

ADAPTIVE AR SPECTRAL ESTIMATION BASED ON MULTI-BAND DECOMPOSITION OF THE LINEAR PREDICTION ERROR WITH VARIABLE FORGETTING FACTORS

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ABSTRACT

A new method for adaptive autoregressive spectral estimation based on the least-squares criterion with multi-band decomposition of the linear prediction error and analysis of each band through independent variable forgetting factors is presented. The proposed method localizes the forgetting factor adaptation scheme in the frequency domain and in the time domain, in the sense that variations on the statistics of the input signal are independently evaluated for each band along the time. In this paper, a new forgetting factor adaptation technique depending exclusively on the input signal is introduced and applied to the multi-window analysis of the linear prediction error structure to generate time-varying autoregressive spectral estimates. An improvement on the fidelity of estimates is shown in computer experiments which compare the proposed method with conventional and multi-band least-squares methods with fixed forgetting factors.

1. INTRODUCTION

Time-varying autoregressive (AR) spectral estimation is not only an important subject of practical interest, since most signals encountered in nature are nonstationary (*e.g.* speech), but is also challenging from the theoretical point of view, since it involves several still unanswered mathematical questions.

Application of a variable forgetting factor (VFF) in connection with the least-squares (LS) optimization criterion has been used to improve the tracking ability of time-varying systems [1, 2, 3, 4, 5]. The functions that estimate the degree of nonstationarity of the signal being analyzed for the forgetting factor (FF) adaptation techniques proposed so far depend on the residual power. Furthermore, whenever a change in the spectrum is detected, even if this change is related only to a frequency-localized part of the input signal, the FF is updated for all components.

Recently, multi-band decomposition of the linear prediction error through a filter bank and subsequent analysis of each band with a different window was shown to give rise to AR spectral estimates with higher fidelity to the true underlying spectrum than those obtained through conventional “single-window” LS methods, particularly when the statistics of the signal being analyzed presents a different behavior for distinct bands of the spectrum [6, 7, 8, 9]. However, multi-band decomposition-based algorithms developed until now use time-invariant FFs.

In this paper, a new adaptation technique for the FF exclusively based on the estimated autocorrelation of the input signal is introduced and applied to the multi-band decomposition structure with independent VFFs, to estimate the time-varying AR spectrum of nonstationary signals.

The structure of this paper follows. In Section 2, the LS method based on multi-band decomposition of the linear prediction error is presented. In Section 3, a new technique based on the estimated autocorrelation values of the input signal for computing the degree of nonstationarity is introduced and a recursive LS (RLS) algorithm is derived. In Section 4, computer experiments comparing the performance of the proposed method with conventional and multi-band LS methods with fixed FFs (FFFs) are shown. Section 5 presents our conclusions.

2. THE LS METHOD BASED ON MULTI-BAND DECOMPOSITION OF THE LINEAR PREDICTION ERROR

In conventional LS methods, for the exponentially windowed case, the following cost function is minimized:

$$\mathcal{F}_M(n) = \sum_{i=1}^n \lambda^{n-i} f_M^2(i) \quad (1)$$

$$f_M(i) = 1 + \sum_{k=1}^M a_k(n) u(i-k), \quad (2)$$

for real values of the *a posteriori* forward prediction error $f_M(i)$ [5]. The exponential window constant λ is a value between 0 and 1, the AR model coefficients at time-index n are represented by $a_k(n)$, $1 \leq k \leq M$, and $u(i)$ is the input signal. A block diagram scheme for (1) is depicted in Fig. 1(a).

The multi-band decomposition-based LS method, as can be inferred from [6, 7, 8, 9], for the exponentially windowed case, minimizes the following cost function

$$\mathcal{F}_M(n) = \sum_{j=1}^B \mathcal{F}_M^{(j)}(n) \quad (3)$$

$$\mathcal{F}_M^{(j)}(n) = (1 - \lambda^{(j)}) \sum_{i=1}^n (\lambda^{(j)})^{(n-i)} \left(f_M^{(j)}(i) \right)^2 \quad (4)$$

$$f_M^{(j)}(i) = h^{(j)}(i) * f_M(i), \quad (5)$$

which is schematically shown in Fig. 1(b) for the 4-channel case. The total cost function, defined in (3), is the sum

