JOINT BLIND EQUALIZATION WITH A SHELL PARTITION-BASED CMA FOR QAM SIGNAL CONSTELLATIONS

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ABSTRACT

For quadrature amplitude modulation (QAM) systems, the joint blind equalization (JBE) algorithm, concatenation of two different equalization modes, has been widely used. After the eye is opened by the blind equalization mode, the decision-directed (DD) mode follows to improve the equalization performance with fast convergence. This paper proposes a shell partition-based CMA (SPCMA) which acts as the DD mode in the JBE. Computer simulation results for 16-QAM show the effectiveness of the proposed JBE.

1. INTRODUCTION

In contrast to point-to-point systems, blind equalization has been employed in multipoint communication systems in which transmission of a training sequence is not required [1]. The joint blind equalization (JBE) algorithm is composed of the blind equalization mode and decision-directed (DD) mode, which are used in initial and fine equalization, respectively. Among the JBEs, the concatenation algorithm of the constant modulus algorithm (CMA) and the DD algorithm is well known [2], [3]. The performance of the CMA is independent of the carrier phase error. Thus the CMA is suitable under the closed-eye condition as a blind equalization mode, whereas the DD algorithm is applied to the open-eye condition for fine and stable equalization.

For signal constellations exhibiting the constant modulus property, e.g., the frequency shift keying (FSK), the performance of the CMA is reasonable. On the other hand, the equalization performance degrades for the multi-level signal such as the quadrature amplitude modulation (QAM) because all signal constellation points are projected onto a single modulus [4]. In order to improve the performance of the CMA for QAM signals, the multi-modulus CMA [5], [6] and the dual-mode Godard algorithm (DMGA) [7] employed multiple moduli rather than a single modulus. They allocate a modulus to each subset of signal constellation points depending on the equalizer output power. However, these algorithms may not guarantee the convergence under a severely distorted channel condition and the decision boundaries between adjacent subsets have not been specifically described. In the multi-modulus CMA and the DMGA, the decision boundaries of adjacent subsets were simply chosen by a value bisecting adjacent moduli or by an ad hoc manner.

This paper proposes a shell partition-based CMA (SPCMA) in which shell boundaries are determined by maximum likelihood (ML) estimation. The SPCMA is applied to the DD mode of the JBE, resulting in the proposed JBE that concatenates the CMA with the SPCMA. In Section 2, the shell partitioning method by ML estimation is proposed and in Section 3, compared with the JBE constructed by the concatenation of the CMA and the DD algorithm, the proposed JBE is described. In Section 4 simulation results for 16-QAM are presented and a conclusion is given in Section 5.

2. SHELL PARTITIONING

To construct a method that allocates a single modulus to each subset, we should compute decision boundaries of the equalizer output. Let us assume that the distribution of the equalizer output z_n, conditioned on the independent identically distributed (i.i.d.) original input a_n, is Gaussian. The distribution of the original signal should not be Gaussian, because the blind deconvolution cannot be accomplished [8]. Under the open-eye condition at the switching time to the DD mode and with the proper dynamic range of an equalizer, we assume that the equalizer output z_n approaches one of the signal constellations. Usually, the dynamic range is adjusted by the automatic gain control (AGC). When the equalizer output is stable in the bounded input and bounded output (BIBO) sense, the Gaussian assumption is justified by the central limit theorem (CLT). Applying the above assumptions, the joint probability density function (pdf) $p(z_R, z_I)$ at a signal constellation point can be written as

$$p_{z_R,z_I}(z_R,z_I) = \frac{1}{2\pi\sigma_R\sigma_I} \exp\left(-\frac{(z_R-\mu_R)^2}{2\sigma_R^2} - \frac{(z_I-\mu_I)^2}{2\sigma_I^2}\right) \quad (1)$$

where the subscripts R and I denote the real and imaginary components, respectively. Constants μ_R (μ_I) and σ_R (σ_I) represent the average and standard deviation of the random variable (RV) z_R (z_I), respectively.

