CMA BEAMFORMING FOR MULTIPATH CORRELATED SOURCES

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ABSTRACT

Blind adaptive beamforming is often used in communication systems to combat co-channel interference. Among a number of techniques, the constant modulus algorithm (CMA) has proven to be an effective tool for blind beamforming of uncorrelated signals. Unfortunately, CMA beamformers encounter problems for correlated signals and interferences. In this paper, we consider correlated co-channel input signals as a result of multipath. We present two adaptation methods based on CMA to capture distinct signal sources. One approach is to use an orthogonal projection constraint on the beamformer parameters. The other approach relies on the independence of source signals and exploit a Gram-Schmidt orthogonalization at beamformer outputs. The performance of our methods will be shown through computer simulations.

1. INTRODUCTION

Adaptive beamforming finds a number of applications in areas such as radar, sonar, and communication systems. Adaptive arrays can be used to place nulls in the direction of interferences while enhancing the desired signal. In recent years, much attention has been given to the Constant Modulus Algorithm (CMA) for adaptive beamforming of unknown signals [1]. CMA has shown to be an effective tool for blind signal beamforming and for multi-source beamforming when signals are uncorrelated [2]. Barry and Batra ([3],[4]) , and Tugnait [5] also proposed the CM array for MIMO communications system with uncorrelated sources.

In a multipath environment, a desired signal arriving at the antenna through secondary path may be joined by interferences with similar angles of arrival. The existence of multipath hence tends to result in multipath correlated signals arriving at the antenna array. As a result, CMA beamformer needs to reject the coherent interferences and should not cancel the desired signal ([6], [7]). To combat correlated signal sources, Shynk et al. ([8],[9],[10]) have used (cascade and parallel) multi-stage CM array to enhance the desired signal based on direction of arrival (DOA) information obtained from first stage CMA beamformers. Since the first stage CMA may not be able to correctly estimate the DOA in coherent environment, this approach will be dependent on the performance of the first CMA array that tends to be unreliable in estimating DOA in a multipath environment. In this paper, we are proposing two new blind adaptive beamforming techniques for multiple signal arrivals that are correlated due to multipath interferences. To ensure that different CMA beamformer captures distinct signals, orthogonality constraints are placed on multiple beamformers for multiple signal capturing.

2. PROBLEM FORMULATION

Consider an N-element antenna array with L independent narrow-band signals impinging on our sensors. The complex received signal vector of length N can be written as:

$$\mathbf{X}(k) = \mathbf{A}\mathbf{a}(k) + \mathbf{n}(k) \tag{1}$$

where A is the array response matrix, $\mathbf{a}(k) = [a_1(k), \ldots, a_L(k)]^T$ is the narrow-band input signal vector, and $\mathbf{n}(k) = [n_1(k), \ldots, n_L(k)]^T$ is the additive white Gaussian noise vector. The array response matrix of a uniformly calibrated array with element distance d can be written as:

$$\mathbf{A} = \begin{bmatrix} 1 & \cdots & 1 \\ \exp^{-j\phi_1} & \cdots & \exp^{-j\phi_L} \\ \vdots & \vdots & \vdots \\ \exp^{-j(N-1)\phi_1} & \cdots & \exp^{-j(N-1)\phi_L} \end{bmatrix} \tag{2}$$

where $\phi_i = 2\pi(d/\lambda)\sin(\theta_i)$, λ is the common wavelength of the sources, $\{\theta_i\}$ are the sources directions of arrival (DOAs).

Given a beamformer weight vector $\mathbf{w}(k)$, the output of our beamformer is $y(k) = \mathbf{w}^H(k)\mathbf{X}(k)$, where $\mathbf{w}(k) = [w_1(k), \ldots, w_N(k)]^T$ is the beamformer complex weight vector. The CMA adaptively solves for the optimum weight vector $\mathbf{w}$ without any training sequence by minimizing the well known constant modulus (CM) cost function:

$$J(\mathbf{w}) = E\{(|y(k)|^2 - 1)^2\} \tag{3}$$

A stochastic gradient search method can be used to minimize the CM cost function $J(\mathbf{w})$ by adaptively adjusting the weight vector $\mathbf{w}$ according to:

$$\mathbf{w}(k+1) \quad = \quad \mathbf{w}(k) - \mu(|y(k)|^2 - 1)\mathbf{X}(k)y^*(k)$$

where μ is a small positive step size.

