ESTIMATING THE DERIVATIVE OF MODULO-MAPPED PHASES

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ABSTRACT

The paper considers the problem of phase unwrapping which means generating absolute phase values from noisy, modulo-2π mapped phase ‘observations’. Phase unwrapping is the central key element in any kind of interferometric application. Nearly all known phase unwrapping techniques try to unwrap the mapped phases by a sequence of differentiating, taking the principal value of the discrete derivative and integrating again. This procedure, conceptually appealing as it may appear, however, yields strongly biased phase derivatives and thus strongly biased phase estimates. It can be shown mathematically, that computing the discrete derivative of noisy modulo-2π mapped phase yields estimates of the unambiguous discrete derivative, which are always biased towards lower absolute values. The bias clearly depends on the phase slope itself as well as on the coherence or on the signal to noise ratio (SNR), respectively. Considering the practical application of Synthetic Aperture Radar Interferometry, the paper presents the theoretical analysis, and gives some numerical results.

1. INTRODUCTION

The determination of the unambiguous phase from noisy observations of complex angularly modulated signals is an unsolved problem in general, especially if phase and amplitude are mutually uncorrelated or even independent. This is clearly the case for a complex SAR interferogram. In terms of signal theory a SAR interferogram can be considered as a complex, simultaneously amplitude and phase modulated 2D signal with non Gaussian error statistics. Usually the wanted interferometric phase is obtained by a simple $\tan^{-1}$ operation, delivering phase values within the principal interval (e.g. $-\pi$, π). These phases contain all the information needed for the generation of digital terrain elevation maps of observed areas but they do not contain that information in an unambiguous way, as any absolute phase offset (an integer multiple of 2π) is lost. Furthermore they are subject to phase noise coming from the superimposed amplitude noise in real and imaginary part of the InSAR image. In terms of signal theory phase unwrapping is *simply* a two dimensional phase demodulation problem. Nearly all classical approaches to phase unwrapping, known from optical interferometry apply a sequence of differentiating, taking the principal value of the discrete derivative and integrating again along specified paths. The paper will show that all these approaches yield biased estimates, especially when combined with Linear Least Squares techniques which are commonly used to reduce stochastic phase errors.

2. THEORY AND CONCEPTS

Let the observed phase be related to the unambiguous phase by:

$$y_\varphi(k) = \left[\varphi(k) + \tilde{e}_\varphi(k)\right]_{|2\pi} \tag{1.}$$

where φ(k) is the true unambiguous phase at time or point k, $\tilde{e}_\varphi(k)$ is the true phase error and the bracket indicates the operation of taking the principal value of the argument phase term in a way that:

$$\tilde{\varphi} = [\varphi]_{|2\pi} = \varphi \pm n \cdot 2\pi \in (-\pi, \pi] \quad and: \quad |\tilde{\varphi}| \le \pi \tag{2.}$$

As a result, the observed phase always lies within the base interval (-π,π]. Forming the discrete derivative yields:

$$\begin{aligned}\Delta_\varphi(k) &= \left[y_\varphi(k+1) - y_\varphi(k)\right]_{|2\pi} \\ &= \left[\left[\varphi(k+1) + \tilde{e}_\varphi(k+1)\right]_{|2\pi} - \left[\varphi(k) + \tilde{e}_\varphi(k)\right]_{|2\pi}\right]_{|2\pi}\end{aligned} \tag{3.}$$

Here we have only used the fact that adding any integer multiple of 2π does not change the result of a modulo-2π operation. With the same reasoning we further obtain:

$$\begin{aligned}\Delta_\varphi(k) &= \Bigg[\left[\left[\varphi(k+1)\right]_{|2\pi} + \left[\tilde{e}_\varphi(k+1)\right]_{|2\pi}\right]_{|2\pi} \\ &\quad - \left[\left[\varphi(k)\right]_{|2\pi} + \left[\tilde{e}_\varphi(k)\right]_{|2\pi}\right]_{|2\pi}\Bigg]_{|2\pi} \\ &= \left[\left[\varphi(k+1)\right]_{|2\pi} - \left[\varphi(k)\right]_{|2\pi} + e_\varphi(k+1) - e_\varphi(k)\right]_{|2\pi}\end{aligned} \tag{4.}$$