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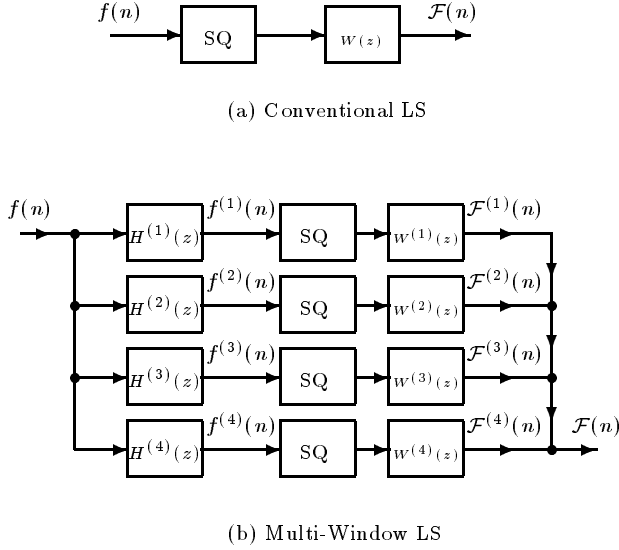


Figure 1. Block-diagram representation of the cost function definition for (a) conventional LS methods and (b) the proposed multi-window LS method when $B = 4$. The “SQ” symbol represents the square operation and $W(z)$ is the z -transform of the windowing function $w(k)$.

of partial cost functions, given in (4), which contain band-specific information on the statistical behavior of the analyzed signal. Each band of the linear prediction error is windowed with a suitable value $\lambda^{(j)}$, $j = 1, \dots, B$. By using a power complementary filter bank $h^{(j)}(n)$, $j = 1, \dots, B$, we can guarantee a uniform response of the system for all frequencies in the sense that $E[\mathcal{F}_M(n)] = cE[f_M^2(n)]$, where $E[\cdot]$ denotes the mean value operator (refer to the Appendix for a proof of this statement).

Minimization of (3) gives rise to a set of augmented normal equations:

$$\Phi_{M+1}(n) \mathbf{a}_M(n) = [\mathcal{F}_M(n) \ 0 \dots 0]^T \quad (6)$$

$$\Phi_{M+1}(n) = \sum_{j=1}^B (1 - \lambda^{(j)}) \sum_{i=1}^n (\lambda^{(j)})^{n-i} \mathbf{u}_{M+1}^{(j)}(i) \left(\mathbf{u}_{M+1}^{(j)}(i) \right)^T \quad (7)$$

$$\mathbf{u}_{M+1}^{(j)}(n) = [u^{(j)}(n) \ u^{(j)}(n-1) \ \dots \ u^{(j)}(n-M)]^T \quad (8)$$

$$u^{(j)}(n) = h^{(j)}(n) * u(n). \quad (9)$$

The correlation matrix shown in (7) satisfies the order-update property [8, 9], consequently an algorithm similar to the Levinson RLS algorithm [10], can be used to solve (6). The autocorrelation vector, which is required as the input to the Levinson RLS algorithm, can be evaluated as

$$\mathbf{c}_{M+1}(n) =$$

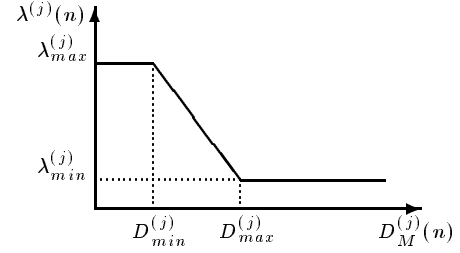


Figure 2. Relation between $D_{min}^{(j)}$, $D_{max}^{(j)}$, $\lambda_{min}^{(j)}$ and $\lambda_{max}^{(j)}$.

$$= \sum_{j=1}^B (1 - \lambda^{(j)}) \sum_{i=1}^n (\lambda^{(j)})^{n-i} u^{(j)}(i) \mathbf{u}_{M+1}^{(j)}(i) \quad (10)$$

$$= \sum_{j=1}^B \left(\lambda^{(j)} \mathbf{c}_{M+1}^{(j)}(n-1) + (1 - \lambda^{(j)}) u^{(j)}(n) \mathbf{u}_{M+1}^{(j)}(n) \right). \quad (11)$$

The basic advantage of the multi-band LS method as compared to conventional LS methods, is that time resolution and frequency resolution can be traded off along the spectrum enabling the use of a properly chosen window for each band of the analyzed signal in accordance to its statistical behavior. However, all existing algorithms based on multi-window analysis of the linear prediction error use VFFs $\lambda^{(j)}$ which must be chosen *a priori* based on previous knowledge of the input signal. Also, the tracking ability of multi-band decomposition-based methods can be improved by using VFFs. In the following, a new adaptation technique for the FF is introduced and an RLS algorithm for the evaluation of the AR model parameters is presented.

