ITERATIVE TOTAL LEAST SQUARES FILTER IN ROBOT NAVIGATION

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ABSTRACT

In the robot navigation problem, noisy sensor data must be filtered to obtain the best estimate of the robot position. The discrete Kalman filter, which usually is used for prediction and detection of signal in communication and control problems has become a commonly used method to reduce the effect of uncertainty from the sensor data. However, due to the special domain of robot navigation, the Kalman approach is very limited. Here we propose the use of a Iterative Total Least Squares Filter which is solved by applying the Lanczos bidiagonalization process. This filter is very promising for very large data information and from our experiments we can obtain more precise accuracy than the Kalman filter.

1. INTRODUCTION

The discrete Kalman filter [8], which usually is used for prediction and detection of signal in communication and control problems has become a commonly used method to reduce the effect of uncertainty from the sensor data. Due to the fact that most of function in applications are non-linear, the extended Kalman filter which linearizes the function by taking a first order Taylor expansion is introduced and this linear approximation is then used as the Kalman filter equation [1, 9].

Due to the domain of robot navigation, several problems often occur when we apply either the Kalman or the extended Kalman filter. An underlying assumption in any least squares estimation is that the entries in the data matrix are error-free [2, 7] which means that the time intervals at which measurements are taken are exact. But in many actual applications, sampling error, human errors, and instrument errors may preclude the possibility of knowing the data matrix exactly. In some cases, the errors in data matrix can be at least as great as the measurements errors. At this moment, applying the Kalman filter will give very poor results. And also the linearization process of the extended Kalman filter has the potential to introduce significant error into the problem [2]. The extended Kalman is not guaranteed to be optimal or even converge because it needs a very good initial estimate of the solution. In some cases, it can easily fall into a local minimum when this initial guess is poor which is the type of situation faced by robot navigators.

In this paper, we propose a new Iterative Total Least Squares Filter (ITLS) which does not require numerous measurements to converge because the camera images in robot navigation is a time consuming process and take the errors in data matrix into consideration. Recently Boley and Sutherland describe a Recursive Total Least Squares Filter (RTLS) which is very easily to update [2]. In some ways, that is still a time consuming algorithm. Here we apply the Lanczos bidiagonalization process which is more computationally attractive to solve the total least squares problems. The experiments indicate that this approach can achieve a greater accuracy with promising computational cost.

The paper is organized as follows. In section 2, we will describe the Iterative Total Least Squares (ITLS) algorithm. We present our experimental results in section 3. Finally we offer some comments and remarks.

2. ITERATIVE TOTAL LEAST SQUARES ALGORITHM

Given an over-determined system of equation $Ax = b$, the TLS problem, in its simplest form, is to find the smallest perturbation to A and b to make the system of equations compatible. Specifically, we find an error matrix E and vector r such that for some vector x

$$\min_{E,r} \|(E,\ r)\|_F, \qquad (A+E)x = b + r.$$

The vector x corresponding to the optimal (E, r) is called the TLS solution.

The most common algorithms to compute the TLS solution are based on Singular Value Decomposition (SVD), a computationally expensive matrix decomposition [6]. A very complete survey of computational aspects and analysis of the TLS problem is given Van Huffel and Vandewalle in [12]. Recently Van Huffel [11] presented some iterative methods based on inverse iteration and Chebyshev acceleration, which compute a basis of a singular subspace associated with the smallest singular values. Their convergence properties, the convergence rate and the operation counts per iteration step are analyzed that they are highly dependent on the gap of singular values. Also some recursive TLS filters have been developed for application in signal processing [3, 4, 15].

