A NEW APPROACH TO THE COMPENSATION OF ALIASING IN TRANSFORM AND SUBBAND CODERS

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ABSTRACT

Filter banks for transform and subband coding are usually designed so as to achieve perfect reconstruction without considering distortions induced by inevitable or even desired effects such as filter implementation, subband quantization, and transmission. Other design algorithms minimize the distortion using more or less realistic models of quantizers. In contrast, we propose a new type of alias compensation based on the reasonable assumption that perfect or even near-perfect reconstruction in general cannot be attained if the subband signals are manipulated in some way. Therefore, a mostly time-invariant behaviour of the overall system seems to be more desirable. The proposed algorithm is capable of designing a compensation filter bank which reduces aliasing while the desired time-invariant part of the original system is preserved as far as possible.

1. INTRODUCTION

The design of filter banks for transform and subband coding has been treated in numerous publications (see [1] for a survey). Most design methods try to achieve perfect or near-perfect reconstruction, i.e. the ideal filter bank should behave like a pure delay element. However, quantization of coefficients and data in fixed point filters, coding of subband signals, and noisy transmission channels induce additional distortions such as aliasing. Recent design approaches [2, 3] consider some of these effects using models of quantizers and minimize the overall distortion of the system. Obviously, also these methods strive for a kind of near-perfect reconstruction.

In many applications aliasing distortions cause extremely disturbing effects whereas time-invariant distortions can be tolerated to a certain degree. Therefore, it is often more desirable to attain a mostly time-invariant implemented system, i.e. aliasing should be reduced noticeably. Furthermore, the original time-invariant part of the system should be preserved as far as possible, since it represents the actually desired system behaviour. In this contribution, we introduce a new algorithm for the design of post-processing alias compensation filter banks which meets these requirements.

2. THE NEW ALGORITHM FOR ALIAS COMPENSATION

The new algorithm is based on a polyphase (PP) representation of filter banks. It can be shown [1, 4] that an M-channel filter bank with input signal $v(k)$ and output signal $y(k)$ can be represented equivalently as a time-invariant multi-input multi-output (MIMO) system with input signals $v(M\kappa+\nu_1)$, $\nu_1 = 0 \dots M-1$ and output signals $y(M\kappa+\nu_2)$, $\nu_2 = 0 \dots M-1$. The MIMO system can be described by the $M \times M$ matrix $\mathbf{H}^{(\mathrm{p})}(z)$ of its partial transfer functions, the so-called PP matrix.

Since not all distortions due to lossy coding, quantization, and transmission can be calculated analytically, we resort to a method for measuring the properties of multirate systems [5, 6]. This method allows to determine the matrix of partial frequency responses $\mathbf{H}^{(\mathrm{p})}(e^{j\Omega_\lambda})$ as well as the PSD of noise-like nonlinear signal components at discrete frequencies $\Omega_\lambda = 2\pi\lambda/L$, $\lambda = 0 \dots L-1$. The frequency response matrix comprises aliasing as well as time-invariant distortions. Therefore, it is possible to incorporate the above-mentioned disturbing effects directly into the design process. However, the influence of nonlinear components will be neglected in this paper.

In the following, we consider an M-channel filter bank with aliasing and with PP matrix $\mathbf{H}^{(\mathrm{p})}(z)$. This system can be divided into two subsystems. The first one is an alias-free, i.e. time-invariant system which can be described by its transfer function $H(z)$ or equivalently by a pseudocirculant PP matrix $\mathbf{H}_{\mathrm{TI}}^{(\mathrm{p})}(z)$ [4, 7]. The second subsystem with PP matrix $\mathbf{H}_{\mathrm{A}}^{(\mathrm{p})}(z)$ is responsible for aliasing. Therefore, we have

$$\mathbf{H}^{(\mathrm{p})}(z) = \mathbf{H}_{\mathrm{TI}}^{(\mathrm{p})}(z) + \mathbf{H}_{\mathrm{A}}^{(\mathrm{p})}(z).$$

The necessary operations for separating the time-invariant subsystem can be derived easily from special properties of modulation matrices [1, 4]. In the following, this separation of time-invariant subsystems will be denoted by the operator $\mathcal{P}\{\bullet\}$.

