PERFORMANCE COSTS FOR THEORETICAL MINIMAL-LENGTH EQUALIZERS

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ABSTRACT

The length and complexity of a linear equalizer filter is highly dependent on the nature of the channel effects it must mitigate. The governing design rules are typically stated in terms of the channel's temporal characteristics, i.e. impulse response duration. Equalizer implementational complexity is a principal limiting factor for high bandwidth data communication applications, and consequently there is motivation for reexamining accepted design guidelines.

Recently, it was demonstrated in [1] that for relatively benign conditions on the effective channel and transmitter pulse shaping, there exists a linear equalizer that perfectly mitigates intersymbol interference, and whose span matches that of the composite distortion. Our paper examines the implications of the minimal-length equalizer in light of accepted design rules, and shows that a tangible loss in performance can be assigned to this complexity reduction. Actual line-of-sight microwave radio channels are used to demonstrate the nature of the performance loss.

INTRODUCTION

This paper examines the apparent inconsistency between traditional design rules and results reported in [1], further elaborated in [2], regarding the minimal-complexity linear equalizer. Specifically, we quantify the impact on performance as a function of equalizer length, and show that in important situations, i.e. for long delay specular multipath channels, a design with minimal length may be subject to significant performance degradation. This is demonstrated using measured channel responses for actual microwave links.

The conclusion is that under the more serious multipath conditions shown here, significant losses may impact receiver performance, losses traceable to "noise gain" and synchronization sensitivities. Such cases indicate that the more traditional means of equalizer design are often more robust than those described in [1].

BACKGROUND

The role of an equalizer in a digital communication context is to restore the ideal Nyquist pulse shaping that has been disrupted by the transmission channel and transmitter preshaping. The composite pulse distortion seen at the receiver gives rise to *intersymbol interference* (ISI) that degrades performance in the form of the receiver's *symbol error rate* (SER). Figure 1 depicts the basic model: linear channel, additive white noise, followed by an equalizing filter.

For purposes of this discussion we draw on two representative channels, shown in Figures 2 and 3. Both are drawn from measurements taken from actual microwave

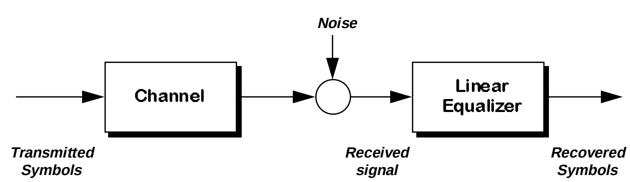

Figure 1. Basic Equalization Model

radio links as part of an Applied Signal Technology study of equalizer requirements.[1] Shown in (a) are the two components of the complex impulse response, and in (b) the magnitude frequency response for each channel. Figure 2 shows the more benign of the two, denoted as G301 with minor, concentrated dispersion due to short multipath; the second figure, Channel G501, represents a more serious time differential in its reflected path, and hence a more challenging equalization task. In both cases, the passband distortion takes the form of a "rippling" effect. In all but the most pathological alignment situations, the

[1] *The data for these channels is available for evaluation on line. The sample impulse responses shown here are drawn from the most significant portion of the original data.*

