CONJUGATE GRADIENT METHOD FOR ADAPTIVE DIRECTION-OF-ARRIVAL ESTIMATION OF COHERENT SIGNALS

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ABSTRACT

A method for the direction-of-arrival (DOA) estimation of coherent signals is proposed, based on the adaptive version of Pisarenko's harmonic retrieval method. It is known that for the DOA estimation of coherent signals, the computed covariance matrix of the sensor array must be spatially smoothed to preserve its full rank. Adaptive algorithms using the Conjugate Gradient (CG) methods can take advantage of this pre-processing by incorporating the available smoothed matrix into the algorithm. The proposed algorithm uses the CG algorithm presented in [3] in combination with spatial and temporal smoothing techniques. Our simulations show that the proposed algorithm has a fast convergence rate even when the input signals are coherent. Due to the use of an updated covariance matrix at each time instant, no internal iterations are used as in regular CG methods, resulting in a more efficient algorithm than previously proposed CG methods.

1. INTRODUCTION

Recently, Conjugate Gradient (CG) methods have been suggested for adaptive filtering and spectral estimation [1, 2, 3, 4, 5]. In all these methods, it is necessary to estimate the covariance matrix of the input data vector, which is usually obtained by ensemble averaging, using a rectangular data window as in [1, 2, 4] or an exponentially decaying data window as in [2, 3, 5].

In array signal processing, for the estimation of the direction-of-arrival (DOA) of coherent signals, the covariance matrix must be explicitly computed by averaging the covariance matrices of subarrays [8, 9, 11, 12]. This is also known as Spatial Smoothing. The computation of the spatially smoothed covariance matrix is necessary because the presence of coherent signals results in a rank-deficient matrix. By applying spatial smoothing techniques, the rank of the covariance matrix can be restored. In this case, Conjugate Gradient methods are especially suitable to implement a DOA estimator due to the availability of the covariance matrix. In [7], the Conjugate Gradient method was used with spatial smoothing to solve the beamforming problem. It was shown, in their particular implementation, that spatial and temporal smoothing techniques give comparable results for uncorrelated signals. Here, the CG algorithm presented in [3] is applied to the DOA estimation problem. It will be shown that, using spatial and temporal smoothing techniques to compute the covariance matrix, the convergence of the adaptive DOA estimator is fast, even for coherent signals. Furthermore, by allowing the covariance matrix to vary between iterations as described in [2, 3], computationally more efficient algorithms can be obtained.

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2. DIRECTION-OF-ARRIVAL (DOA) ESTIMATION

First, consider that p incoherent narrow-band planar wavefronts are impinging on a uniformly spaced linear array of omnidirectional sensors. The data vector of length M can be described as

$$\mathbf{y}(n) = \mathbf{x}(n) + \mathbf{v}(n) \tag{1}$$

where $\mathbf{v}(n)$ is a white additive noise vector and $\mathbf{x}(n)$ is the signal vector. (1) can be rewritten as

$$\mathbf{y}(n) = \mathbf{A}\mathbf{s}(n) + \mathbf{v}(n) \tag{2}$$

where $\mathbf{s}(n)$ is the vector with the complex amplitude of the p signals, and $\mathbf{A}$ is the $M \times p$ Vandermonde matrix defined as

$$\mathbf{A} = [\mathbf{a}(\theta_1) \;\; \mathbf{a}(\theta_2) \;\; \cdots \;\; \mathbf{a}(\theta_p)] \tag{3}$$

where

$$\mathbf{a}(\theta_i) = [1 \;\; e^{j\phi_i} \;\; e^{j2\phi_i} \;\; \cdots \;\; e^{j(M-1)\phi_i}]^T. \tag{4}$$

