

EFFICIENT EQUALIZATION OF NONLINEAR COMMUNICATION CHANNELS

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ABSTRACT

Nonlinear intersymbol interferences (ISI) often arise in voiceband communication channels at high transmission rates or in satellite channels due to nonlinearities in the power amplifiers. Proposed equalizers for the cancellation of these nonlinear interferences are mainly based on the Volterra series expansion, which is an elegant but very complex model. This paper presents a decision feedback equalizer (DFE) which is based on a new nonlinear filter structure. It is composed only of linear tapped delay line filters and multipliers. Hence, the complexity of this still very general structure is comparable to linear filtering. Simulation of data transmission over a telephone channel show that the proposed DFE clearly outperforms the conventional DFE and is also superior to the Volterra DFE with a comparable complexity.

1. INTRODUCTION

In digital communication channels nonlinear intersymbol interferences (ISI) often arise at high transmission rates. It is the dominant impairment on many voiceband telephone channels at data rates above 4800 bps [1]. Also satellite channels suffer from nonlinear ISI because of the nonlinearities in the power amplifiers [2].

Several attempts have been made to compensate these interferences. Most equalizers and cancelers are based on the Volterra series expansion, e.g. [3] and [4], which is a general but very complex description of a nonlinear system. The 3rd order equivalent lowpass Volterra model for a bandpass nonlinearity writes [2]

$$y_n = \sum_{i=0}^{N_A-1} A(i)x_{n-i} + \sum_{i,j,k=0}^{N_C-1} C(i,j,k)x_{n-i}x_{n-j}x_{n-k}^* \quad (1)$$

where $A(i)$ and $C(i,j,k)$ are the complex-valued 1st and 3rd order filter weights, respectively. Due to the bandpass characteristic even order nonlinearities can be neglected and the conjugate complex of the input signal is required in the 3rd

order part. This is because only the specified components fall into the transmission band.

The major drawback of the Volterra approach is the enormous complexity. Even when the symmetry in the 3rd order Volterra coefficients is exploited (1) still requires $N_A + N_C^2(N_C+1)/2$ filter weights where N_A and N_C denote the filter lengths of the linear and cubic part, respectively. It turned out that not the same subset of coefficients $C(i,j,k)$ is significant for a variety of channels [3]. Hence, it is necessary to reduce the filter length N_C in order to cope with the complexity of the Volterra filter.

2. EFFICIENT NONLINEAR FILTER STRUCTURE

This paper presents a new decision feedback equalizer (DFE) which is based on a nonlinear filter structure with a complexity comparable to linear filtering.

The forward and feedback parts of the proposed DFE are implemented with the nonlinear filter structure of Fig. 1. It is composed only of linear tapped delay line (TDL) transversal filters and multipliers. The output symbol y at time n is obtained from the input symbols x through

$$y_n = \sum_{i=0}^{N_A-1} A(i)x_{n-i} + \sum_{i=0}^{N_C^p-1} C_p(i) \sum_{j=0}^{N_C^a+N_C^m-2} C_3(j)x_{n-i-j} \sum_{k=0}^{N_C^m-1} C_m(k) \cdot \sum_{l=0}^{N_C^a-1} C_1(l) x_{n-l-k-i} \sum_{m=0}^{N_C^a-1} C_2(m) x_{n-m-k-i} \quad (2)$$

A is the complex weight vector of the linear part and the weight vectors C_i build the 3rd order nonlinear part of the filter structure. (2) is again the equivalent lowpass model of a 3rd order bandpass nonlinearity.

The overall 3rd order filter length N_c equals $N_C^a + N_C^m + N_C^p - 2$ and (2) requires only $N_A + 3N_C^a + 2N_C^m + N_C^p - 1$ weights. The performance of this nonlinear filter has been already demonstrated in different applications, e.g. loudspeaker modeling

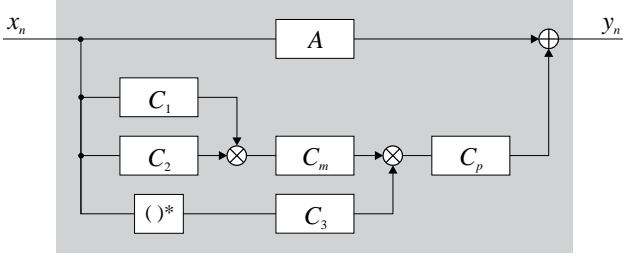


Fig. 1: Equivalent lowpass model of the 3rd order nonlinear filter structure, composed of linear TDL filters and multipliers

and linearization [5] and the compensation of nonlinear sensor distortions [6].