For wireless communication systems, multipath propagations result in correlated sources arrive at our sensors. Consider a multipath scenario where m correlated signals arrive at our N sensors from L ($L > m$) distinct angles $\{\theta_1 \ldots \theta_L\}$. Let p_i be the power of the $i-$th source signal. Denote the arrival signal vector as $\mathbf{a}(k)$. Let $\mathbf{s}(k)$ represent the vector of normalized source signals $\mathbf{s}(k) = [\mathbf{s}_1(k), \ldots, \mathbf{s}_m(k)]^T$ in which $s_i(k)$ are zero mean, independent, with unit variance (power). Then the arrival signal vector under multipath will be a instantaneous "mixing" of original m signals. This mixing can be represented by an $L \times m$ mixing matrix Φ. Thus, the $N \times 1$ complex received vector can be written as:

$$\begin{aligned} \mathbf{X}(k) &= \mathbf{A}\,\Phi\,\mathbf{s}(k) + \mathbf{n}(k) \\ &= \mathcal{H}\,\mathbf{s}(k) + \mathbf{n}(k) \end{aligned} \tag{4}$$

where $\mathbf{n}(k)$ is the zero mean, additive Gaussian noise independent at each of our sensors. Without loss of generality, we can assume that angles $\{\theta_i\}$ are distinct and the number of arrivals L is greater than the number of the sources m. If the number of sensors $N \geq L$, then $\mathbf{A}$ is full column rank. For practical purposes, assume that the mixing multipath $L \times m$ matrix Φ has full column rank. This assumption is equivalent to assume that the original signals $\mathbf{s}(k)$ are separable from $\Phi\,\mathbf{s}(k)$. Thus, we can conclude that $N \geq m$ and $\mathcal{H}$ has full column rank m.

3. MULTIPLE BEAMFORMERS

As the array of antennas receive m independent sources with multipath propagation, the objective of array processing is to capture all m independent sources separately. In order to capture all m sources, we need to construct m beamformers and design additional strategy so that no two beamformers will converge to identical source, i.e., $\mathbf{w}_i \neq \mathbf{w}_j, \quad i \neq j$. Clearly, we need to drive our parameter vectors of different beamformers to obtain different stationary points in order to separate all the incoming signals. Here we present two different CMA parameter constraints for this purpose.

3.1. Gram-Schmidt CMA (GS-CMA)

In this technique, we exploit the fact that all source signals $\{s_i(k)\}$ are orthogonal to one another (*before* transmission). Thus, the ideal capturing of m different source signals at the receiver requires that all beamformer outputs be orthogonal to one another. To impose this signal orthogonality constraint, we apply a Gram-Schmidt orthogonalization procedure.

Define

$$R_x = E\{\mathbf{X}(k)\mathbf{X}^H(k)\} \tag{5}$$

which is positive definite. To achieve signal separation and orthogonality we require that

$$E\{y_i(k)y_j^*(k)\} = E\{\mathbf{w}_i^H\mathbf{X}(k)\mathbf{X}^H(k)\mathbf{w}_j\} = \mathbf{w}_i^H R_x \mathbf{w}_j = 0. \tag{6}$$

Now we can define an inner-product in the $\mathbf{w}$ space

$$< \mathbf{w}_i,\ \mathbf{w}_j >=: \mathbf{w}_j^H R \mathbf{w}_i. \tag{7}$$

To implement the Gram-Schmidt orthogonalization with CMA, $\mathbf{w}_i$ updated by CMA should be projected onto the space orthogonal to $\{\mathbf{w}_1,\ \mathbf{w}_2,\ \ldots,\ \mathbf{w}_{i-1}\}$ as defined by the inner-product.

In actual algorithm implementation, the Gram-Schmidt CMA (GS-CMA) is implemented as follows:

1. Initialize all the weight vector as $\mathbf{w}_i = \mathbf{e}_i$, where $\mathbf{e}_i$ is the i-th standard basis vector and start CMA adaptation.
2. At each iteration, orthogonalized the weight vectors based on the Gram-Schmidt orthogonalization procedure

$$\mathbf{w}_i = \mathbf{w}_i - \sum_{j=1}^{i-1} \frac{\mathbf{w}_j^H R_x \mathbf{w}_i}{\mathbf{w}_j^H R_x \mathbf{w}_j} \mathbf{w}_j, \quad i = 2,\ 3,\ \ldots,\ m. \tag{8}$$

Ideally, the objective is for beamformers to capture different signals with additive noise so that

$$\mathbf{w}_i^H \mathcal{H}\mathcal{H}^H \mathbf{w}_j = 0. \tag{9}$$

It should be noted that since the array noise $\mathbf{n}(k)$ is present, the Gram-Schmidt procedure in fact results in

$$\mathbf{w}_i^H \mathcal{H}\mathcal{H}^H \mathbf{w}_j + \sigma^2 \mathbf{w}_i^H \mathbf{w}_j = 0. \tag{10}$$

Hence the retrieved signals are not completely orthogonal. Nonetheless, for small noise variance σ^2, the effect of the biased beamforming should be insignificant.