3. THE MULTI-BAND DECOMPOSITION-BASED METHOD WITH VFF AND ITS APPLICATION TO AR SPECTRAL ESTIMATION

In this paper, we are proposing a new FF adaptation scheme conceptually different from methods derived so far [1, 2, 3, 4, 5] in that it is an exclusive function of the input signal. It can be summarized as follows

$$\lambda^{(j)}(n) = \left[1 - \frac{1}{L^{(j)}(n)} \right]_{\lambda_{min}^{(j)}}^{\lambda_{max}^{(j)}} \quad (12)$$

$$L^{(j)}(n) = \frac{N^{(j)}}{D_M^{(j)}(n)} \quad (13)$$

$$D_M^{(j)}(n) = \frac{1}{I^{(j)}} \left\| \sum_{i=0}^{I^{(j)}-1} \left(\mathbf{c}_{M,I^{(j)}}^{(j)}(n-i) - \mathbf{c}_{M,I^{(j)}}^{(j)}(n-I^{(j)}-i) \right) \right\|^2 \quad (14)$$

$$\mathbf{c}_{M,I^{(j)}}^{(j)}(n) = [c_{0,I^{(j)}}^{(j)}(n), c_{1,I^{(j)}}^{(j)}(n), \dots, c_{M-1,I^{(j)}}^{(j)}(n)]^T \quad (15)$$

Table 1. An RLS algorithm for AR spectral analysis based on multi-band decomposition of the linear prediction error with variable forgetting factors.

For each time-index $n = 1, 2, 3, \dots$ compute:

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For  $j = 1, 2, \dots, B$ 
    Calculate  $u^{(j)}(n)$  using (9)
    Evaluate  $D_M^{(j)}(n)$ ,  $L^{(j)}(n)$  and  $\lambda^{(j)}(n)$  using (14), (13) and
    (12), respectively
     $\mathbf{c}_M^{(j)}(n) = \lambda^{(j)}(n)\mathbf{c}_M^{(j)}(n-1) + (1 - \lambda^{(j)})u^{(j)}(n)\mathbf{u}_M^{(j)}(n)$ ,
End
 $\mathbf{c}_M(n) = \mathbf{c}_M^{(1)}(n) + \mathbf{c}_M^{(2)}(n) + \dots + \mathbf{c}_M^{(B)}(n)$ 
 $\mathcal{F}_0(n) = \mathcal{B}_0(n) = c_0(n)$ 
 $\mathbf{a}_0(n) = \mathbf{g}_0(n) = 1$ 

For  $m = 1, 2, \dots, M$  compute :
     $\mathbf{C}_{m-1}(n) = [c_1(n), \dots, c_m(n)]\mathbf{g}_{m-1}(n-1)$ 
     $K_m^f(n) = -\mathbf{C}_{m-1}(n)/\mathcal{B}_{m-1}(n-1)$ 
     $K_m^b(n) = -\mathbf{C}_{m-1}(n)/\mathcal{F}_{m-1}(n)$ 
     $\mathcal{F}_m(n) = \mathcal{F}_{m-1}(n) + K_m^f(n)\mathbf{C}_{m-1}(n)$ 
     $\mathcal{B}_m(n) = \mathcal{B}_{m-1}(n-1) + K_m^b(n)\mathbf{C}_{m-1}(n)$ 
     $\mathbf{a}_m(n) = \begin{bmatrix} \mathbf{a}_{m-1}(n) \\ 0 \end{bmatrix} + K_m^f(n) \begin{bmatrix} 0 \\ \mathbf{g}_{m-1}(n-1) \end{bmatrix}$ 
     $\mathbf{g}_m(n) = \begin{bmatrix} 0 \\ \mathbf{g}_{m-1}(n-1) \end{bmatrix} + K_m^b(n) \begin{bmatrix} \mathbf{a}_{m-1}(n) \\ 0 \end{bmatrix}$ 
End