Even though it performs a very good approximation to the general Volterra filter, the complexity is significantly reduced and comparable to linear filtering. This is shown in Fig. 2 where the number of weights is compared for the Volterra filter (1) and for the approximation (2) as a function of the filter length.

The weights of the proposed filter structure can be determined with block-orientated algorithms or adaptively. It can be shown that well known algorithms for the linear TDL filter can be employed. They only must be performed in an iterative manner [7].

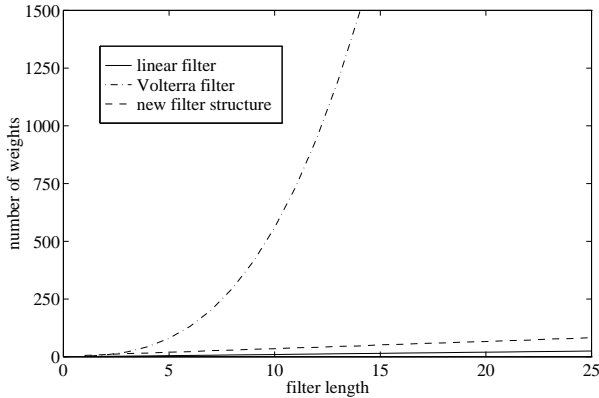


Fig. 2: Complexity of the new filter structure compared to a linear filter and to the 3rd order Volterra filter

3. SIMULATION OF VOICEBAND DATA TRANSMISSION

To show the performance of the new filter structure data transmission over a telephone channel was simulated. The base-band equivalent communication model is depicted in Fig. 3.

3.1. Nonlinear channel

The output of the shaping filter is

$$x(t) = \text{Re} \left\{ e^{j2\pi f_c t} \sum_n a_n s(t-nT) \right\} \quad (3)$$

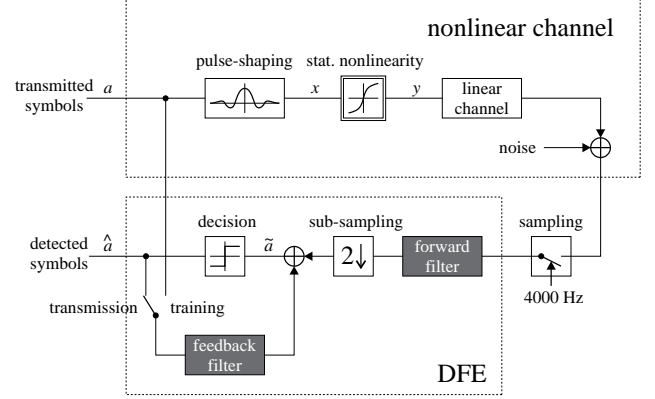


Fig. 3: Nonlinear voiceband data transmission

where a_n are 64-QAM symbols, f_c is the carrier frequency and T is the symbol period. The shaping function $s(t)$ is a square-root raised cosine with a rolloff factor of 0.4.

The channel exhibits nonlinear AM/AM and AM/PM conversion, i.e. the input signal

$$x(t) = A(t) \cos(2\pi f_c t + \varphi(t)) \quad (4)$$

produce the output signal

$$y(t) = \Phi(t)A(t) \cos(2\pi f_c t + \varphi(t) + \psi(t)) \quad (5)$$

with

$$\Phi(t) = 1 - \frac{\zeta(t)}{6} + \frac{7\zeta^2(t)}{360} - \frac{\zeta^3(t)}{540}, \quad (6)$$

$$\psi(t) = \frac{\zeta(t)}{15} + \frac{\zeta^2(t)}{100} \quad (7)$$

and

$$\zeta(t) = 0.3125 \frac{A^2(t)}{\langle A^2(t) \rangle} \quad (8)$$

where $\langle A^2 \rangle$ represents the average power of the signal x .

This is a pretty realistic model of the nonlinearity in the telephone channel: Equ. (6) and (8) is the 3rd order inverse of an optional non-linear encoder in the V.34 standard which was included in order to cope with AM/AM distortion [8]. Additional AM/PM conversion, Equ. (7), is included to be more general.

The effect of this nonlinearity on the 64-QAM signal is shown in Fig. 4. Note that the order of nonlinearity with respect to $A(t)$ is higher than the order of the used DFEs.

The telephone channel itself is assumed to be linear with the characteristics depicted in Fig. 5.