$e_\varphi(k+1) - e_\varphi(k)$ are the phase errors at points k and k+1, respectively, mapped into the base interval (-π,π]. The stochastic properties of these errors, namely distribution density and second order moments are known. Now again using the same identities in equation 4 as before we can further write:

$$\begin{aligned}\Delta_\varphi(k) &= \left[\left[\varphi(k+1) - \varphi(k)\right]_{|2\pi} + \left[e_\varphi(k+1) - e_\varphi(k)\right]_{|2\pi}\right]_{|2\pi} \\ &= \left[\delta_\varphi(k) + \left[e_\varphi(k+1) - e_\varphi(k)\right]_{|2\pi}\right]_{|2\pi}\end{aligned} \tag{5.}$$

$\delta_\varphi(k)$ is the true discrete phase derivative, the modulus of which is supposed to be always smaller than π. Equation 5 clearly expresses the error which we commit when forming the discrete derivative from modulo-2π mapped noisy data. If there were no phase error present the result would be totally correct, but since phase errors always occur in normal interferograms, we commit a systematic error when ‘differentiating’ modulo-2π mapped data. In the next step we will investigate the stochastic features of the phase error difference in the second term of equation 5. Then we will evaluate, how a modulo operation, which occurs twice, changes the known distribution density of a random variable. Let us introduce the non-mapped phase error difference variable by:

$$\tilde{\delta}_e(k) = e_\varphi(k+1) - e_\varphi(k) \tag{6.}$$

Obviously the numerical values can vary between ±2π, as any of the terms in the difference can vary between ±π. Assuming that the phase errors of two subsequent phase samples are independent random variables the resulting distribution density of the phase difference is the correlation product of the individual phase distribution densities. Thus we have:

$$\begin{aligned} f_{\tilde{\delta}_e(k)}(\xi) &= f_{e_\varphi(k+1)}(-\xi) * f_{e_\varphi(k)}(\xi) \\ &= \int_{-\infty}^{\infty} f_{e_\varphi(k+1)}(u) \cdot f_{e_\varphi(k)}(\xi+u) du \end{aligned} \tag{7.}$$

Conceptually equation 7 can be solved since the individual terms are known from [1,2]. The phase error distribution density, given there, is:

$$f_{e_\varphi(k)}(\varepsilon) = \frac{1}{2\pi} \cdot \frac{1-|\gamma|^2}{1-|\gamma|^2 \cdot \cos^2(\varepsilon)} \cdot \left[1 + \frac{|\gamma| \cdot \cos(\varepsilon) \cdot [\pi - \arccos(|\gamma| \cdot \cos(\varepsilon))] \cdot}{\sqrt{1-|\gamma|^2 \cdot \cos^2(\varepsilon)}} \right] \tag{8.}$$

The phase error difference $\tilde{\delta}_e(k)$ given in equation 6 and showing values between ±2π is now mapped into the base interval between ±π. The functional mapping is:

$$\delta_e(k) = \begin{cases} \tilde{\delta}_e(k) + 2\pi & \tilde{\delta}_e(k) \in (-2\pi, -\pi] \\ \tilde{\delta}_e(k) \quad \text{if} & \tilde{\delta}_e(k) \in (-\pi, \pi] \\ \tilde{\delta}_e(k) - 2\pi & \tilde{\delta}_e(k) \in (\pi, 2\pi] \end{cases} \tag{9.}$$

The distribution density of the ±π-mapped phase error difference $\delta_e(k)$ can be obtained by writing:

$$\begin{aligned} f_{\delta_e(k)}(\xi) = & \int_{-2\pi}^{-\pi} f_{\delta_e(k)/\tilde{\delta}_e(k)}(\xi/u) \cdot f_{\tilde{\delta}_e(k)}(u) du \\ & + \int_{-\pi}^{\pi} f_{\delta_e(k)/\tilde{\delta}_e(k)}(\xi/u) \cdot f_{\tilde{\delta}_e(k)}(u) du \\ & + \int_{\pi}^{2\pi} f_{\delta_e(k)/\tilde{\delta}_e(k)}(\xi/u) \cdot f_{\tilde{\delta}_e(k)}(u) du \end{aligned} \tag{10.}$$