With $z_R = \sqrt{c}\cos\phi$, $z_I = \sqrt{c}\sin\phi$, $c = z_R^2 + z_I^2 \geq 0$, and $\sigma_{z_R}^2 = \sigma_{z_I}^2 = \sigma^2$, the pdf $p_c(c)$ of the equalizer output power c can be expressed as

$$p_c(c) = \frac{1}{2\sigma^2}\exp\left(-\frac{c+\mu_R^2+\mu_I^2}{\sigma_z^2}\right) I_0\left(\frac{\sqrt{(\mu_R^2+\mu_I^2)c}}{\sigma^2}\right) \quad (2)$$

where σ_z^2 is identical to $2\sigma^2$ and I_0 represents the modified Bessel function defined as

$$I_0(x) = \frac{1}{2\pi}\int_0^{2\pi} \exp(x\cos\theta)d\theta. \quad (3)$$

The RV c follows the noncentral chi-square distribution with two degrees of freedom ($\chi^2(2)$) [9].

To utilize the Gaussian characteristic of an equalizer output sequence $\{z_n\}$ in equalization, let us define the subset G_k, $k \geq 1$, in the 2-D complex equalizer output domain. The Gaussian distributed equalizer outputs generated by a specific signal constellation point belong to the same subset. Applying ML estimation to the equalizer output power

$$L(|z|^2) = \frac{p(|z|^2|G_k)}{p(|z|^2|\bar{G_k})} \gtrless \frac{P(\bar{G_k})}{P(G_k)} \quad (4)$$

we obtain the decision boundary d_k in the 1-D real domain, where $\bar{G_k}$ is the complement of G_k, $p(|z|^2|G_k)$ is the pdf of the equalizer output power conditioned on G_k, and $P(G_k)$ denotes the probability that the given equalizer output power $|z|^2$ corresponds to G_k. Thus, each shell S_k is partitioned by computed decision boundaries. Fig. 1 shows the decision boundaries computed by ML estimation for 16-QAM.

3. PROPOSED JBE

Fig. 2 shows the JBE system, in which the coefficients of the transversal filter $\{w_n\}$ are updated by the blind equalization mode or DD mode. Let us briefly review the JBE that concatenates the CMA with the DD algorithm. The CMA, the blind equalization mode in the JBE, is a special case of Godard's algorithm [10]. The tap-weights of the CMA are iteratively updated by [2]

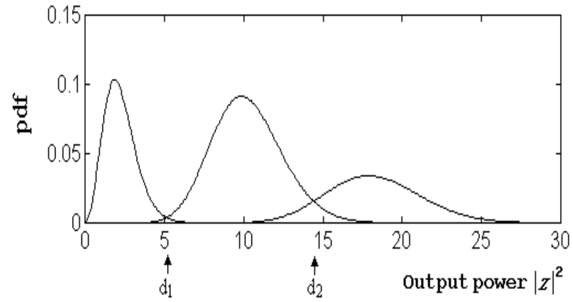

Figure 1: Decision boundaries computed by ML estimation for 16-QAM.

$$W_{n+1} = W_n - \alpha X_n^* z_n(|z_n|^2 - R) \quad (5)$$

where W_n is an equalizer tap-weight vector, X_n is an input vector of an equalizer, α is a gain constant, the subscript n denotes the time index, and the superscript * represents the complex conjugate. The constant modulus R is given by

$$R = \frac{\mathrm{E}[|a_n|^4]}{\mathrm{E}[|a_n|^2]}. \quad (6)$$

When the signal constellation points for 16-QAM are located at $\{\pm1, \pm3\}$ on the real and imaginary axes, the modulus R is equal to 13.2 [3]. Also, the update algorithm of the DD mode in the JBE is written as [11]

$$W_{n+1} = W_n - \alpha X_n^*[(z_{n,R} - \mathrm{dec}(z_{n,R})) + j(z_{n,I} - \mathrm{dec}(z_{n,I}))] \quad (7)$$

where the function dec($\cdot$) denotes a decision device.