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$$\begin{aligned}
 c_{k, I^{(j)}}^{(j)}(n) &= \frac{1}{I^{(j)}} \sum_{i=0}^{I^{(j)}-1} u^{(j)}(n-i)u^{(j)}(n-i-k) \quad (16) \\
 &= c_{k, I^{(j)}}^{(j)}(n-1) + \\
 &\frac{1}{I^{(j)}} \left(u^{(j)}(n)u^{(j)}(n-k) - u^{(j)}(n-I^{(j)})u^{(j)}(n-I^{(j)}-k) \right), \quad (17)
 \end{aligned}$$

where $[*]_{\lambda_{min}^{(j)}}^{\lambda_{max}^{(j)}}$ denotes that $*$ is bounded above by $\lambda_{max}^{(j)}$ and

also bounded below by $\lambda_{min}^{(j)}$, as shown in Fig. 2.

The facts that inspired the formulation of the function accounting for nonstationarities of the input signal, $D_M^{(j)}(n)$, are: 1) nonstationarity is inherent to the analyzed signal, so a direct function of it or of its moments should be able to identify spectral changes; 2) the autocorrelation function and the energy spectral density form a Fourier pair, so it should be possible to measure spectral variations from the former; 3) for wide-sense-stationary correlation-ergodic signals, the estimated autocorrelation function for each lag k approaches the true autocorrelation value as the window length tends to infinity [5].

The complete RLS algorithm based on the multi-band scheme with VFFs can be found in Table 1. The first part of the algorithm (from the beginning until the evaluation of the total estimated autocorrelation vector $\mathbf{c}_M(n)$) corresponds to the proposed multi-band LS solution with VFFs, and the second part of the algorithm (from the initialization of $\mathcal{F}_0(n)$ until the end) is based on the Levinson RLS algorithm and was derived in [10].

It should be noted that the FF adaptation technique presented in this section has some limitations. Particularly, in practical applications $N^{(j)}$ must be a time-varying quantity accounting for power variations of the input signal for each band. One possible approach to tackle this problem is to use a power-normalized form of the estimated autocorrelation given in (16). Furthermore, when *a priori* knowledge of

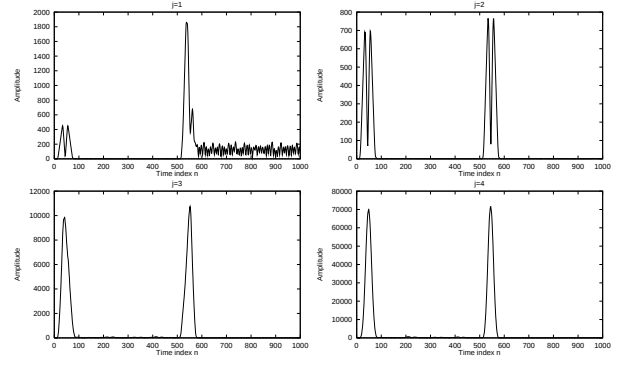


Figure 3. Plots of the nonstationarity detection function $D_M^{(j)}(n)$, $j = 1, \dots, 4$, for the signal analyzed in Fig. 4. For all bands $I^{(j)} = 20$.

the input signal is limited, choices of $D_{min}^{(j)}$ and $D_{max}^{(j)}$ are a difficult task. To reduce the influence of these bounds, the following algorithm may be used to update $\lambda^{(j)}(n)$:

$$\lambda^{(j)}(n) = \lambda_{min}^{(j)} + |\cos(\alpha^{(j)}(n))| \left(\lambda_{max}^{(j)} - \lambda_{min}^{(j)} \right) \quad (18)$$

where

$$\alpha^{(j)}(n) = \arctan \left(\frac{\delta D_M^{(j)}(n)}{\delta n} \right). \quad (19)$$