Our algorithm approximates the alias-free subsystem and simultaneously reduces aliasing by means of a post-processing M-channel FIR compensation filter bank. This filter bank has a PP matrix $\mathbf{C}^{(\mathrm{p})}(z)$ of degree N. The degree can be arbitrarily chosen so as to fit the design purposes. It is well-known [1, 4] that the PP matrix of the compensated overall system is given by the product of the involved PP matrices, i.e.

$$\begin{aligned}\mathbf{H}_{\mathrm{C}}^{(\mathrm{p})}(z) &= \mathbf{C}^{(\mathrm{p})}(z)\cdot\mathbf{H}^{(\mathrm{p})}(z) = \sum_{\nu=0}^{N}\mathbf{C}_{\nu}^{(\mathrm{p})}z^{-\nu}\mathbf{H}^{(\mathrm{p})}(z)\\ &= \begin{bmatrix}\mathbf{C}_0^{(\mathrm{p})} & \mathbf{C}_1^{(\mathrm{p})} & \dots & \mathbf{C}_N^{(\mathrm{p})}\end{bmatrix}\cdot\begin{bmatrix}\mathbf{H}^{(\mathrm{p})}(z)\\ z^{-1}\mathbf{H}^{(\mathrm{p})}(z)\\ \vdots\\ z^{-N}\mathbf{H}^{(\mathrm{p})}(z)\end{bmatrix}\end{aligned}$$

with constant matrices $\mathbf{C}_{\nu}^{(\mathrm{p})}$. As the new algorithm is derived in the time-domain, the time-domain PP matrices $\mathbf{h}_{\nu}(k)$ corresponding to $z^{-\nu}\mathbf{H}^{(\mathrm{p})}(z)$, $\nu = 0\dots N$ are required. They can be easily calculated from measurement results using inverse DFTs

$$\mathbf{h}_{\nu}(k) = \frac{1}{L}\sum_{\lambda=0}^{L-1}\mathbf{H}^{(\mathrm{p})}(e^{j\Omega_\lambda})e^{j\Omega_\lambda(k-\nu)},\ k = 0\dots L-1.$$

The resulting $(N+1)\cdot M^2L$ time-domain values are then arranged in $M\times ML$ matrices

$$\mathbf{h}_{\nu} = \begin{bmatrix}\mathbf{h}_{\nu}(0) & \mathbf{h}_{\nu}(1) & \dots & \mathbf{h}_{\nu}(L-1)\end{bmatrix}.$$

The corresponding time-domain PP matrix $\mathbf{h}_{\mathrm{C}}$ of the overall system then is given by

$$\mathbf{h}_{\mathrm{C}} = \begin{bmatrix}\mathbf{C}_0^{(\mathrm{p})} & \mathbf{C}_1^{(\mathrm{p})} & \dots & \mathbf{C}_N^{(\mathrm{p})}\end{bmatrix}\cdot\begin{bmatrix}\mathbf{h}_0\\ \vdots\\ \mathbf{h}_N\end{bmatrix} = \mathbf{C}\cdot\mathbf{H}$$

with the matrix $\mathbf{C}$ of compensation filter bank coefficients. Now it is possible to compute these coefficients such that the PP matrix $\mathbf{h}_{\mathrm{C}}$ of the compensated system approximates the PP matrix $\mathbf{h}_{\mathrm{TI}} = \mathcal{P}\{\mathbf{h}_0\}$ of the alias-free subsystem in a least squares sense, i.e. the Frobenius norm

$$\|\mathbf{h}_{\mathrm{C}} - \mathbf{h}_{\mathrm{TI}}\|_{\mathrm{F}}^2 = \mathrm{trace}\left\{(\mathbf{h}_{\mathrm{C}} - \mathbf{h}_{\mathrm{TI}})^{\mathrm{T}}(\mathbf{h}_{\mathrm{C}} - \mathbf{h}_{\mathrm{TI}})\right\}$$

has to be minimized. This yields the desired coefficient matrix

$$\mathbf{C} = \mathbf{h}_{\mathrm{TI}}\cdot\mathbf{H}^{\mathrm{T}}\left(\mathbf{H}\cdot\mathbf{H}^{\mathrm{T}}\right)^{-1}$$

of the compensation filter bank.

In general, the alias-free subsystem of the resulting overall system will differ to some extent from the original one. Moreover, the alias components will be reduced but not perfectly compensated. We therefore apply the procedure iteratively to the compensated system in order to improve the accuracy of the approximation and the reduction of aliasing. The algorithm therefore can be outlined as follows:

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1. Calculate $\mathbf{h}_0$ and $\mathbf{H}$ from measurement results and initalize $i = 0$ and
$$\mathbf{h}_{\mathrm{TI}}^{(0)} = \mathcal{P}\{\mathbf{h}_0\}.$$
2. Approximate $\mathbf{h}_{\mathrm{TI}}^{(i)}$ by
$$\mathbf{h}_{\mathrm{C}}^{(i)} = \mathbf{C}\cdot\mathbf{H} = \mathbf{h}_{\mathrm{TI}}^{(i)}\cdot\mathbf{H}^{\mathrm{T}}\left(\mathbf{H}\cdot\mathbf{H}^{\mathrm{T}}\right)^{-1}\mathbf{H}$$
in a least squares sense.
3. Separate the time-invariant subsystem
$$\mathbf{h}_{\mathrm{TI}}^{(i+1)} = \mathcal{P}\{\mathbf{h}_{\mathrm{C}}^{(i)}\}$$
of the compensated overall system.
4. Stop the iteration if the aliasing distortions
$$\left\|\mathbf{h}_{\mathrm{C}}^{(i)} - \mathcal{P}\left\{\mathbf{h}_{\mathrm{C}}^{(i)}\right\}\right\|_{\mathrm{F}}^2$$
are sufficiently compensated, otherwise set $i = i+1$ and return to step 2.

Using eigenvalue theory and Kronecker product notation, it can be shown that the algorithm always converges. Furthermore, it is possible to avoid the iteration and calculate the final alias-free subsystem directly from a subset of the eigenvectors of a matrix which can be derived from measurement results.

First results indicate that our algorithm is capable of finding an alias-free overall system presuming that one exists. Obviously, this comprises the design of synthesis filter banks in perfect reconstruction systems with a given analysis part. If a perfect compensation of aliasing is definitely not possible, the algorithm at least yields a reduction in a least squares sense.

3. EXAMPLES

In order to demonstrate the properties of the proposed method we apply it to some design problems. For a visualization of our measurement results we employ the bifrequency transfer function $H(e^{j\Omega_2}, e^{j\Omega_1})$ which relates different frequencies Ω_1 at the input and Ω_2 at the output of the system. The time-invariant subsystems correspond to the function on the main diagonal in the bifrequency plots whereas the frequency responses on the other diagonals are alias-components.

3.1. Perfect Reconstruction Filter Bank

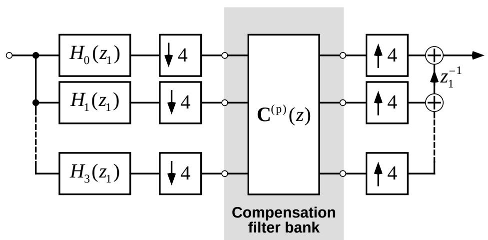

Figure 1: Compensated 4-channel filter bank without synthesis filters

The first example starts with the analysis part of a 4-channel filter bank. The synthesis part for a perfect reconstruction is known. Nevertheless, we use a simple parallel to serial conversion instead and obtain the measurement results shown in Fig. 2a. Obviously, the original time-invariant subsystem is not a pure delay. Moreover, significant aliasing appears on the secondary diagonals. Now we use the new algorithm for the design of a compensation filter bank having the same degree ($N = 28$) as the original synthesis filter bank. Actually, the resulting filters and the original synthesis filters are identical. Thus, the overall system (Fig. 1) is a perfect reconstruction filter bank, i.e. the time-invariant subsystem of the compensated filter bank (Fig. 2b) is a delay and in addition the aliasing is compensated perfectly. It has to be emphasized that the algorithm did not require any a-priori knowledge of the overall system. In particular, it was not necessary to know if perfect reconstruction is possible at all.

3.2. DCT Coder

The next example is a simple 8-channel DCT coder (Fig. 3) with a different wordlength in each channel. Although the ideal system achieves perfect reconstruction, large aliasing distortions arise due to the subband quantization (Fig. 4a). For the compensation of these distortions we design a post-processing filter bank of

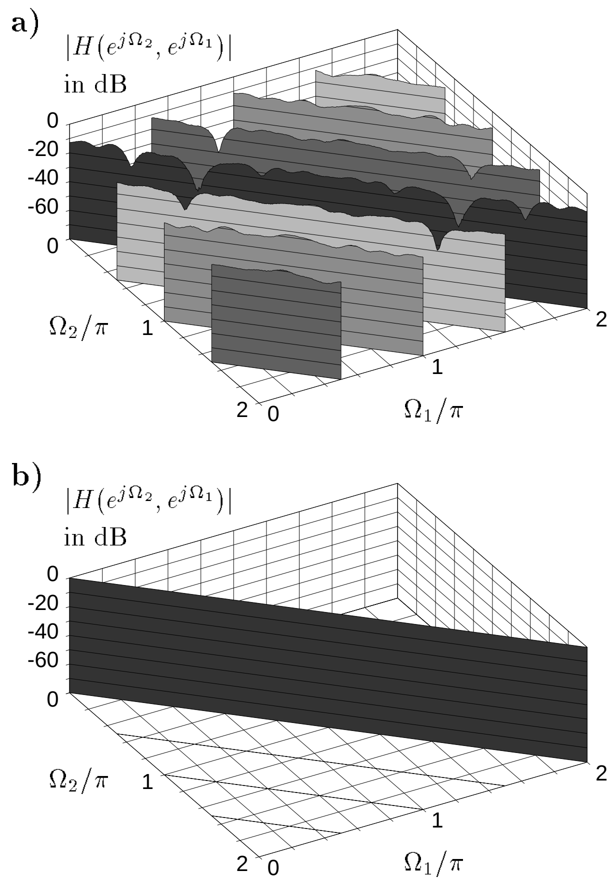

Figure 2: 4-channel filter bank without synthesis filters: Measured bifrequency transfer function without (a) and with (b) compensation of aliasing distortions