3.2. Two Step CMA (TS-CMA)

When statistical information such as signal orthogonality cannot be applied, different methods must be used to ensure that different beamformer parameters do not converge to identical directions.

Consider a simple case of 2 correlated sources arriving at the array. To achieve $\mathbf{w}_1 \neq \mathbf{w}_2$, we are going to decompose the second weight vector $\mathbf{w}_2$ into 2 parts

$$\mathbf{w}_2 = \mathbf{w}_1 u_2 + \mathbf{w}_1^{\perp} \tag{11}$$

where $\mathbf{w}_1^{\perp}$ is realized by :

$$\begin{aligned} \mathbf{B} &=: \left(\mathbf{I} - \frac{\mathbf{w}_1\mathbf{w}_1^H}{\mathbf{w}_1^H\mathbf{w}_1}\right) \\ \mathbf{w}_1^{\perp} &= \mathbf{B}\mathbf{v}_2 \end{aligned} \tag{12}$$

Thus $\mathbf{w}_2$ consists of:

$$\mathbf{w}_2 = \mathbf{w}_1 u_2 + \mathbf{B}\mathbf{v}_2 \tag{13}$$

We will then adapt $\mathbf{w}_2$ by adjusting u_2 and $\mathbf{v}_2$ separately. To avoid common convergence, we use a two step approach by first adapt $\mathbf{w}_1$ and $\mathbf{v}_2$. The adaptation of u_2 follows the approximate convergence of $\mathbf{v}_2$ by checking its average magnitude of the stochastic gradient.

Similarly, we can extend the two step CMA (TS-CMA) adaptive beamforming algorithm to the case of m beamformers. Define

$$\begin{aligned} \mathbf{B}_l &= \mathbf{I} - \mathbf{W}_{l-1}\left(\mathbf{W}_{l-1}^H\mathbf{W}_{l-1}\right)^{-1}\mathbf{W}_{l-1}^H \\ \mathbf{W}_{l-1} &= [\mathbf{w}_1\ \ \mathbf{w}_2\ \ \ldots\ \ \mathbf{w}_{l-1}\ ] \end{aligned} \tag{14}$$

where $l = 2, \ldots,\ m$. Separate

$$\mathbf{w}_l = \mathbf{W}_{l-1}\mathbf{u}_l + \mathbf{B}_l\mathbf{v}_l \tag{15}$$

where where $\mathbf{u}_l$ and $\mathbf{v}_l$ are two vectors to be adjusted in two steps. One can use the gradient search method to adaptively solve for the two vectors $\mathbf{u}_i$ ad $\mathbf{v}_i$ for the i-th beamformer. Here are two steps to our algorithm for each beamformer.

1. Run CMA by first freezing $\mathbf{u}_p$.
2. Continue CMA by allowing $\mathbf{u}_p$ adaptation after step one converges.

Because the two-step CMA (TS-CMA) does not rely on source signal independence, it is also applicable when sources signals are highly correlated. This particular feature may be useful in radar and sonar detection of return signals from the same excitation pulse.

3.3. Blind Equalization of MIMO channels

Although both methods theoretically may be applied to a multiple-input multiple-output (MIMO) dynamic system such as

$$\mathbf{x}(k) = \mathbf{H}(z)\mathbf{s}(k) + \mathbf{n}(k),$$

there exists, however, an intrinsic problem for blind equalization of MIMO channels. Since the system now has memory, it is now possible for a beamformer filter to capture a signal $s_i(k)$ or its delay $s_i(k-d)$. Because each individual element in the original source signals $\mathbf{s}(k)$ represents an i.i.d. sequence in communications, neither orthogonality nor different filter parameters can guarantee the capturing of different signals sources by separate beamformer filters. Hence, the capturing of the same signal at different delay becomes an intrinsic problem for the MIMO blind equalization system.

Although non-trivial extensions to MIMO equalization are necessary, when the MIMO channel memory is short, capturing of identical signals at different delays becomes less likely. Thus, both of our methods are more likely to be successful for MIMO equalization when the channel memory is short.