We now consider computing an approximate solution to the total least squares problem using the Lanczos process [14]. By the TLS solution is determined by the left singular vector v_{n+1} of (A, b). This leads us to consider applying the Lanczos bidiagonalization described in [14] to (A, b). We put

$$\beta_1 u_1 = b, \quad \alpha_1 \begin{pmatrix} v_1^1 \\ v_1^2 \end{pmatrix} = \begin{pmatrix} A^T \\ b^T \end{pmatrix} b. \tag{1}$$

and for $j = 1, 2, \ldots$

$$\beta_{j+1} u_{j+1} = (A,\ b) \begin{pmatrix} v_j^1 \\ v_j^2 \end{pmatrix} - \alpha_j u_j,$$
$$\alpha_{j+1} \begin{pmatrix} v_{j+1}^1 \\ v_{j+1}^2 \end{pmatrix} = \begin{pmatrix} A^T \\ b^T \end{pmatrix} u_j - \beta_j v_{j-1},$$

where $\alpha_{j+1} \geq 0$ and $\beta_{j+1} \geq 0$ are determined so that $\|u_{j+1}\|_2 = \|v_{j+1}\|_2 = 1$.

After k steps we have computed

$$V_k = (v_1, \ldots, v_k), \qquad U_{k+1} = (u_1, \ldots, u_{k+1}),$$

and a bidiagonal matrix $B_k \in \mathbf{R}^{(k+1)\times k}$. The matrix form of the first recurrence is

$$(A,\ b) \begin{pmatrix} V_k^1 \\ V_k^2 \end{pmatrix} = U_{k+1} B_k. \tag{2}$$

How is this related to the Lanczos process on A? We first note that the properties $u_k \in \tilde{\mathcal{K}}_k$ where $\tilde{\mathcal{K}}_k = span\{b, (AA^T)b, \ldots, (AA^T)^{k-1}b\}$, and $U_k^T U_k = I$, uniquely determine these vectors, and hence they are identical to the vectors u_k generated by the Lanczos process applied to A. Further it holds that $v_k^1 \in \mathcal{K}_k$ where $\mathcal{K}_k = span\{A^T b, \ldots, (A^T A)^{k-1} A^T b\}$ but these vectors will differ from the vectors v_k generated in the previous process.

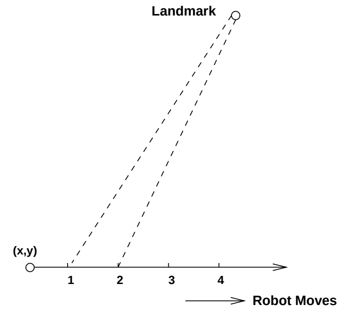

Figure 1: Diagrams of measurement

The SVD of B_k can be computed cheaply by the standard implicit QR algorithm [7, 13]. If B_k can be expressed as $B_k = P_{k+1} \Omega_k Q_k^T$ then from (2) we have

$$(A,\ b) \begin{pmatrix} V_k^1 \\ V_k^2 \end{pmatrix} (Q_k e_k) = \omega_k (U_{k+1} P_{k+1} e_k).$$

Hence with

$$\begin{pmatrix} z_k \\ \gamma_k \end{pmatrix} = \begin{pmatrix} V_k^1 \\ V_k^2 \end{pmatrix} Q_k e_k,$$

the approximate TLS solution is given by $x_k = -z_k / \gamma_k \in \mathcal{K}_k$. Note that we only need the last singular vector $Q_k e_k$ to compute x_k, but the vectors v_k need to be saved or regenerated when x_k is computed.

3. EXPERIMENTAL RESULTS

To compare the performance with the Kalman filter and the Recursive Total Least Squares Filter(RTLS) in practice, we run our iterative approach of Total Least Squares Filter for one set of experiments suggested in [2].

In the set of experiments, we simulate a simple robot navigation problem typical of that faced by an actual mobile robot [1, 5, 9]. The robot has identified a single landmark in a two-dimensional environment and knows the landmark location on a map. It does not know its own position. It moves in a straight line and with a known uniform velocity. Its goal is to estimate its own start position relative to the landmark by measuring the visual angle between its direction of heading and the landmark. Measurements are taken periodically as it moves. Figure 1 gives a simple diagram of the problem. Assume that the landmark is located at (0,0), that the y coordinate of the robot's start position does not change as the robot moves and that the robot knows what side of the landmark it is on. To map this robot-based system to the ground coordinate system, it suffices to know only the robot's compass heading from a kind of internal compass. We