degree 0, i.e. with a constant PP matrix $\mathbf{C}^{(p)}(z) = \mathbf{C}_0^{(p)}$. The inverse DCT and the compensation thus can be performed simultaneously using a modified IDCT matrix. The results are shown in Fig. 4b. Obviously, the time-invariant subsystem remains almost unchanged. In contrast, the alias-components which appear on the secondary diagonals, are reduced significantly by 19.18 dB.

4. CONCLUSIONS

We proposed a new algorithm for the compensation of aliasing in implemented filter banks. It is based on measurement results and does not require any knowledge of the internal system structure. As opposed to other design methods, disturbing effects such as quantization, coding, and noisy transmission channels are inherently incorporated into system design.

In general, perfect or near-perfect reconstruction

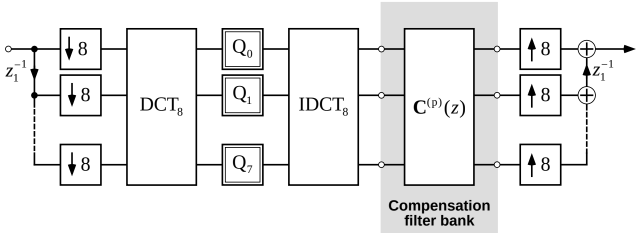

Figure 3: Compensated 8-channel DCT coder with scalar quantizers

cannot be achieved in implemented filter banks. We therefore abandon these goals and try to preserve the original time-invariant subsystem while aliasing is compensated as far as possible.

The algorithm can be employed for a subsequent compensation of aliasing in already implemented systems or for the design of synthesis filter banks with respect to additional distortions induced by quantization or coding. A pre-processing version of the compensation filter bank can be easily derived by solving the transpose problem. In this case, the compensation can be considered as a kind of signal forming in order to prevent aliasing in advance. Further improvements such as a spectral weighting of alias components and the inclusion of noise-like distortions into the optimization are possible. Our method therefore is a highly flexible way of improving the performance and design of filter banks including implementation aspects.

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5. REFERENCES

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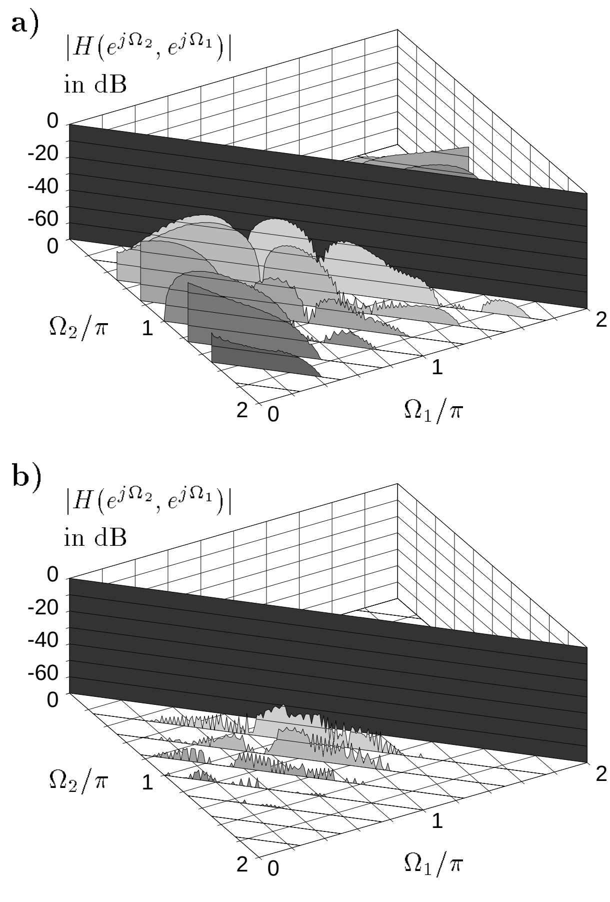

Figure 4: 8-channel DCT coder with unequal quantization: Measured bifrequency transfer function without (a) and with (b) compensation of aliasing distortions

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[1] A postscript version of this paper can be obtained from the author's WWW page