Using the correct functional mapping in each of the intervals and exploiting that the conditional probability density of a functionally mapped variable only consists of a Dirac impulse we have:

$$f_{\delta_e(k)}(\xi) = \int_{-2\pi}^{-\pi} \delta(\xi - (u+2\pi)) \cdot f_{\tilde{\delta}_e(k)}(u) du$$

$$+ \int_{-\pi}^{\pi} \delta(\xi - u) f_{\tilde{\delta}_e(k)}(u) du + \int_{\pi}^{2\pi} \delta(\xi - (u-2\pi)) f_{\tilde{\delta}_e(k)}(u) du$$

$$= \left[f_{\tilde{\delta}_e(k)}(\xi - 2\pi) + f_{\tilde{\delta}_e(k)}(\xi) + f_{\tilde{\delta}_e(k)}(\xi + 2\pi) \right] \cdot rect\left[\frac{\xi}{2\pi}\right] \tag{11.}$$

Now we return to the sum in equation 5 and introduce the non-mapped discrete difference by:

$$\tilde{\Delta}(k) = \delta_\varphi(k) + \left[e_\varphi(k+1) - e_\varphi(k) \right]_{|2\pi} = \delta_\varphi(k) + \delta_e(k) \tag{12.}$$

Further introducing the conditional density of $\tilde{\Delta}(k)$ conditioned on the fact that the true phase derivative takes on the value δ_0 we may write:

$$f_{\tilde{\Delta}(k)/\delta_\varphi(k)}(\xi/\delta_0) = f_{\delta_e(k)}(\xi - \delta_0) \tag{13.}$$

In the last equality we have used the fact that the ±π-mapped phase error difference $\delta_e(k)$ is independent of the true phase derivative $\delta_\varphi(k)$. Substituting equation 11 into 13 we obtain:

$$\begin{aligned} f_{\tilde{\Delta}(k)/\delta_\varphi(k)}(\xi/\delta_0) &= rect\left[\frac{\xi - \delta_0}{2\pi}\right] \\ &\cdot \left[f_{\tilde{\delta}_e(k)}(\xi - \delta_0 - 2\pi) + f_{\tilde{\delta}_e(k)}(\xi - \delta_0) + f_{\tilde{\delta}_e(k)}(\xi - \delta_0 + 2\pi) \right] \end{aligned} \tag{14.}$$

This is the conditional distribution density of $\tilde{\Delta}(k)$ conditioned on the fact that the true phase derivative takes on the value δ_0. This variable is ±π-mapped again to yield $\Delta_\varphi(k)$ (Equ. 5). Now utilizing the same arguments and reasoning as before, we get the final result for the conditional density of the 'mapped' phase derivative, conditioned on the fact that the true phase derivative takes on the value δ_0:

$$\begin{aligned} & f_{\Delta_\varphi(k)/\delta_\varphi(k)}(\xi/\delta_0) \\ & = \left[f_0(\xi - \delta_0 - 2\pi) + f_0(\xi - \delta_0) + f_0(\xi - \delta_0 + 2\pi) \right] \cdot rect\left[\frac{\xi}{2\pi}\right] \end{aligned} \tag{15.}$$

where the short hand expression $f_0(\xi)$ has been utilized for convenience. This expression is the 2π-cutout of the sum of three shifted replicas of the distribution density:

$$\begin{aligned} f_0(\xi) = & \left[f_{\tilde{\delta}_e(k)}(\xi - 2\pi) + f_{\tilde{\delta}_e(k)}(\xi) + f_{\tilde{\delta}_e(k)}(\xi + 2\pi) \right] \\ & \cdot rect\left[\frac{\xi}{2\pi}\right] \end{aligned} \tag{16.}$$

We now introduce the bias error of the 'mapped' derivative: $e(k) = \Delta_\varphi(k) - \delta_\varphi(k)$ and evaluate the conditional density of this error conditioned on $\delta_\varphi(k) = \delta_0$. From probability theory we know that:

$$\begin{aligned} & f_{e(k)/\delta_\varphi(k)}(\xi/\delta_0) \cdot d\xi \\ & = P\left\{ \omega : e(k,\omega) \in (\xi, \xi + d\xi] / \delta_\varphi(k,\omega) = \delta_0 \right\} \\ & = P\left\{ \omega : \Delta_\varphi(k,\omega) \in (\xi + \delta_0, \xi + \delta_0 + d\xi] / \delta_\varphi(k,\omega) = \delta_0 \right\} \\ & = f_{\Delta_\varphi(k)/\delta_\varphi(k)}(\xi + \delta_0 / \delta_0) \cdot d\xi \end{aligned} \tag{17.}$$