The vector $\mathbf{a}(\theta_i)$ is called the "steering vector" and the electrical angle ϕ_i of the incident wave is given by

$$\phi_i = 2\pi \frac{d}{\lambda} sin\theta_i = \omega_0 \tau_i$$

where $\lambda = c/f_0$, c is the speed of propagation, $\omega_0 = 2\pi f_0$, $\tau_i = \frac{d}{c} sin\theta_i$, d is the uniform sensor spacing, and θ_i is the angle of incidence (direction-of-arrival) of the signal. The covariance matrix of the input data vector is given by [8, 9]

$$\mathbf{R} = E[\mathbf{y}(n)\mathbf{y}(n)^H] = \mathbf{A}\mathbf{S}\mathbf{A}^H + \sigma_v^2 \mathbf{I} \tag{5}$$

where $(\cdot)^H$ denotes the transpose conjugate, $\mathbf{S} = E[\mathbf{s}(n)\mathbf{s}(n)^H]$ is a diagonal $p \times p$ matrix whose elements specify the power of each signal and $\mathbf{A}\mathbf{S}\mathbf{A}^H$ has full rank p

for uncorrelated sources. Usually we have $p < M$, so that the minimum eigenvalue of $\mathbf{R}$ is σ_v^2.

The eigenvector $\mathbf{q}_{min}$ corresponding to the minimum eigenvalue σ_v^2 is orthogonal to the columns of the matrix $\mathbf{A}$. Furthermore, the angles of the roots of a polynomial, whose coefficients are the elements of $\mathbf{q}_{min}$, are the harmonic frequencies contained in $\mathbf{R}$, i.e., the electrical angles. When the source signals are uncorrelated, the matrix $\mathbf{ASA}^H$ has full rank (p) and the electrical angles can be estimated from the elements of $\mathbf{q}_{min}$, as in Pisarenko's harmonic retrieval method. From the electrical angles ϕ_i, the direction-of-arrival of the wavefronts θ_i can be determined.

3. IMPLEMENTING DOA ESTIMATION OF COHERENT SIGNALS USING SPATIAL SMOOTHING

When the input signals are coherent (perfectly correlated) or highly correlated, the rank of $\mathbf{ASA}^H$ will drop [8, 11, 12]. Because of the Vandermonde structure of $\mathbf{A}$, no linear combination of steering vectors (in the case of correlated signals) can result in another steering vector. Consequently, the electrical angles cannot be estimated from $\mathbf{q}_{min}$. In order to avoid the collapse of the rank of $\mathbf{R}$ and consequently the rank of $\mathbf{ASA}^H$, spatial smoothing methods have been proposed that guarantee full rank for the smoothed $\mathbf{R}$ [8, 11, 12, 13].

A simple method of Spatial Smoothing (SS) consists of the averaging of the covariance matrices of subarrays. The method is described as follows:

Consider p completely coherent sources, where a "snapshot" of the M sensor outputs at any time instant is given by

$$\mathbf{y}(n) = [y_1(n),\ y_2(n),\ \ldots\ ,\ y_M(n)]^T.$$

Define k subarrays of length $p+1$ as

$$\begin{aligned}
\mathbf{z}_1(n) &= [y_1(n),\ y_2(n),\ \ldots\ ,\ y_{p+1}(n)]^T \\
\mathbf{z}_2(n) &= [y_2(n),\ y_3(n),\ \ldots\ ,\ y_{p+2}(n)]^T \\
&\ \ \vdots \\
\mathbf{z}_k(n) &= [y_k(n),\ y_{k+1}(n),\ \ldots\ ,\ y_M(n)]^T
\end{aligned}$$

then compute a spatially smoothed covariance matrix as

$$\mathbf{R}_{SS} = \frac{1}{k}\sum_{i=1}^{k} E[\mathbf{z}_i(n)\mathbf{z}_i(n)^H]. \tag{6}$$

It has been shown in [8, 11, 12, 13] that the matrix $\mathbf{R}_{SS}$, also called the forward spatially smoothed covariance matrix [8], has full rank p when $k \geq p$. After computing the spatially smoothed covariance matrix of the subarrays, it is still possible to use time averaging (temporal smoothing) as shown in [3], resulting in

$$\mathbf{R}(n) = \lambda_f \mathbf{R}(n-1) + \mathbf{R}_{SS}. \tag{7}$$

4. THE ADAPTIVE IMPLEMENTATION OF PISARENKO'S HARMONIC RETRIEVAL METHOD USING THE CG ALGORITHM

The CG algorithm presented in [2] is reproduced here for convenience:

Set initial conditions: $\mathbf{w}(0) = \mathbf{0}$, $\mathbf{g}(0) = \mathbf{b}(0)$, $\mathbf{p}(1) = \mathbf{g}(0), n = 1$.