4. COMPUTER SIMULATIONS

In the first set of simulations, we consider a scenario of $N = 6$ antenna elements with equal spacing $d = 0.5\lambda$. Three zero mean, unit variance, independent sources s1, s2, and s3 (FM, QPSK, and 16QAM, respectively) impinge on our sensors at $\theta_1 = -35°$, $\theta_2 = +15°$ and $\theta_3 = +40°$, respectively. The average single SNR is 20dB. The average signal power is the average of the arrival signal power at the array elements. The multipath mixing matrix in this set of simulations is:

$$\mathbf{\Phi} = \begin{bmatrix} 1.00 & 0.00 & -.50 \\ 0.11 & 1.00 & 0.80 \\ -.50 & 0.20 & 1.00 \end{bmatrix}.$$

Figure 1 shows the beampattern of our first beamformer using GS-CMA method. The three vertical lines in this figure represent the three distinct arrival angles of our sources. As we can see from this figure, the DOA information obtained from the first stage CMA beamformer is not accurated in this case. If we use this estimate DOA to initilize other beamformers in a parallel structure [9], we may not be able to successffuly capture distinct sources.

The performance measures in our analysis are: SINR (Signal to Interference and Noise Ratio) and MSE (Mean Square Error). Figures 2 and 3 show the results of our new adaptive beamforming techniques GS-CMA and TS-CMA, respectively. To ensure that each beamformer will not start from an identical point, we initialize our weight vectors in both GS-CMA and TS-CMA method to be zero except for $\mathbf{w}_1(1) = \mathbf{w}_2(2) = \mathbf{w}_3(3) = 1$. From both figures, we see that each beamformer suppress the other two other sources by an amount of approximately from -30dB to -40dB. We can also see that the MSE of our beamformers are from -15dB to -20dB. Thus, we have demonstrated that by placing constraints on our complex weight vectors, each beamformer sucessfully captured a distinct source. Notice that CMA is also successful for a non-constant modulus signal (s3 or 16QAM).

In the second set of simulations, we consider a 2-input 2-output communication system. Our two inputs are QPSK and 16QAM (s1 and s2, respectively). The average SNR is 20dB. The channel transfer function is

$$\mathbf{H}(z) = \begin{bmatrix} 1 + 0.7z^{-1} & 0.7 \\ -0.3 + 0.4z^{-1} & 0.9 + 0.1z^{-1} \end{bmatrix}.$$

Notice that $\mathbf{H}(z)$ has a very short memory. We use two 7-tap equalizers in this case and apply both methods: GS-CMA and TS-CMA to this system. The performance measures are also SINR and MSE. Figures 4 and 5 show the results of our methods when applied to this system. From both figures, we see that in both methods, the first equalizer suppress the output power of the 16QAM source by an amount of approximately -45dB while the second equalizer suppress the output power of the QPSK source by an amount of approximately -30dB. The MSE of all equalizers are approximately -20dB. Thus we have demonstrated that GS-CMA and TS-CMA can be extended to a Multi-Input Multi-Output communication system.

5. CONCLUSION

In this paper, we propose two new blind adaptive techniques for both beamformer and Multi-Input Multi-Output channels. We introduce a Gram-Schmidt procedure (GS-CMA) based on source statistics for signal separation. We also propose a two-step adaptation (TS-CMA) that is independent of source statistics. The GS-CMA method exploits the fact that all input signals are uncorrelated before arriving at our sensors therefore the output at our beamformer must be also uncorrelated. The TS-CMA method exploits the fact that the beamformer filter of the $i-th$ beamformer (or equalizer) can be expressed as a linear combination of other beamformer filters and the orthogonal subspace span by them. The constraints in both methods enable us to ensure that each beamformer (or receiver) will not lock on to some identical source. Our computer simulations demonstrate the performance of our methods in multipath scenarios of both constant and non-constant modulus signals.