Exploiting the identity of equ. 15 in equation 17 we obtain the wanted conditional density:

$$f_{e(k)/\delta_\varphi(k)}(\xi/\delta_0) = [\,f_0(\xi-2\pi) \;+\; f_0(\xi) + f_0(\xi+2\pi)] \cdot rect\left[\frac{\xi+\delta_0}{2\pi}\right] \tag{18.}$$

with $f_0(\xi)$ given in equation 16. Figure 1 demonstrates the meaning of equation 18 for an arbitrary density:

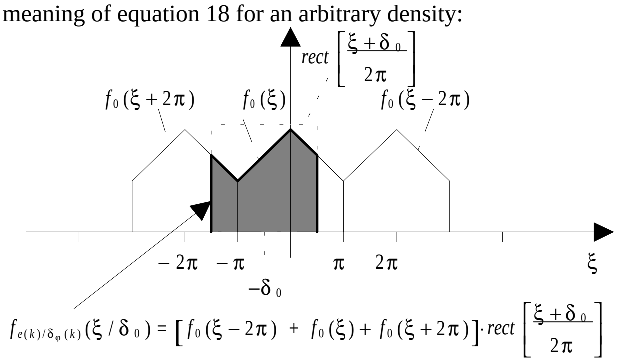

Figure 1: The generation of the conditional error distribution

With the help of figure 1 the conditional expectation $E_{\delta_0}\{e(k)\} = E\{e(k)/\delta_\varphi(k) = \delta_0\}$ of the bias error is readily calculated:

$$E_{\delta_0}\{e(k)\} = \int_{-\delta_0-\pi}^{-\delta_0+\pi} \xi \cdot f_{e(k)/\delta_\varphi(k)}(\xi/\delta_0)\cdot d\xi \tag{19.}$$

1. As our first case we will consider the interval $-\pi < \delta_0 \le 0$. For this case we can subdivide the integral into the following two parts:

$$\begin{aligned} E_{\delta_0}\{e(k)\} &= \int_{-\delta_0-\pi}^{\pi} \xi \cdot f_0(\xi)\cdot d\xi + \int_{\pi}^{-\delta_0+\pi} \xi \cdot f_0(\xi-2\pi)\cdot d\xi \\ &= \int_{-\pi}^{\pi} \xi \cdot f_0(\xi)\cdot d\xi + 2\pi \cdot \int_{-\pi}^{-\delta_0-\pi} f_0(u)\cdot du = 2\pi \cdot F_0(|\delta_0| - \pi) \end{aligned} \tag{20.}$$

2. The second case is given by: $0 < \delta_0 \le \pi$. Here the following sequence of operations is valid:

$$\begin{aligned} E_{\delta_0}\{e(k)\} &= \int_{-\delta_0-\pi}^{-\pi} \xi \cdot f_0(\xi+2\pi)\cdot d\xi + \int_{-\pi}^{-\delta_0+\pi} \xi \cdot f_0(\xi)\cdot d\xi \\ &= \int_{-\pi}^{\pi} \xi \cdot f_0(\xi)\cdot d\xi - 2\pi \cdot \int_{-\delta_0+\pi}^{\pi} f_0(u)\cdot du = -2\pi \cdot F_0(\delta_0 - \pi) \end{aligned} \tag{21.}$$

Since $f_0(\xi) = f_0(-\xi)$ is an even density function, symmetric around zero, the corresponding distribution $F_0(\xi)$ will show the following symmetry: $F_0(-\xi) = 1 - F_0(\xi)$. From inspecting equations 20 and 21, respectively, we conclude that the conditional mean of the bias error is an odd function with respect to the nominal value δ_0:

$$E\{e(k)/\delta_\varphi(k) = -\delta_0\} = -E\{e(k)/\delta_\varphi(k) = \delta_0\} \tag{22.}$$

From equations 20, 21 we conclude that knowing $F_0(\xi)$ is completely sufficient for determining the bias error. If we furthermore restrict us to the case of phase slopes between $\pm\pi$, we can utilize equation 16 and write:

$$\begin{aligned} F_0(\xi) &= \int_{-\pi}^{\xi} f_{\tilde{\delta}_e(k)}(u+2\pi)du + \int_{-\pi}^{\xi} f_{\tilde{\delta}_e(k)}(u)du + \int_{-\pi}^{\xi} f_{\tilde{\delta}_e(k)}(u-2\pi)du \\ &= F_{\tilde{\delta}_e(k)}(\xi+2\pi) + F_{\tilde{\delta}_e(k)}(\xi) + F_{\tilde{\delta}_e(k)}(\xi-2\pi) - 1 \end{aligned} \tag{23.}$$