$$\begin{aligned}
\alpha(n) &= \eta\frac{\mathbf{p}(n)^T\mathbf{g}(n-1)}{\mathbf{p}(n)^T\mathbf{R}(n)\mathbf{p}(n)} && (8)\\
\mathbf{w}(n) &= \mathbf{w}(n-1) + \alpha(n)\mathbf{p}(n) && (9)\\
\mathbf{g}(n) &= \lambda_f\mathbf{g}(n-1) - \alpha(n)\mathbf{R}(n)\mathbf{p}(n) && (10)\\
&\quad + \mathbf{x}(n)(d(n) - \mathbf{x}(n)^T\mathbf{w}(n-1)) && (11)\\
\beta(n) &= \frac{(\mathbf{g}(n)-\mathbf{g}(n-1))^T\mathbf{g}(n)}{\mathbf{g}(n-1)^T\mathbf{g}(n-1)} && (12)\\
\mathbf{p}(n+1) &= \mathbf{g}(n) + \beta(n)\mathbf{p}(n) && (13)
\end{aligned}$$

where $\alpha(n)$ is the step size that minimizes a cost function $f(\mathbf{w})$, defined as $f(\mathbf{w}) = \frac{1}{2}\mathbf{w}(n)^T\mathbf{R}(n)\mathbf{w}(n) + \mathbf{b}(n)^T\mathbf{w}(n)$ (see [2]), $\beta(n)$ provides quasi $\mathbf{R}$-conjugacy for the direction vector $\mathbf{p}(n)$, $\mathbf{g}(n)$ is the residual vector defined as $\mathbf{g}(n) = -\nabla f(\mathbf{w})^T$, $\mathbf{R}(n)$ is the estimated covariance matrix of the input data vector $\mathbf{x}(n)$, and η in (8) controls the convergence of the algorithm as described in [2].

This algorithm was used in [3] to implement an adaptive version of Pisarenko's harmonic retrieval method. Other adaptive implementations can be found in [4, 10]. Here we use it to implement an adaptive DOA estimator for coherent sources. The algorithm becomes

Set initial conditions: $\bar{\mathbf{w}}(0) = [1, 0, ..., 0]^T$, $\mathbf{g}(0) = [-1, 0, ..., 0]^T$, $\mathbf{p}(1) = \mathbf{g}(0)$, $n = 1$.

$$\begin{aligned}
\alpha(n) &= \eta\frac{\mathbf{p}(n)^H\mathbf{g}(n-1)}{\mathbf{p}(n)^H\mathbf{R}(n)\mathbf{p}(n)} && (14)\\
\mathbf{w}(n) &= \bar{\mathbf{w}}(n-1) + \alpha(n)\mathbf{p}(n) && (15)\\
\bar{\mathbf{w}}(n) &= \mathbf{w}(n)/\|\mathbf{w}(n)\| && (16)\\
\mathbf{g}(n) &= \frac{1}{\|\mathbf{w}(n)\|}[\lambda_f\mathbf{g}(n-1) - \alpha(n)\mathbf{R}(n)\mathbf{p}(n) \\
&\quad -\frac{1}{k}\sum_{i=1}^{k}\mathbf{z}_i(n)\mathbf{z}_i(n)^H\bar{\mathbf{w}}(n-1)] && (17)\\
\beta(n) &= \frac{(\mathbf{g}(n)-\mathbf{g}(n-1))^H\mathbf{g}(n)}{\mathbf{g}(n-1)^H\mathbf{g}(n-1)} && (18)\\
\mathbf{p}(n+1) &= \mathbf{g}(n) + \beta(n)\mathbf{p}(n) && (19)
\end{aligned}$$

where $\|\mathbf{w}_k\| = (\mathbf{w}_k^H\mathbf{w}_k)^{1/2}$, $\mathbf{R}(n)$ is the covariance matrix defined in (7), and the covariance matrices of subarrays used in $\mathbf{R}_{SS}$ are estimated using their instantaneous versions. After the convergence of the algorithm, $\bar{\mathbf{w}}(n)$ will converge to $\pm\mathbf{q}_{min}$ as shown in [3].

Note that the algorithm doesn't provide exact $\mathbf{R}$-conjugacy for the direction vector $\mathbf{p}(n)$ due to the use of a variable $\mathbf{R}$ for each time instant, and due to the weight vector normalization. In this situation, the algorithm will not converge in finite steps as in the regular CG methods [6]. Therefore it is preferable not to use internal iterations per time instant, reducing the complexity of the algorithm. The use of time averaging (temporal smoothing) in addition to the spatial smoothing improves the performance of the algorithm by reducing the estimation noise. The effect of using various window lengths for the computation of the time averaged matrix $\mathbf{R}$ has been shown in [3].