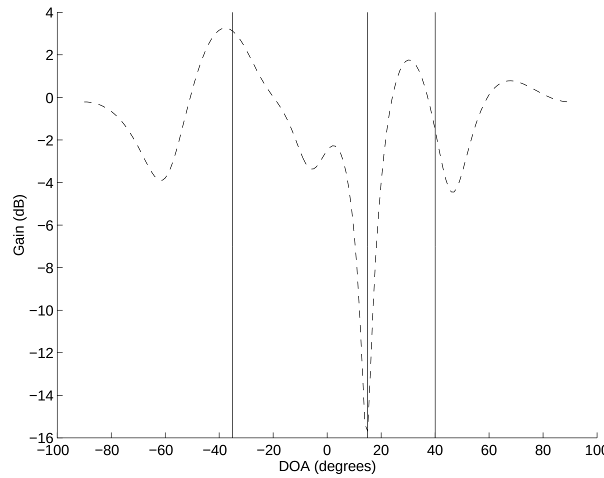

Figure 1. Beampattern for the first beamformer

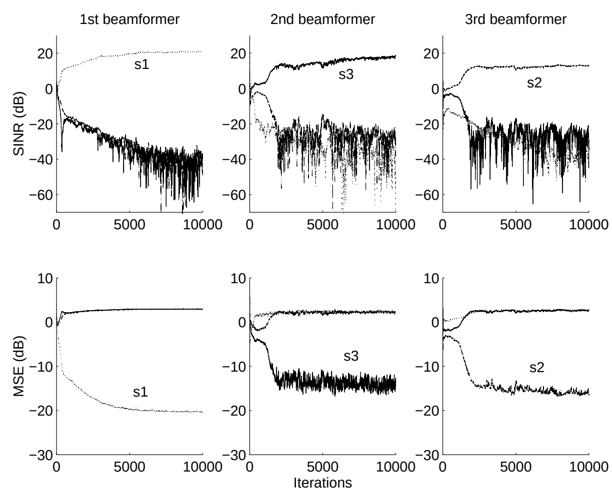

Figure 2. GS-CMA for multiple beamformers

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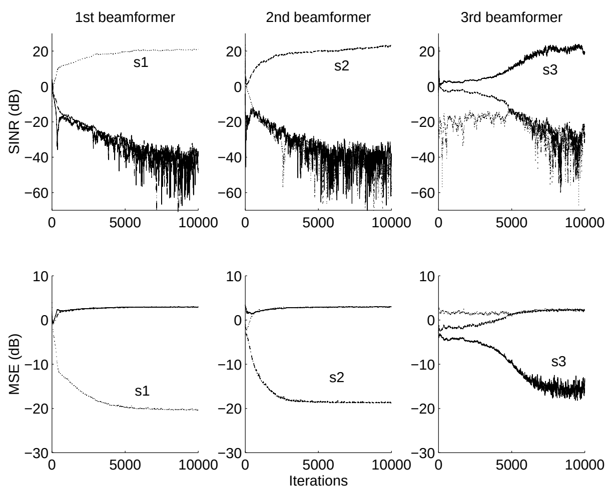

Figure 3. TS-CMA for multiple beamformers

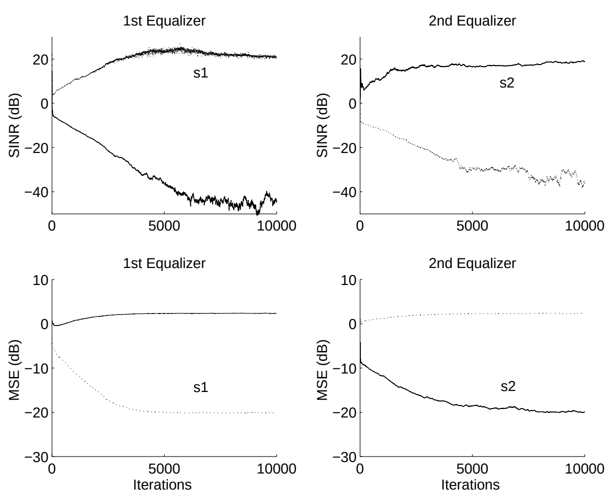

Figure 4. GS-CMA for MIMO channels

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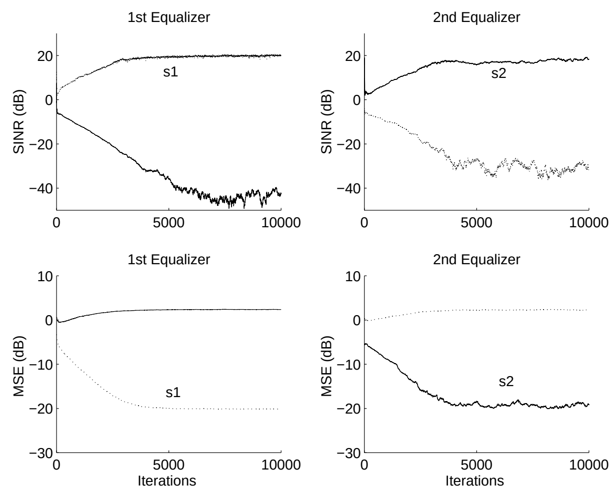

Figure 5. TS-CMA for MIMO channels

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6. ACKNOWLEDGEMENT

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