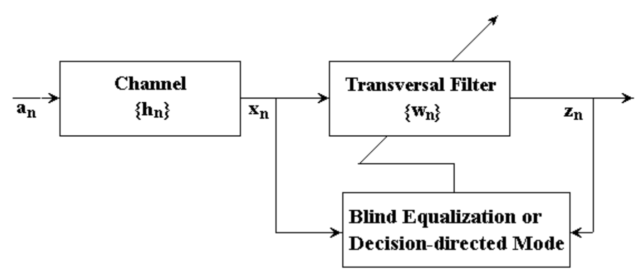

Figure 2: Joint blind equalization system.

Table 1: Statistical Characteristics of a Test Channel

time delay (symbol)	attenuation (dB)	phase (degree)	Gaussian noise (dB)
1	-11	45	30
3	-20	-10	
8	-29	30	

In the proposed JBE, the SPCMA with the computed decision boundary d_k by ML estimation in (4), follows the CMA. A shell is defined by the region between adjacent decision boundaries. Then, the SPCMA replaces the DD algorithm of the JBE, resulting in the proposed JBE that concatenates the CMA with the SPCMA. The update algorithm of the SPCMA is represented as

$$W_{n+1} = W_n - \alpha X_n^* z_n(|z_n|^2 - R_k) \tag{8}$$

where the modulus R_k is determined by

$$R_k = \frac{\mathrm{E}[|a_n \in G_k|^4]}{\mathrm{E}[|a_n \in G_k|^2]}. \tag{9}$$

The modulus R_k is set to 2, 10, and 18 for 16-QAM. In the SPCMA, for the equalizer output power $|z_n|^2$ computed at each iteration, we determine the shell S_k on which the equalizer output power lies. Next, the tap-weights are updated by putting R_k corresponding to the shell S_k into (8).

4. SIMULATION RESULTS AND DISCUSSIONS

Table 1 lists statistical characteristics of a test channel based on the Rummler's three-path model, in which the channel contains multi-path fading and additive Gaussian noise. In computer simulations, the performance of the CMA, the JBE, and the proposed JBE is compared for 16-QAM, in terms of scatter diagrams and convergence characteristics. In the JBE, when the mean square error (MSE) is negligibly small, i.e., in the open-eye condition, the blind equalization mode is switched to the DD mode. In simulation, the DD mode initiates when the MSE reaches -6.2 (dB) which corresponds to the variance $\sigma_z^2 = 0.24$. When the MSE is equal to -6.2 (dB) the decision boundaries d_1 and d_2 for 16-QAM, marked by arrows in Fig. 1, are 5.2 and 14.5, respectively. It is observed that the performance of the proposed JBE is not severely affected, if the decision boundaries are chosen in the range of $0.16 \leq \sigma_z^2 \leq 0.52$.

Fig. 3 shows scatter diagrams for 16-QAM. It shows that both the JBE and the proposed JBE have good equalization performance. The comparable results of the proposed JBE results from the fact that the SPCMA has an optimal criterion for QAM signals. The gain constants of the SPCMA are set four times as large as those of the CMA in the proposed JBE.