3.2. Decision feedback equalizer

The DFE is composed of a forward filter, a decision device and a feedback filter. The forward and feedback filters are implemented either with linear TDL filters ("conventional

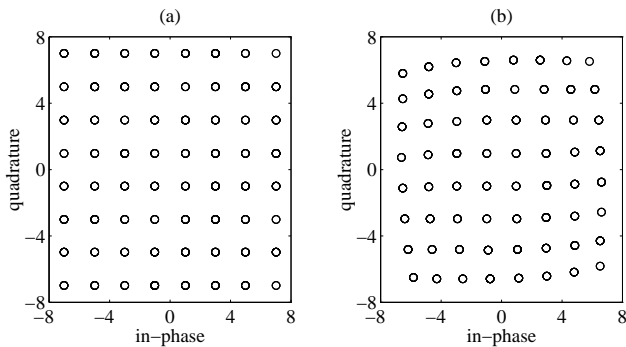


Fig. 4: 64-QAM constellation before (a) and after (b) the static nonlinearity

DFE”), Volterra filters (“Volterra DFE”) or with the filter structure of Fig. 1 (“new DFE”). The forward filter is implemented as a fractionally spaced equalizer with a sampling rate of $2/T$.

4. PERFORMANCE COMPARISON

Simulation was performed with the following parameters: The carrier frequency f_c was set to 2000 Hz and the symbol period T equaled 0.5 ms which correspond to 2000 baud.

The forward and feedback filters of the DFE had the same filter lengths. The corresponding values for the conventional, Volterra and new DFE, respectively, are given in Table 1. The cubic partial filter lengths for the new DFE were $N_C^a = 13$, $N_C^m = 3$ and $N_C^p = 11$. The filter lengths of the 3rd order parts of the Volterra DFE were chosen in order to provide almost the same complexity as the new DFE.

	N_A	N_C	number of weights
conventional DFE	25	-	$2 \cdot 25 = \mathbf{50}$
Volterra DFE	25	5	$2(25+75) = \mathbf{200}$
new DFE	25	25	$2(25 + 55) = \mathbf{160}$

Table 1: Filter lengths and complexity of the different DFEs

The determination of the optimal filter weights was done in the training mode (see Fig. 3) where it is assumed that the transmitted symbols a_n are known at the receiver. A least squares algorithm was performed on a set of $L = 10000$ symbols. The objective of this block-oriented algorithm is to minimize the accumulated squared error between the transmitted symbols and the detected symbols before decision

$$\mathcal{E} = \sum_{n=0}^{L-1} (a_n - \tilde{a}_n)^2. \quad (9)$$

It should be noted that the determination of the coefficients can be also performed adaptively. [7] presents a LMS algorithm

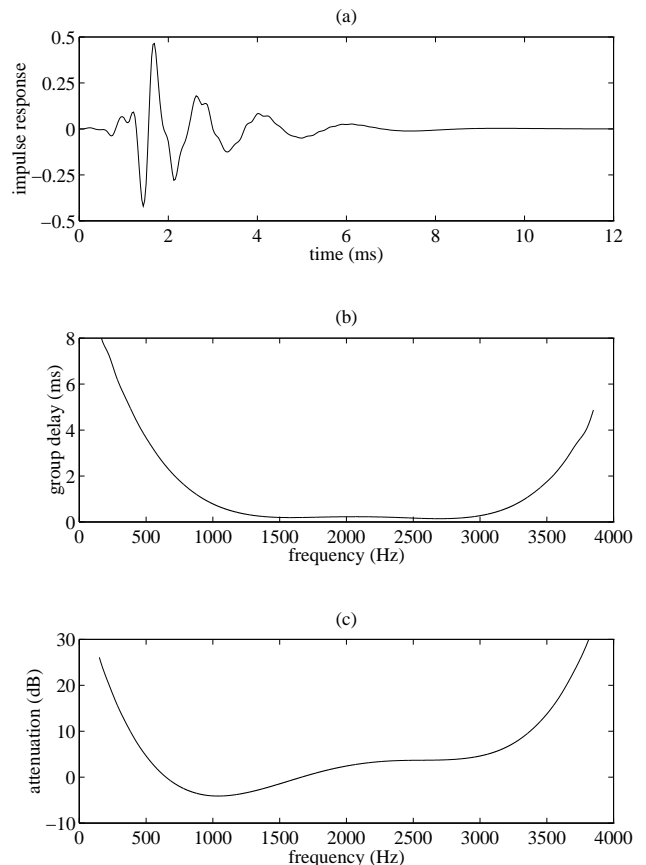


Fig. 5: Impulse response (a), group delay (b) and attenuation (c) of the linear telephone channel

for the new filter structure.