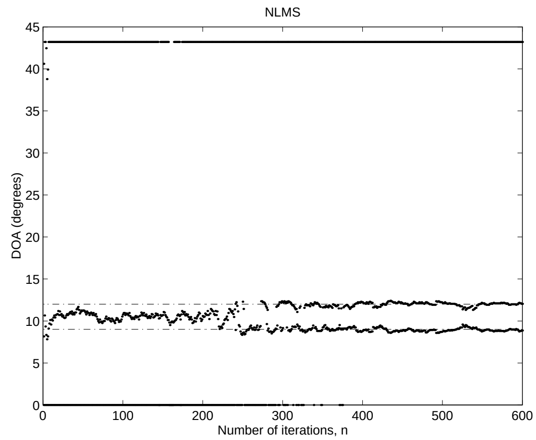

Fig. 1. DOA estimates for two uncorrelated signals at $9°$ and $12°$. NLMS algorithm.

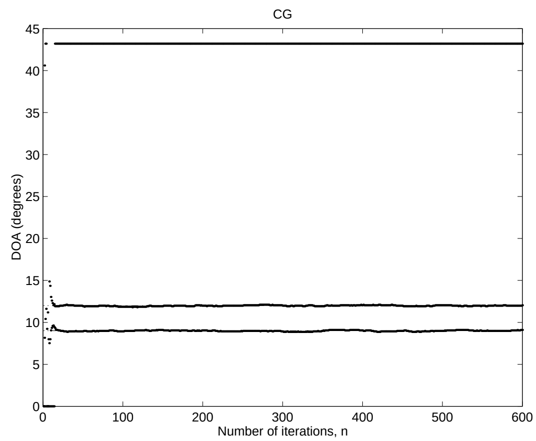

Fig. 2. DOA estimates for two uncorrelated signals at $9°$ and $12°$. CG algorithm.

In (6) only a forward SS covariance matrix is computed. It is also possible to incorporate a backward SS covariance matrix to increase the effective aperture of the sensor array, thus reducing the number of sensors necessary to implement spatial smoothing [8, 9, 13].

The performance of the DOA estimator can be further improved by using the unconstrained CG algorithm shown in [3]. The unconstrained CG algorithm provides a better convergence rate and less estimation noise.

5. SIMULATIONS

Consider a test setting similar to the one described in [14] where two closely spaced equal-power uncorrelated plane waves are impinging on an 4-sensor uniform linear array with SNR=20. The uncorrelated receiver noise is white, zero mean, and with unit variance. The signal sources are kept at fixed angles of $9°$ and $12°$. Only time averaging is used in the computation of $\mathbf{R}$. Figs. 1 and 2 compare the DOA estimates of the normalized LMS algorithm with $\mu_{NLMS} = 0.1$ and the proposed CG algorithm with $\lambda_f = 0.95$ and $\eta = 0.7$, respectively. The mean and the standard deviation of the estimates, after the convergence of the algorithms, are shown in Table I, where results using the RLS algorithm with $\lambda_f = 0.95$ are also shown for comparison purposes. It can be seen that the CG algorithm performance is as good as the performance of the RLS algorithm, for uncorrelated sources.

Table I. Simulation results for the CG, the NLMS, and the RLS algorithms.

	θ_1		θ_2	
	mean	std	mean	std
NLMS	8.9735	0.1840	11.9433	0.2225
CG	9.0304	0.0455	11.9866	0.0362
RLS	9.0368	0.0469	11.9878	0.0453

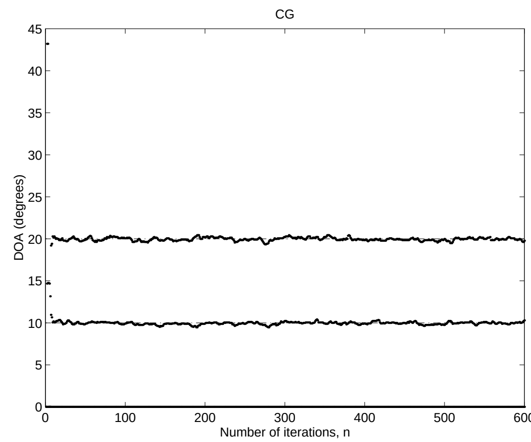

Fig. 3. DOA estimates for two coherent signals at $10°$ and $20°$. CG algorithm.

