented in the similar manner. Figure 6 shows how five of

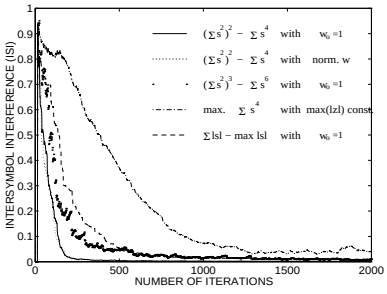

Figure 6. Comparison of five algorithms out of the proposed class of algorithms. Input is 4-level QAM, Channel impulse response is [.04 -.05 .07 -.21 -.5 .72 .36 .21 .03 .07], with 13 adaptive weights and no noise.

the newly presented algorithms compare with each other in the areas of convergence speed and minimum achieved ISI. From the figure it can be easily seen that the fastest convergence as well as the lowest residual ISI is achieved by the $(\sum_i s_i^2)^2 - \sum_i s_i^4$ cost function's algorithm. It is also the least computationally complex algorithm of the above proposed class. We now compare the $(\sum_i s_i^2)^2 - \sum_i s_i^4$ cost function's algorithm to the existing ones through simulations. From Figure 7 we can see that the performance of the new algorithm is superior to [9] and similar to Godard's without the local minima.

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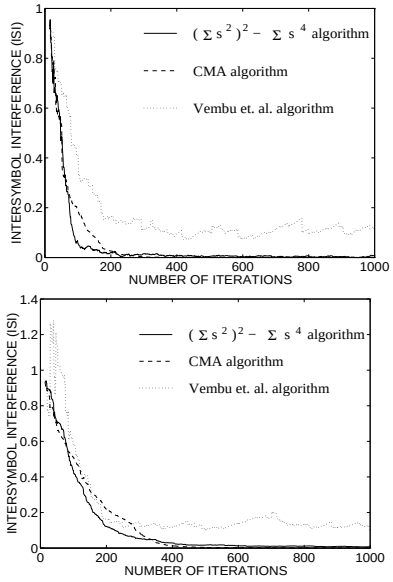

Figure 7. Comparison of the three available algorithms. Input is 4-level QAM, Channel impulse response is [.04 -.05 .07 -.21 -.5 .72 .36 .21 .03 .07], with no added noise (top) and 15db SNR additive white noise (bottom), with 13 adaptive weights.

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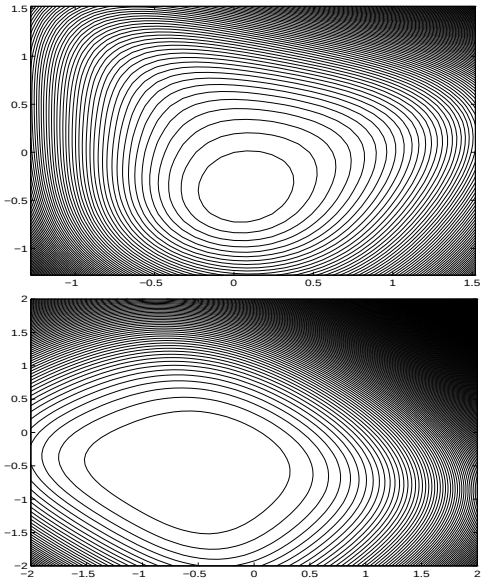

Figure 3. Comparison of the contours of the $\|s\|_2^4 - \|s\|_4^4$ for channel impulse response [.04 -.05 .07 -.21 -.5 .72 .36 .21 .03 .07] (top) and [.5 1 .5] (bottom) and equalizer of length 3 with middle weight fixed at 1.

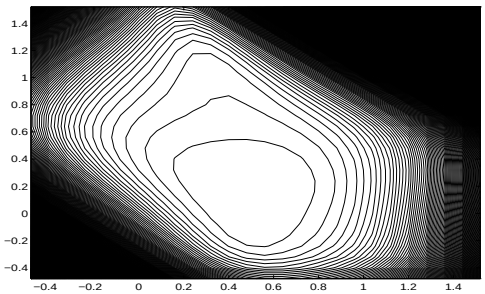

Figure 4. The contour plot of $\|s\|_4^2 - \|s\|_8^8$ cost function for channel impulse response [1 -.6 .36] and equalizer of length 3 with middle weight fixed at 1.