The performance of the different DFEs is compared in the transmission mode (see Fig. 3) where the detected symbols are used as the input to the feedback filter.

Fig. 6 shows the bit error rate (BER) for the different DFEs as a function of the signal-to-noise ratio (SNR). Due to the short memory lengths of the cubic filter parts the Volterra DFE only performs better than the conventional DFE at higher SNR. Below 24 dB the performance even deteriorates. The error propagation feature of the DFE seems to be more critical for nonlinear filter structures.

As stated in [7] the determination of the optimal weights for the new filter structure is a nonlinear optimization problem with the possibility of local minima. Hence, identification was performed with 50 different initial values for the filter weights and Fig. 6 shows the performance of the best and worst solution of these optimization runs. It is seen that the problem of local minima is well behaved because the difference in performance is less than 0.3 dB.

At a BER of 10^{-6} the worst of the new DFEs outperforms the Volterra DFE by more than 2 dB and its performance is only about 1 dB worse than the performance of a conventional DFE operating on a corresponding linear channel.

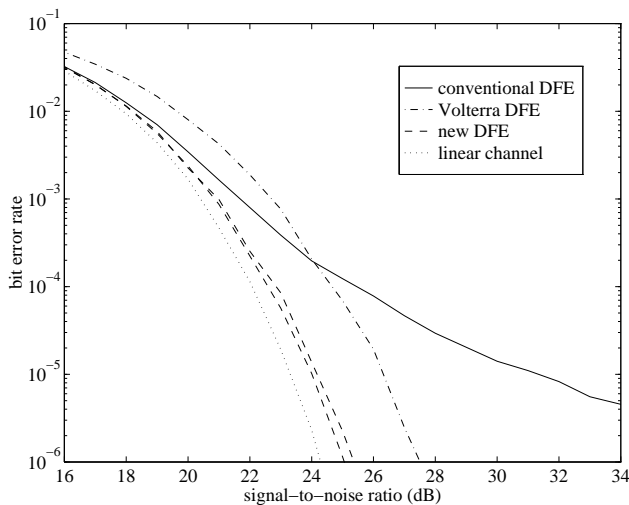


Fig. 6: Bit error rates of the conventional DFE, the Volterra DFE and the new DFE. For comparison also the bit error rate for the corresponding linear channel is shown

The new DFE clearly outperforms the conventional DFE. The reason for the superiority is shown in Fig. 7 where the constellations of the equalized symbols before decision (\tilde{a} in Fig. 3) are compared for a SNR of 30 dB. The new DFE is superior in restoring the nonlinearly deformed original 64-QAM symbol constellation.

5. DISCUSSION

A new DFE for the cancellation of nonlinear ISI was presented. It clearly outperforms the conventional DFE and is also superior to a Volterra DFE with comparable complexity. The underlying filter structure is very general and can be used for a variety of nonlinearities. Note especially that this structure can be extended in order to cope also with 2nd order interferences [5].

It has turned out that nonlinear equalization is particularly effective for QAM modulation where the different symbols have varying amplitudes. Another observation is that the improvements become more significant at higher SNR. At low SNR the impairment of the additive noise dominates the nonlinear distortions. These facts suggest the application of the new DFE for data transmission over telephone channels even though it might be also used in radio communications.

The final conclusion is that one should deliberately accept nonlinearities in the communication channel in order to increase the transmission rate. The resulting nonlinear ISI might be equalized by appropriate receivers. Improvements in processor speed open the way to the implementation of nonlinear filter structures as presented in this paper.

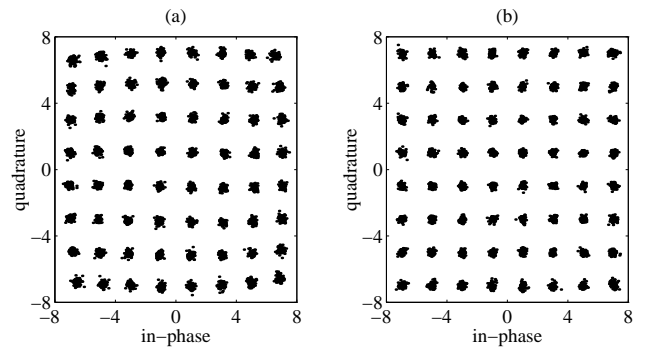


Fig. 7: Symbol constellation of the equalized symbols \tilde{a} before decision: conventional DFE (a) and the new DFE (b) at a SNR of 30 dB

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