4. COMPUTER EXPERIMENTS

The test signal used in the experiments described in this section is composed of two spatially-close low-frequency sinusoids (340 Hz and 740 Hz) which start when a high-frequency sinusoid (4000 Hz) stops. (sampling rate = 10 kHz). White noise was added. The system used to filter the linear prediction error is a 4-channel linear-phase near-perfect-reconstruction filter bank designed in [11] with stop-band attenuation of 59 db. Plots of the detection function $D_M^{(j)}(n)$, $j = 1, \dots, 4$, are depicted in Fig. 3. Note that, ideally, there should be no spikes in the plots of $D_M^{(2)}(n)$ and $D_M^{(3)}(n)$, but in practice, the stopband attenuation will govern the influence of signals in neighboring bands. Note also that even though this influence is observed, the peak amplitude values are much lower than the related peaks in the bands where these nonstationarities really occur, and this influence may be neutralized by proper choice of the clipping values $D_{min}^{(j)}$ and $D_{max}^{(j)}$. In Fig. 4, from left to right, AR spectral estimates for the system described above through conventional LS with three different FF values and multi-band decomposition-based LS with FFFs and VFFs are plotted at every 15-samples interval.

Careful analysis of the results displayed in Fig. 3 and Fig. 4 enlighten the following points:

- nonstationarities are precisely detected by $D_M^{(j)}(n)$;
- the estimated high-frequency component fastly vanishes for the proposed VFF multi-band method;
- during the stationary parts of the input signal, a stable response similar to that obtained with long-windowed RLS methods is achieved at no expense of speed of convergence with the proposed method.

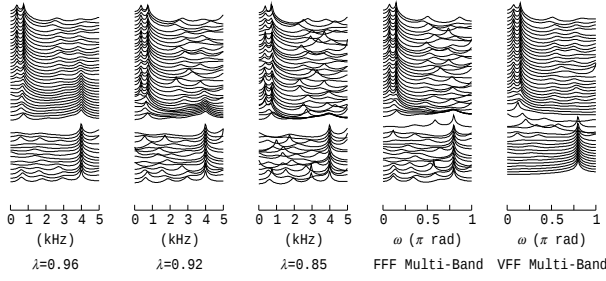


Figure 4. Time-varying AR spectral estimates obtained through conventional RLS with FFFs, multi-band decomposition-based method with FFFs (0.96, 0.92, 0.85, 0.85) and VFFs (rightmost plot).

5. CONCLUSIONS

A new AR spectrum estimation method based on the LS criterion through multi-window analysis of the linear prediction error with independent VFFs was presented. The proposed method trades off time resolution and frequency resolution of the spectral analyzer along the frequency spectrum in accordance with the frequency-localized time-varying statistics of the analyzed signal, giving rise to estimates that represent the true underlying spectrum with more fidelity than conventional LS methods. A new technique for adaptation of the FF exclusively based on the input signal was introduced and an RLS algorithm was derived. Computer experiments comparing the performance of multi-band decomposition-based (with fixed and variable FFs) and conventional RLS methods were shown.

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A PROOF THAT POWER COMPLEMENTARINESS OF THE ANALYSIS BANK ENSURES A UNIFORM RESPONSE OF THE ADAPTIVE ALGORITHM

A proof that power complementariness of the filter bank used in the multi-band method is a sufficient condition for the validity of the relation $E[\mathcal{F}_M(n)] = cE[f_M^2(n)]$ is in the following.

Proof:

From the definition of the cost function, given in equations (3)–(5), we have

$$E[\mathcal{F}_M(n)] = \sum_{j=1}^B E[\mathcal{F}_M^{(j)}(n)] \quad (20)$$

$$= \sum_{j=1}^B E \left[\sum_{i=1}^n (\lambda^{(j)})^{n-i} \left(f_M^{(j)}(i) \right)^2 \right] \quad (21)$$

$$= \sum_{j=1}^B (1 - \lambda^{(j)}) \sum_{i=1}^n (\lambda^{(j)})^{n-i} \frac{1}{2\pi} \int_{-\pi}^{\pi} P_{f_M}(\omega) |H^{(j)}(e^{j\omega})|^2 d\omega, \quad (22)$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} P_{f_M}(\omega) \sum_{j=1}^B |H^{(j)}(e^{j\omega})|^2 d\omega, \quad (23)$$

where $P_{f_M}(\omega)$ is the power spectrum of $f_M(n)$. If the analysis bank satisfies the power complementary property, that is,

$$\sum_{j=1}^B |H^{(j)}(e^{j\omega})|^2 = c, \quad (24)$$

where c is a positive constant, then it can be seen from (23) that

$$E[\mathcal{F}_M(n)] = cE[f_M^2(n)]. \quad (25)$$

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