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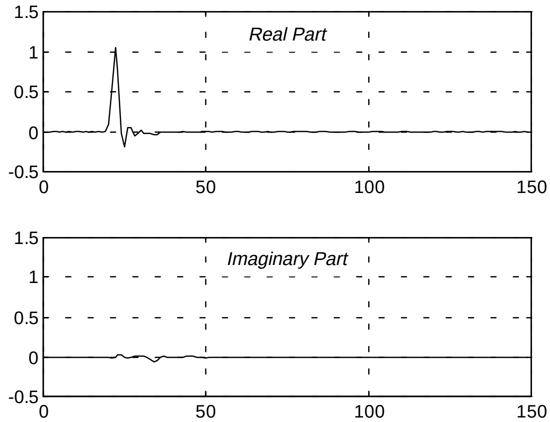
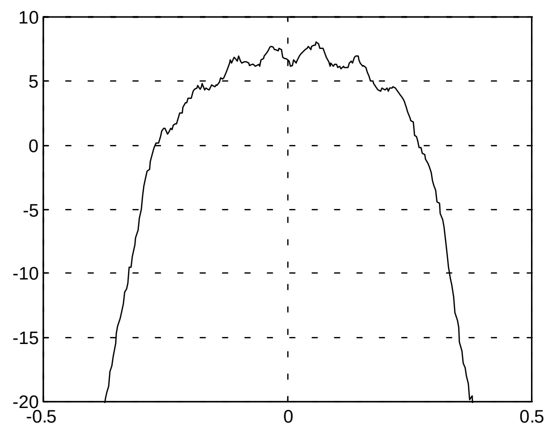
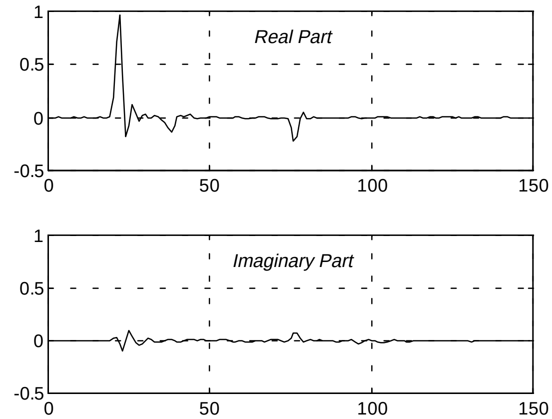


a) *Channel impulse response*

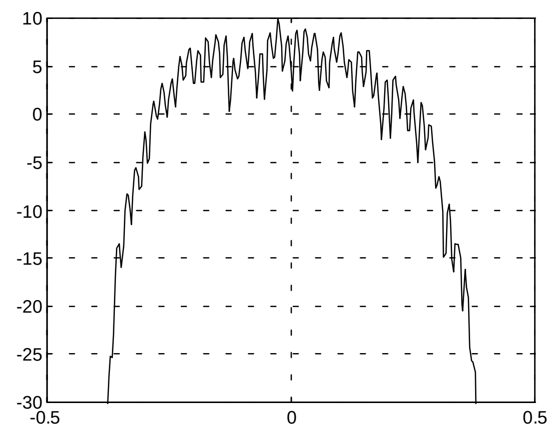


b) *Magnitude response*

Figure 2. Channel G301



a) *Channel impulse response*



b) *Magnitude response*

Figure 3. Channel G501

reflected path will be as shown here, i.e. weaker than the direct path.

The traditional and intuitive approach to specifying a linear equalizer is to consider compensation of the channel in the frequency domain. This "inversion" approach dictates that the equalizing filter have sufficient time duration to compensate for the principal elements of the passband distortion. In cases involving multipath propagation as those above, this approach calls for a decaying, reberatory time response. To illustrate with a trivial case, consider the representation of simple single multipath, with level α and differential delay Δ:

$$G(z) = 1 + \alpha z^{-\Delta},$$

whose reciprocal takes the form

$$H(z) = 1 - \alpha z^{-\Delta} + \alpha^2 z^{-2\Delta} - \alpha^3 z^{-3\Delta} \ldots$$

Thus, we might expect the ideal linear equalizer to approach an impulse response having an infinite number of decaying terms spaced by the path delay interval.

For the cases we have shown in Figures 2 and 3, when a least squares-based equalizer is used, we find that its impulse response does indeed follow this basic trend. The resulting design approach for these typical multipath situations considers the finite length equalizer as an approximation to the ideal. To do a credible job, the equalizer should be allowed sufficient length to encompass the bulk of the energy of the inverse channel response. The logical extension is that the longer the equalizer, the better the performance. Engineering practice has found that limiting the equalizer span to about three times the chan nel path delay results (i.e. through the α^3 term) will provide sufficient ISI reduction for moderately dense modulation. See [2] for further discussion of practical strategies for equalizer length specification.