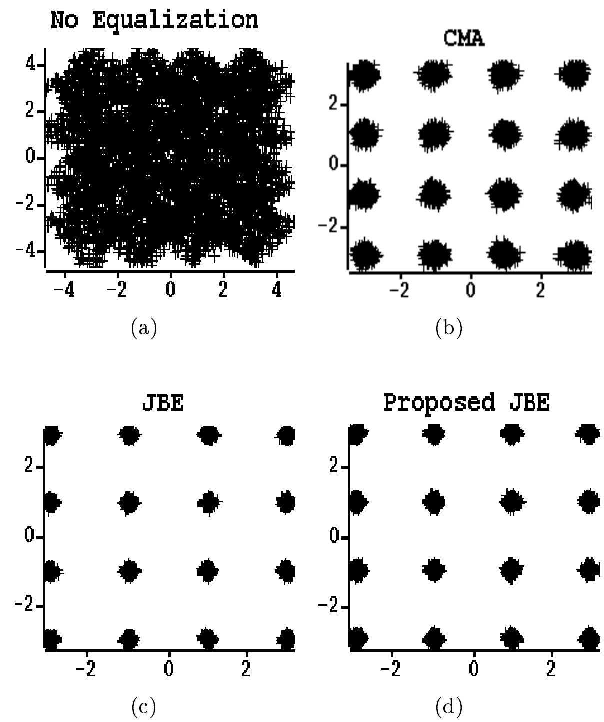

Figure 3: Scatter diagrams of an equalizer output for 16-QAM. (a) No equalization. (b) CMA. (c) JBE (CMA+DD). (d) Proposed JBE (CMA+SPCMA).

The convergence trajectory of each equalization algorithm is shown in Fig. 4 where arrows marked along the x axis denote the switching time to the DD mode. It shows that the performance of the proposed JBE is similar to that of the JBE.

In practice, after initial equalization is accomplished by the CMA, the JBE cannot be switched into the DD mode at once because the carrier should be locked for stable operation of the DD algorithm [3], [6]. On the contrary, as the SPCMA is independent of the carrier phase error, the proposed JBE can be immediately switched from the CMA to the SPCMA. So, it is not necessary that the carrier be locked at the switching time. Also, the carrier recovery is fast done when the SPCMA is employed [6]. The size of the region of attraction (ROA) in which a global minimum is reached is quite large in the SPCMA [5].

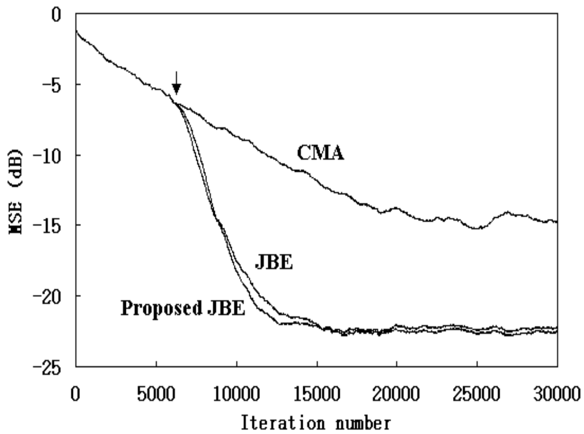

Figure 4: Convergence trajectory for 16-QAM.

For QAM signals, the carrier recovery is operated only when the equalizer output is regarded as one of outmost signal constellation points [3], [6]. Applying the shell partitioning algorithm the carrier recovery can operate not only for the shell that contains the outmost signal constellation points but for the shells which contain four symmetric diagonal constellation points. For instance, the carrier recovery is applicable to S_3 containing the outmost constellation points $(\pm 3, \pm 3)$ and the shell S_1 corresponding to $(\pm 1, \pm 1)$ for 16-QAM.

Moreover, the algorithmic similarity of the CMA and the SPCMA leads to easy hardware implementation of the proposed JBE.

5. CONCLUSIONS

Approximating the distribution of an equalizer output as Gaussian, the decision boundaries of the SPCMA are obtained by ML estimation. Based on the shell partitioning algorithms, a JBE which concatenates the CMA with the SPCMA is proposed. The proposed JBE achieves fast switching to the DD mode without full compensation for carrier phase error. The equalization performance of the proposed JBE is comparable with that of the JBE, and the proposed JBE can be easily applied to high order QAM signals. Further research will focus on hardware implementation of the proposed JBE.

6. REFERENCES

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