where in the last equality we have only used the usual symmetry properties. For convenience we will furtheron restrict ourselves to the case of positive slopes so that we can substitute equation 23 into equation 21 and write:

$$\begin{aligned} E\{e(k)/\delta_\varphi(k) = \delta_0\} &= -2\pi \cdot F_0(\delta_0 - \pi) \\ &= -2\pi \cdot \left[F_{\tilde{\delta}_e(k)}(\delta_0+\pi) + F_{\tilde{\delta}_e(k)}(\delta_0-\pi) + \underbrace{F_{\tilde{\delta}_e(k)}(\delta_0-3\pi)}_{=0} - 1 \right] \\ &= -2\pi \cdot \left[F_{\tilde{\delta}_e(k)}(\delta_0+\pi) - F_{\tilde{\delta}_e(k)}(\pi-\delta_0) \right] = -2\pi \cdot \int_{\pi-\delta_0}^{\delta_0+\pi} f_{\tilde{\delta}_e(k)}(\xi)d\xi \end{aligned} \tag{24.}$$

The distribution density is periodic with respect to 4π. This means that we can expand it in a Fourier series:

$$f_{\tilde{\delta}_e(k)}(\xi) = \sum_{m=-\infty}^{\infty} d_m \exp\left[j\frac{m\cdot\xi}{2} \right] \tag{25.}$$

Substituting equation 25 into 24 we readily obtain:

$$\begin{aligned} E\{e(k)/\delta_\varphi(k) = \delta_0\} &= -2\pi \int_{\pi-\delta_0}^{\delta_0+\pi} \sum_{m=-\infty}^{\infty} d_m \exp\left[j\frac{m\cdot\xi}{2} \right] d\xi \\ &= -2\pi \sum_{m=-\infty}^{\infty} d_m \int_{\pi-\delta_0}^{\delta_0+\pi} \exp\left[j\frac{m\xi}{2} \right] d\xi = -4\pi\delta_0 \sum_{n=-\infty}^{\infty} d_m j^m si\left(\frac{m\delta_0}{2} \right) \end{aligned} \tag{26.}$$

where the Fourier coefficients are given by:

$$d_m = \frac{1}{4\pi} \cdot \int_{-2\pi}^{2\pi} f_{\tilde{\delta}_e(k)}(v) \cdot \exp\left[-j\frac{mv}{2} \right] dv \tag{27.}$$

Now we approximately calculate d_m by FFT-techniques by:

$$d_m \cong \frac{1}{N} \cdot \begin{cases} \Phi_{\tilde{\delta}_e(k)}(m) & 0 \le m < \frac{N}{2} \\ & \text{for} \\ \Phi_{\tilde{\delta}_e(k)}(N-m) & -\frac{N}{2} \le m \le -1 \end{cases} \tag{28.}$$
$$where\text{: } \Phi_{\tilde{\delta}_e(k)}(m) = \sum_{n=0}^{N-1} f_{\tilde{\delta}_e(k)}(n) \cdot \exp\left[-j2\pi\frac{nm}{N} \right]$$

$f_{\tilde{\delta}_e(k)}(n)$ is the sampled continuous density $f_{\tilde{\delta}_e(k)}(\xi)$ where:

$$f_{\tilde{\delta}_{e(k)}}(n) = f_{\tilde{\delta}_{e(k)}}(\xi_n = n \cdot \varphi_0)$$
$$where: \ \varphi_0 = \frac{4\pi}{N} \ \text{is the sampling interval} \qquad (29.)$$

The continuous density $f_{\tilde{\delta}_{e(k)}}(\xi)$ is, as indicated by equation 7, the continuous correlation. The discrete equivalent employing the sampled versions of the individual densities is given by:

$$f_{\tilde{\delta}_{e(k)}}(n) = \frac{4\pi}{N} \cdot f_{e_\varphi(k+1)}(-n) * f_{e_\varphi(k)}(n) \qquad (30.)$$

Realizing this discrete convolution as a cyclic convolution we carry over to FFT-techniques by writing:

$$f_{\tilde{\delta}_{e(k)}}(n) = \frac{4\pi}{N} \cdot \sum_{l=0}^{N-1} f_{e_\varphi(k+1)}(l) \cdot f_{e_\varphi(k)}(l+n) \qquad (31.)$$