We can see from Figure 3 that this design rule results in a linear equalizer that is excessive in length, on the order of 150 active coefficients. In recent work [1] showed that significant reduction in equalizer length can be realized by setting aside this inversion interpretation, and instead considering directly the underlying relationships governing distortion in fractionally spaced symbols. This focus on "fractional spacing" in the equalizer structure is key to this result, i.e. having tap spacing one half of the symbol period. Such structures have long been recognized for their benefits in timing recovery [3,4].

In that new result, it was shown that with proper choice of bulk delay, a linear equalizer exists that completely suppresses the ISI and spans only the duration of the channel impulse response. *In other words, in the multipath case, this new approach promises the need for only about 1/3 the complexity of the traditional design.*

PERFORMANCE CHARACTERIZATION

For veterans of digital radio developments this statement is completely counter to experience [5]. For this reason, its reception has ranged from cautious optimism to outright disbelief. To reconcile this disparity we will resort to link performance quantified in terms of receiver SER. For a digital link using 64-level quadrature amplitude modulation (64QAM), SER as a function of SNR will take the form shown as the dashed curve in Figure 4. This represents the performance for the ideal noise-only situation.

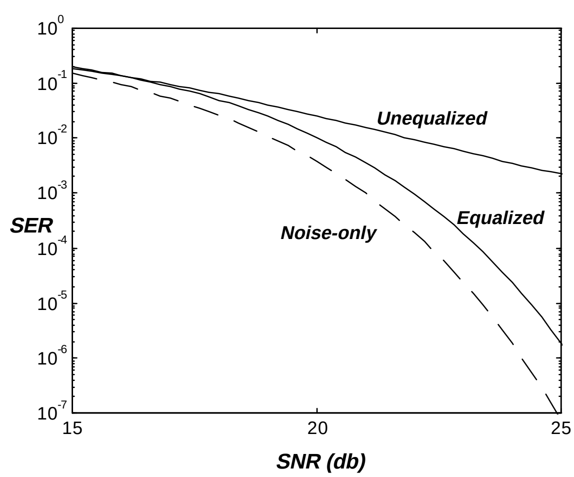


Figure 4. Typical Performance Curve

The dispersive effects of a channel results in an SER curve displaced to the right from the ideal, shown as the top curve in the figure; it follows the ideal curve for high noise levels, then diverges from it as the ISI effects dominate at high SNR levels. By introducing an equalizer, we see that the performance is significantly improved, i.e. the SER curve realizes a gain due to reduction of the ISI.

Performance loss can be quantified by the dB displacement of the SER curve. That is, a loss equates to an improvement in reception that is necessary to provide the equivalent SER. Such improvements may translate to higher transmitter power, better antenna gain, additional modulation complexity, etc., and any one of these factors may have a significant economic impact on a well-designed link even for seemingly small losses. In Figure 4 we see that equalization at a 22 dB SNR results in recovery of more than 3 dB of the losses due to ISI.

The mechanics of this loss relate to 1) incomplete ISI mitigation, and 2) enhancement of the noise, i.e. the level of the additive white noise is scaled by the "energy" of the equalizer taps. In combination, these two effects result in dispersion of the received eye pattern or signal constellation, and in a sense must be optimized jointly to minimize impact on SER. Zero-forcing approaches [6,7,8] that focus on the former are thus most effective in high SNR cases.

LOSS COMPARISON

Given this frame for performance characterization, in the practical model of Figure 1 we expect to encounter losses due to channel dispersion for any equalizer. In this section we examine losses as a function of length, using the design procedure described in [1]. Specifically, for the channels presented earlier we compute the equalizer for a range of lengths and determine their SER performance. This can be presented as in terms of loss as a function of equalizer length, and serve as an indicator of the gain afforded by additional complexity.