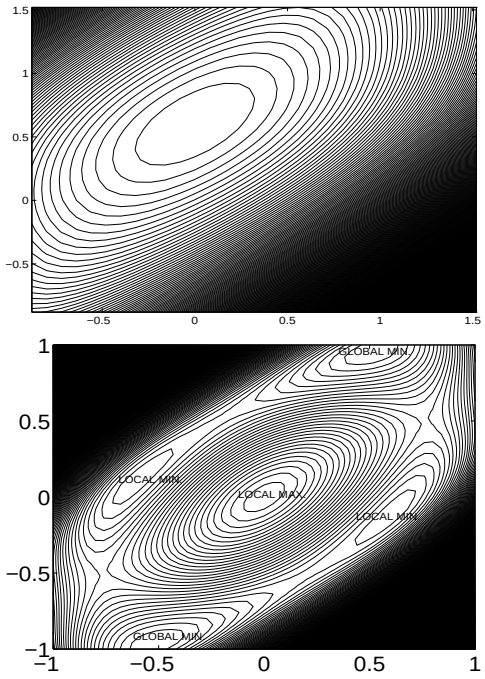

Figure 5. Comparison of the contours of the $\|s\|_2^4 - \|s\|_4^4$ (top) and Godard-2 (bottom) cost functions. Channel impulse response=[1 -.6 .36].

only, we will not need the $|\cdot|$. The input-output relation is $z_i = \sum_j a_{i-j} s_j$ Therefore, by standard manupulation and envoking the zero mean, i.i.d. assumption of the input a_i (note that if a_i's probability density function is symmetric about the origin (this is almost always the case) all the odd moments of a i.e. Ea^3, Ea^5 etc. are equal to zero) we get

$$\begin{aligned} E\{|z_i|^2\} &= \sum_{j_1}\sum_{j_2} E\{a_{i-j_1} a^*_{i-j_2}\} s_{j_1} s_{j_2} \\ &= E\{|a_i|^2\} \sum_j s_j^2 \end{aligned} \tag{12}$$

Similarly,

$$\begin{aligned} E\{|z_i|^4\} &= \sum_{j_1}\sum_{j_2}\sum_{j_3}\sum_{j_4} E\{a_{i-j_1} a^*_{i-j_2} a_{i-j_3} a^*_{i-j_4}\} \cdot \\ &\quad \cdot s_{j_1} s_{j_2} s_{j_3} s_{j_4} \\ &= E\{|a_i|^4\} \sum_j s_j^4 \\ &\quad + 3E\{|a_i|^2\}^2 [(\sum_j s_j^2)^2 - \sum_j s_j^4] \end{aligned} \tag{13}$$

Rearranging and solving (2) and (1) for $\sum_j s_j^4$ and $\sum_j s_j^2$ respectively we get our cost function as a function of z's

$$\begin{aligned} (\sum_j s_j^2)^2 - \sum_j s_j^4 &= [\frac{1}{E\{|a_i|\}^2} E\{|z_i|^2\}]^2 \\ &\quad - \frac{E\{|z_i|^4\} - 3E\{|z_i|^2\}^2}{E\{|a_i|^4\} - 3E\{|a_i|^2\}^2} \end{aligned} \tag{14}$$

intersymbol interference ISI in terms of the joint response s as $ISI = \frac{\sum_i |s_i|^2 - |s|^2_{max}}{|s|^2_{max}}$ [8].

3. A REFINED CLASS OF COST FUNCTIONS

It was shown in [1] that by minimizing the distance between any two norms of s where the p-norm of s is defined as $\{\sum_i |s_i|^p\}^{\frac{\zeta}{p}}$, i.e., $\{\sum_i |s_i|^p\}^{\frac{\zeta}{p}} - \{\sum_i |s_i|^q\}^{\frac{\zeta}{q}}$ where q is any number $> p$ and ζ is any integer > 0, we will arrive at the equalization condition. We examined $\sum_j |s_j| - \max_j |s_j|$ in [1] and showed that it is convex in s. However, this kind of convexity is only sectional (see Fig. 2), and hence may not be transformed into convexity in **w**. In this presentation we refine our previous work [1]. The newly refined class presented here is $\{\sum_i |s_i|^p\}^{\frac{\zeta}{p}} - \{\sum_i |s_i|^q\}^{\frac{\zeta}{q}}$ where p and q are *even* integers and ζ is the least common multiple of p and q. Here are some examples:

$$\{\sum_j |s_j|^2\}^2 - \sum_j |s_j|^4 \tag{3}$$

$$\{\sum_j |s_j|^2\}^3 - \sum_j |s_j|^6 \tag{4}$$

$$\{\sum_j |s_j|^4\}^2 - \sum_j |s_j|^8 \tag{5}$$

Note that due to the above mentioned property of norms there exist many other possibilities of arriving at the equalization condition, for example

$$fix \ \max_s |s_k|, \ minimize \ \sum_j |s_j|^p, \ p < \infty \tag{6}$$

$$minimize \ \& \ then \ fix \ \sum_j |s_j|^2, \ maximize$$

$$\sum_j |s_j|^p, \ p > 2 \tag{7}$$

$$fix \ \sum_j |s_j|, \ maximize \ \sum_j |s_j|^p, \ p > 1 \tag{8}$$

But it is the class represented by (3) - (5) that exhibits unimodality and thus is the focus of this paper.

We now show that $(\sum_j s_j^2)^p - \sum_j s_j^{2p}$ are unimodal for each delay k or unimodal in a wide-sense in s. To see where the extrema are located the partial derivative of $(\sum_j s_j^2)^p - \sum_j s_j^{2p}$ with respect to s_i is taken and set equal to 0.

$$\begin{aligned} \frac{\partial[(\sum_j s_j^2)^p - \sum_j s_j^{2p}]}{\partial s_i} &= 2p(\sum_j s_j^2)^{p-1} s_i - 2p s_i^{2p-1} \\ &= [(\sum_j s_j^2)^{p-1} - s_i^{2(p-1)}] 2p s_i = 0 \end{aligned} \tag{9}$$

Equation (9) has two solutions, one of which corresponds to s = **0**. This solution will not occur because of a fixed equalizer weight. The other solution is the minimum corresponding to perfect equalization. This is shown by the following:

$$s_i^{2(p-1)} = (\sum_j s_j^2)^{p-1}. \tag{10}$$

Taking the $(p-1)st$ root of both sides

$$s_i^2 = (\sum_j s_j^2) \tag{11}$$

The equation (11) only holds when s has at most one nonzero element, i.e., is the desired delta function. Thus $(\sum_j s_j^2)^p - \sum_j s_j^{2p}$ are unimodal in s.

The above proven unimodality in s does not imply unimodality in **w**. However, we have plotted the surfaces of these cost functions for numerous channels and two adaptive weights. All plots turn out to be unimodal. This leads us to believe that the proposed class of cost functions is unimodal (may not be convex) under mild conditions. Figures 3 and 4 show various cost functions for various channels. Clearly, they are all unimodal. Another example of the contour surfaces of the newly proposed and the Godard cost functions in two adaptive weights **w** is shown in Figure 5. From Figure 5 it is clear that Godard's (p=2) algorithm exibits local minima [10]. These minima will cause insufficient ISI removal if the algorithm is not properly initialized. Our cost function's surface however, is unimodal.

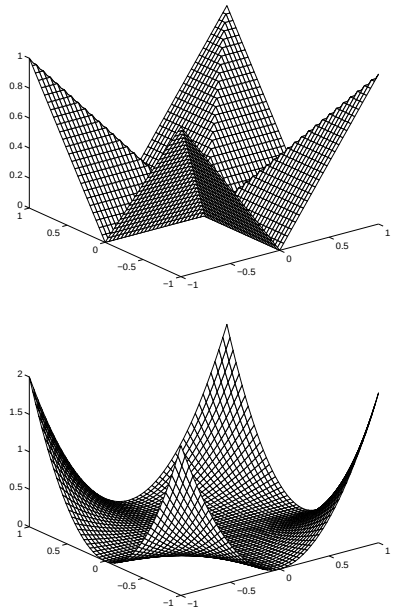

Figure 2. Plot of the cost functions $\sum_j |s_j| - \max |s_j|$ (top) and $\{\sum_j |s_j|^2\}^2 - \sum_j |s_j|^4$ (bottom), in s (s_1 and s_2)

3.1. Implemention

Since s is not available we need to convert our cost function from a function of s's to a function of z's. For QAM signals $a_i \in \mathcal{C}$. This implies that y_i and z_i are also $\in \mathcal{C}$ while s $\in \Re$. Since s $\in \Re$ and we will be using even powers of $|s_j|$