The result of equation 31 can be easily obtained in the frequency domain by letting:

$$\Phi_{\tilde{\delta}_{e(k)}}(m) = \frac{4\pi}{N} \cdot \Phi_{e_\varphi(k+1)}(m)^* \cdot \Phi_{e_\varphi(k)}(m) \qquad (32.)$$

where the Fourier transforms are calculated by::

$$\Phi_{e_\varphi(k+i)}(m) = \sum_{n=0}^{N-1} f_{e_\varphi(k+i)}(n) \cdot \exp\left[-j2\pi \frac{nm}{N}\right] \qquad (33.)$$

Then the rule for approximately evaluating the bias is:

$$\boxed{\begin{array}{l}
E\left\{e(k) / \delta_\varphi(k) = \delta_0\right\} \cong -4\pi\delta_0 \cdot \sum_{m=-\frac{N}{2}}^{\frac{N}{2}} \tilde{d}_m \cdot j^m \cdot si\left(\frac{m \cdot \delta_0}{2}\right) \\
\tilde{d}_m = 4\pi \cdot \begin{cases} \tilde{\Phi}_{e_\varphi(k+1)}(m)^* \cdot \tilde{\Phi}_{e_\varphi(k)}(m) & \text{for } 0 \le m < \frac{N}{2} \\ \tilde{\Phi}_{e_\varphi(k+1)}(N-m)^* \cdot \tilde{\Phi}_{e_\varphi(k)}(N-m) & \text{for } -\frac{N}{2} \le m \le -1 \end{cases} \\
\tilde{\Phi}_{e_\varphi(k+i)}(m) = \frac{1}{N} \cdot \sum_{n=0}^{N-1} f_{e_\varphi(k+i)}(n) \cdot \exp\left[-j2\pi \frac{nm}{N}\right] \\
0 \le \delta_0 \le \pi
\end{array}} \qquad (34.)$$

Finally we obtain the solution for negative phase slopes by equation 22. Equations 34 and 22 provide the final result and form the basic framework for evaluating the bias error depending on the phase slope itself as well as on the form of the densities. These densities depend on the degree of coherence or on the SNR of the interferogram (the quality of the fringes). In the following we will give some quantitative results. We will assume identical distribution densities for two successive points. Figure 2 shows the outcoming bias over the true phase slope evaluated for different degrees of coherence. It is clearly visible that the maximum allowed phase slope that may be estimated with negligible bias strongly depends on the coherence. If the coherence is one, there is no phase slope bias as long as the phase slope is less than π. The other extreme is a coherence of 0.1. Here the slope bias is considerable even for small lopes.

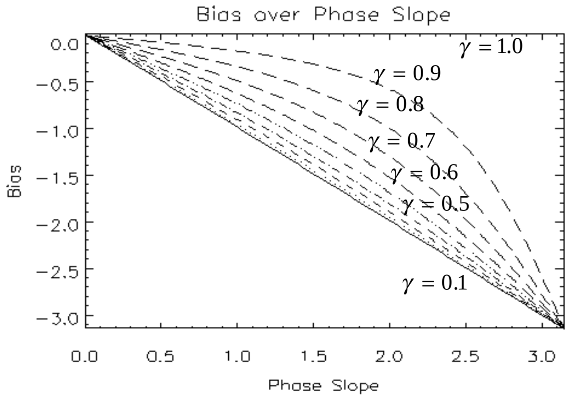

Figure 2: Bias Error over Phase Slope

3.0 CONCLUSIONS: HOW TO SOLVE THE PROBLEM?

Do not apply any filtering to the phase slope!

Any filter operation will produce an estimate which is ‘nearer’ to the conditional mean, which is not identical with the true phase slope. If any filtering is to be applied it should be applied to complex data rather than to the phases or phase slopes.

Correct the bias by subtracting it!

With the results given in equation 34 it should be possible to eliminate the bias by simply subtracting it. The bias estimation of equation 34 is a keypoint to maintain Linear Least Squares phase unwrapping approaches.

Use unbiased estimators!

Clearly the best solution to a problem is an approach which prevents the problem from arising. This can be achieved by applying unbiased phase slope estimators. All these estimators share the common property that they operate on complex data rather than on the phases. A very reasonable approach would be to use an Extended Kalman Filter which does not explicitly differentiate any mapped phases [2,3]. Recently a combination of local slope estimation and Kalman filtering techniques has been proposed [4,5]. This combination yields unbiased and nearly perfectly noisefree unwrapped phases down to coherence values of 0.3 without any prefiltering!

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