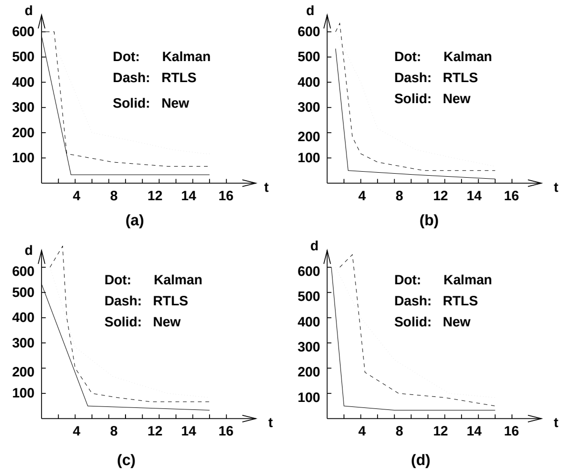

Figure 2: Mean deviation to actual start position

will follow the simple way described in [2] to know the compass heading independently.

In this experiment, we assume that the y coordinate of the robot path was negative, that robot velocity v is 20 per unit of time and that measurements of α are taken at unit time intervals. At any time t_i, we know that

$$\cot(\alpha_i) = \frac{x + t_i * v}{y}$$

where (x, y) is the robot start position and α_i is the angle from the robot heading to the landmark. Random error with a uniform distribution are added to the angle measures and a normally distributed random error to the time measurement. We can formulate the problem so that the data matrix, as well as the measurement vector contained error as follows:

$$A_i = [1 \;\; -\cot(\alpha_i)], \;\; x_i = [x^T \;\; y^T]^T, \;\; b_i = -t_i * v,$$

where, at time t_i, A_i is the data matrix, b_i is the measurement vector, and x_i is the estimated state vector consisting of the coordinates (x, y) of the robot start position. The Kalman filter is given an estimated start of $(0, 0)$. The RTLS algorithm and our approach have no estimated start position provided. The leading column of the data matrix is scaled by 100 to reduce the allowed errors. Here we show some results in Figure 2. The mean deviations d of the estimates from the actual start location of $(-460, -455)$ are compared for four different error amounts. Figure 2(a) and (b) have uniformly distributed error in α of $\pm 2°$ and normally distributed error in t with standard deviation with $0, 0.05, 0.1$ and 0.5. Figure 2(c) and (d) have uniformly distributed error in α of $\pm 4°$ and normally distributed error in t with $0, 0.05, 0.1$ and 0.5. Table 1 gives the mean deviation from the actual location after measurements. For all groups of experiments, the new approach, namely iterative approach of Total Least Squares Problems converges more quickly than RTLS filter which is also faster than the classical Kalman filter. Also we can see clearly that the new approach can achieve a closer estimation to the actual location than RTLS and Kalman filters as well.

Table 1: Mean deviation from actual location

Error(α)	Error(t)	0	0.05	0.1	0.5
$\pm 2°$	Kalman	32.47	20.27	24.54	28.56
	RTLS	20.24	15.90	24.81	23.75
	New	20.22	13.62	21.58	20.09
$\pm 4°$	Kalman	21.01	31.80	34.63	36.67
	RTLS	10.11	24.97	32.13	31.39
	New	8.24	18.46	29.38	26.57

From the results, we can demonstrate that in the domain of robot navigation the ITLS filter can provide more accurate estimates in fewer time step than the Kalman and RTLS filters, especially when errors are introduced in both the measurement vector and the data matrix.

4. CONCLUSION

In this paper, we propose a new approach namely Iterative Total Least Squares Filter by using the Lanczos bidiagonalization process. This filter is very suitable for large data information with relatively few readings and makes very few assumption about the data or the problem solved. Compared with the classical Kalman filter, we take the error term of data matrix into consideration and do not take care the initial guess and guarantee the solution to be optimal without falling into a local minimum when the initial solution is poor which is very typical in robot navigation. We apply it use to the robot navigation and the experiments indicate this approach is a successful approach.

5. REFERENCES

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