Consider Figure 5, i.e. a family of four such curves for Channel G501 (Figure 3).[2] Represented are the loss curves for four SNR values: from top to bottom, 28, 25, 20, and 15 dB. We see the following consistent behavior, more pronounced at higher SNRs where residual ISI dominates noise enhancement effects. For this channel with its reflected path delay of about 64 samples, we see that the "minimal-length" equalizer suffers significant losses, nearly nine dB, when compared to the ideal linear equalizer. And as the length approaches 200 taps, the performance loss drops for all SNR levels to a small fraction of a dB.

Several interesting observations come to mind with this figure. First, we see that performance improves in a stepwise fashion with increases in length. This can be explained in terms of the intuitive "reverberation" concept

[2] *For purposes of this discussion, the loss value is with respect to performance of an "infinite" length equalizer.*

mentioned earlier. As the equalizer grows to encompass decaying terms, the traditionalist might predict better ISI suppression, and indeed, we see that performance loss improves dramatically near lengths 75, 110, and 130. This is consistent with the presence of the minor secondary delayed component in the original impulse response (Figure 3a) separated by about 15 samples from the main peak. This example serves to validate the established "3× rule" of practice.

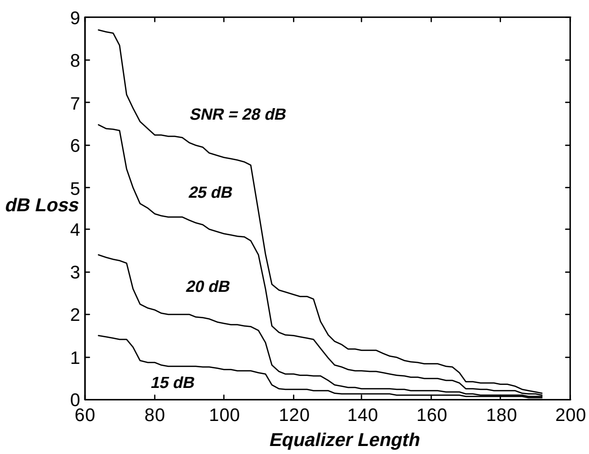


Figure 5. Loss Behavior, Channel G501

The second example focusing on Channel G301 of Figure 2 yields the family of loss curves shown in Figure 6, for SNR levels of 25, 20, and 15 dB. Recall that in this case, the propagation path has only minor delay in its multipath, on the order of 10 samples, so that the impulse response is well-concentrated into a single strong,

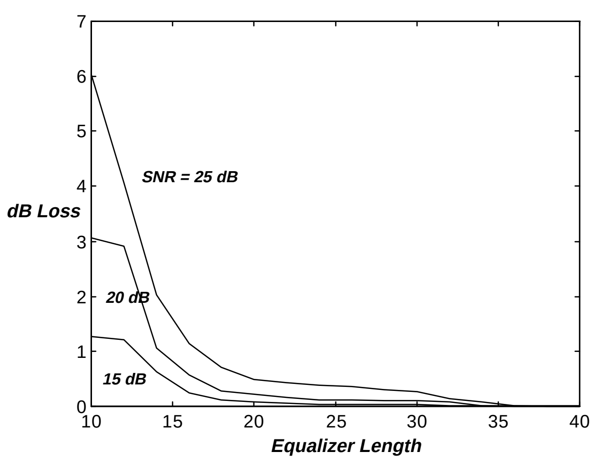


Figure 6. Loss Behavior, Channel G301

dispersive lobe. In this case the curves again show significant loss, as much as 6 dB, for the minimal-length equalizer. However, because of the low reflective level, all but about a half dB is recovered by doubling, not tripling, the equalizer length. Indeed, by the "3×" length, losses are minimal.

CONCLUSION

In this paper we have examined recent discoveries describing a theoretical basis for reduced-complexity equalizer design. Based on ideal zero-forcing conditions, that result proves inappropriate for many practical cases involving noisy, multipath digital radio links. In such cases, an equalizer of minimal length may be subject to an enhancement of noise that significantly degrades receiver performance. The exact interdependence of noise level and equalizer length is not yet clear, but in such cases, intuitive design approaches that specify equalizer length in terms of multipath delay time still appear reliable.

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