A REFINED CLASS OF COST FUNCTIONS IN BLIND EQUALIZATION

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ABSTRACT

The use of gradient descent recursive algorithms in blind adaptive equalization requires a cost function with a unique minimum such that the FIR equalizer setup removes sufficient Intersymbol Interference (ISI). A cost function based on minimizing the difference between the second and the fourth norms of the joint channel-equalizer impulse response, each raised to the fourth power i.e., $\|\cdot\|_2^4 - \|\cdot\|_4^4$ is proposed. An implementable recusive on-line algorithm using the above cost function is also derived for QAM inputs. A sizable array of examples shows that the above class is unimodal in equalizer weights. Extensive simulations show that the performance of the newly proposed algorithms is comparable to the CMA algorithms' performance without the misconvergences.

1. INTRODUCTION

Blind equalization is the on-line recovery of discretely distributed source signals at the output of a channel using only prior knowledge of the source signal's alphabet and statistics. Blind equalization for digital data communication was first introduced by Y. Sato [2] and was generalized in subsequent works by Godard [3], Treichler [4], Benveniste *et al.* [5] [6], and Picci *et al.* [7]. The above adaptive algorithms for parameter updating replace the prediction error in the traditional LMS algorithm with a modified error signal not involving the reference signal or the "training signal". These algorithms can be viewed as generalized "Bussgang" algorithms since upon convergence the equalizer output attains the Bussgang statistical property. The above algorithms are computationally simple and thus easy to implement. Due to their computational simplicity and ease of implementation, these algorithms are widely used in the communication industry. The price of the above simplicity is, however the risk of possible misconvergences. The misconvergences are caused by lack of unimodality (or convexity) of the algorithm's cost functions [3] [4] [10]. Insufficient removal of Intersymbol Interference (ISI) can occur if algorithm's initialization results in a convergence to a stable local minimum.

In this paper a family of cost functions is proposed which is general and promising. This family consists of a number of cost functions, many of which exhibit wide-sense unimodal performance surfaces.

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2. BACKGROUND

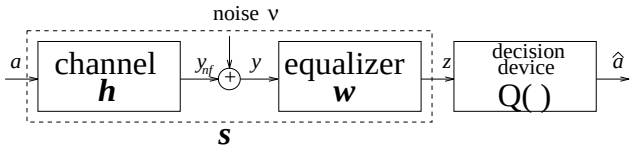

Figure 1. Blind equalization model

Figure 1 depicts a typical (baseband) representation of a time-invariant communication system (assuming operation in the discrete-time domain) where a_i is the digitized input signal (e.g. $\pm 1 \pm j$) and the output signal y_i is the result of convolving a_i with the channel impulse response h_i, as shown by

$$y_{nf,i} = \sum_{j=0}^{M} h_j a_{i-j} \tag{1}$$

where M+1 is the length of the channel (can be finite or infinite) and it is assumed that h_j is zero for $j < 0$ (i.e., the system is causal). Define $\mathbf{w} \triangleq [w_{-N}\ w_{-N+1} \ldots w_{N-1}\ w_N]^T$, the parameter vector of the equalizer weights of length $2N+1$ with middle weight, w_0 set at 1 to prevent the all zero setup [1]. Denote the impulse response of the total channel-equalizer combination as $\mathbf{s}_i = [s_{i+N}, \ldots, s_{i-(N+M)}]$ whose elements are given by

$$s_i = \sum_{j=-N}^{N} h_{i-j} w_j, \tag{2}$$

The original channel input is normally restored by sending the equalizer output z_i into a decision device. The decision device which is typically a quantizer can be simply viewed as a direct map $Q : \Re \rightarrow \mathcal{A}$ where $\mathcal{A}$ is the finite alphabet of which the input data a_i consists of. The objective of the blind equalizer is to adjust $\mathbf{w}$ such that the output sequence $\hat{a}$ is a delayed version of the input sequence a. In other words the total channel-equalizer combination $\mathbf{s}$ (after equalization) should be $\mathbf{s} = \sum_j h_{k-j}\hat{w}_j = \delta_k$ where $\hat{\mathbf{w}}$ is the optimum weight vector and δ_k is the *Kronecker delta* and k is an arbitrary delay. This will be referred to as the equalization condition, which is equivalent to $ISI = 0$. Define