Next, consider the same test, but with two coherent sources at $10°$ and $20°$ and an 8-sensor array. The array is divided into subarrays and spatial and temporal smoothing are used. Here $\lambda_f = 0.8$, $\eta = 0.6$, and $\mathbf{w}(n)$ has length 4. When spatial smoothing is not used, all algorithms tested failed to distinguish the two coherent sources. Figs. 3 and 4 show the performance of the proposed CG algorithm with coherent sources. Consider now the same setting for coherent sources, but only spatial smoothing is used. Fig. 5 shows the performance of the algorithm. Notice the increase in the estimation noise. Finally, consider two fixed sources at $5°$ and $22°$ and one moving source varying from $12°$ to $15°$ in 300 units of time. Fig. 6 shows the tracking capability of the proposed algorithm. Ten independent trials are shown.

6. CONCLUSION

A method for DOA estimation of coherent signals has been described, based on the adaptive version of Pisarenko's harmonic retrieval method. The Conjugate Gradient algorithm presented in [3] was used, taking advantage of the availability of the computed covariance matrix. The simulations show that the proposed algorithm has a fast convergence rate even when the input signals are coherent. Due to the use of an updated $\mathbf{R}$ at each time instant, no internal iterations are used as in regular CG methods [6], resulting in

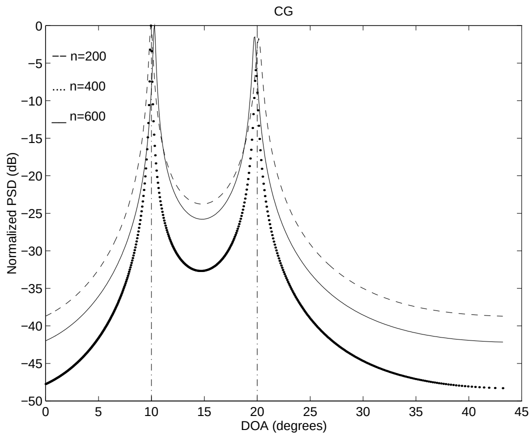

Fig. 4. DOA estimates for two coherent signals at 10° and 20°. CG algorithm.

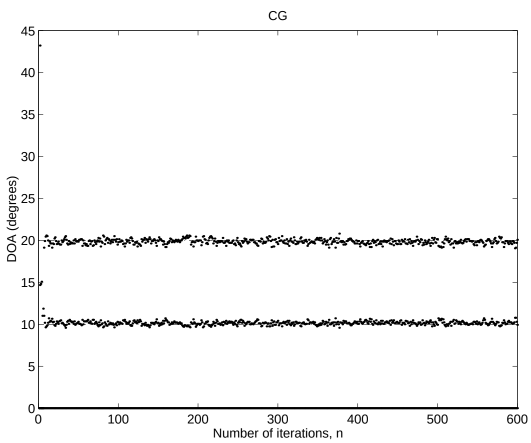

Fig. 5. DOA estimates for two coherent signals at 10° and 20°. CG algorithm with spatial smoothing only.

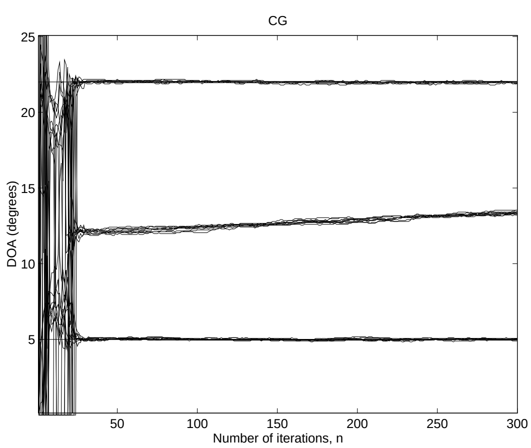

Fig. 6. DOA estimates. CG algorithm tracking moving source. Ten independent trials.

a computationally more efficient algorithm than previously proposed CG methods.

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