A RECONSTRUCTION METHOD FOR HELICAL COMPUTED TOMOGRAPHY

Ariel Rischal, Susan S. Young † *and Mehrdad Soumekh*
Department of ECE, SUNY-Buffalo, Amherst, New York 14260
† Health Imaging Research Laboratory, Eastman Kodak Company
901 Elmgrove Road, Rochester, NY 14653-5740

ABSTRACT

This paper presents a method for volumetric reconstruction from Helical Computerized Tomography (H-CT) data which are collected with a fan beam source. An interpretation of the H-CT data in terms of the Axial Computerized Tomography (A-CT) data is provided. This analysis indicates that the H-CT data for positive and negative detector angles can be combined to form periodically nonuniform hexagonal samples of the A-CT data. A Fourier-based method to reconstruct the A-CT data from this form of data coverage is presented. The target function is then reconstructed using the conventional fan beam computed tomography algorithms for A-CT.

I. INTRODUCTION

This paper is concerned with three-dimensional imaging from the data which are obtained from helical scanning of a target region with a fan beam source. The early three-dimensional or volumetric computed imaging systems were based on step-and-shoot data acquisition procedure. In this scheme, the three-dimensional target function $f(x, y, z)$ was formed by acquiring the fan beam data on a fixed plane, e.g., $z = z_0$; the resultant database was used in a fan beam computed tomography algorithm to form the cross-sectional image $f(x, y, z_0)$. Next, the target was moved in the z (axial) domain by an increment Δ_z. The fan beam data collection and cross-sectional imaging was repeated to form the image $f(x, y, z_0 + \Delta_z)$. This axial step of the target and fan beam illumination was repeated at various discrete axial z values to obtain the desired volumetric image. This imaging scheme is also known as Axial Computed Tomography (A-CT). One of the drawbacks of A-CT, which utilizes discrete target axial motion, is the time to acquire its database which is in excess of 2-3 minutes for a typical diagnostic medical imaging problem. To circumvent this problem, a data acquisition procedure has been suggested in which the target is moved with a constant speed in the axial domain [1-2]; the database for this system is acquired in less than 30 sec for a typical imaging problem. This volumetric image formation method is called Helical Computed Tomography (H-CT); this is due to the fact that the source traces a helical locus on a cylinder with respect to the target.

The existing reconstruction methods from the fan beam H-CT data are based on linear extrapolation/interpolation techniques which combine one or two adjacent helical pitch data of the fan beam source [2]. For this purpose, some form of weighted average of the data from the positive and negative detectors is introduced. A reconstruction method based on viewing the parallel beam H-CT data in terms of hexagonally sampled data has been suggested [1] which combines the data in all helical pitches. However, there is no multidimensional sampling interpretation of and reconstruction for the positive and negative detector samples of the fan beam H-CT data.

In this paper, a method for reconstructing a three-dimensional target function from its fan beam H-CT data is presented. A description of the A-CT and H-CT systems with fan beam sources is provided in Section II. A method for imaging from the H-CT data is discussed in Section III. We first show that the H-CT data for positive and negative detector angles can be interpreted as periodically nonuniform samples of the A-CT data in the axial direction. We then utilize the theory of Fourier interpolation from periodically nonuniform sampled data to reconstruct the A-CT data of the target from its H-CT data. The target function can then be imaged via the conventional fan beam reconstruction algorithms at a fixed axial location. The proposed algorithm could be optimal if the target function is bandlimited in the axial domain. We provide reconstruction results for a simulated three-dimensional head phantom which is composed of ellipsoids. This target function is not bandlimited in the axial domain. In spite of this fact, the Fourier-based interpolation is shown to be more accurate than the linear interpolation-based method. Note that in practice the *finite* size of the aperture of the source/detector yields a *bandlimited* transmit/receive-mode beam pattern in the axial domain [3, pp. 61-65],[4, Chapter 3]. In this case, the *effective* target function, as seen by the imaging system, is bandlimited in the axial domain.

II. VOLUMETRIC COMPUTED IMAGING

A. Axial Computed Tomography

Consider imaging system geometry in A-CT at a fixed axial value z [4, Section 7.4]. The target on the (x, y) plane is irradiated with a fan beam source (transmitting element) and a set of detectors (receiving elements) record the resultant scattering. The source and the detectors are located on a circle of radius R (gantry) which is centered around the target region. The gantry (source and detectors) is rotated continuously with a constant angular speed. We identify the time-dependent angular direction of the source as the gantry moves by θ. For a fixed source angle θ, we denote the detectors' angle domain with respect to the source by α. In medical imaging with X-ray sources, the source is noncoherent and the measured data contain information on the line integrals of the attenuation coefficient distribution in the target region (e.g., see [4]). In the case of coherent sources, the measured data also provide information on the line integrals of the propagation speed (index of refraction) distribution in the target [4]. In either case, the line integrals of a target function (attenuation coefficient and perhaps index of refraction) are measured at discrete values of the source angle θ and the detector angle α. In A-CT, this procedure is repeated at a set of discrete values of z.

The resultant database are the samples of the fan beam A-CT signal which we denote with $s_A(\theta, \alpha, z)$. We identify the sample spacing in the three-dimensional measurement domain of the fan beam A-CT signal via $(\Delta_\theta, \Delta_\alpha, \Delta_z)$. Thus, the measured data corresponds to the samples $s_A(m\Delta_\theta, n\Delta_\alpha, k\Delta_z)$ for some integer values of (m, n, k). Note that for a fixed detector angle $n\Delta_\alpha$, the sampling in the (θ, z) domain is rectilinear. Suppose the target is within a disk of radius X_0; clearly, $X_0 < R$ which is the radius of the gantry. Moreover, the desired spatial resolution in the (x, y) domain is $\Delta_0 \equiv \Delta_x = \Delta_y$. To avoid aliasing (ghosts) in the reconstructed cross-sectional images of A-CT, the following constraints should be satisfied for the source and detector sample spacing $\Delta_\theta \leq \frac{\Delta_0}{X_0}$ and $\Delta_\alpha \leq \frac{\Delta_0}{R}$. Note that the angular sample spacing in the detector α domain is more restrictive than that of the source θ domain since $X_0 < R$. The axial resolution is determined by the axial sample spacing (step) Δ_z which can be chosen to be as small as the axial thickness of the collimated X-ray beam. The domain of the source angle is $\theta \in [0, 2\pi)$, and the detector domain is $\alpha \in [-\alpha_0, \alpha_0]$ where $\alpha_0 \equiv \arcsin(\frac{X_0}{R})$. The size of the measurement domain is (M, N, K), where $M \equiv \frac{2\pi}{\Delta_\theta}$, $N \equiv \frac{2\alpha_0}{\Delta_\alpha} + 1$, $K \equiv \frac{2Z_0}{\Delta_z}$, and $2Z_0$ is the axial size of the target area of interest. There is some redundancy in the above-mentioned fan beam database. One can show that the following relationship exists between the fan beam data for the positive and negative detector angles:

$$s_A(\theta, -\alpha, z) = s_A(\theta + 2\alpha - \pi, \alpha, z). \tag{1}$$

For the discrete data, this redundancy becomes

$$s_A(m\Delta_\theta, -n\Delta_\alpha, \Delta_z) = s_A(m_n\Delta_\theta, n\Delta_\alpha, \Delta_z) \tag{2}$$

(note that $\frac{M}{2}\Delta_\theta = \pi$) where

$$m_n\Delta_\theta \equiv m\Delta_\theta + 2n\Delta_\alpha - \frac{M}{2}\Delta_\theta \tag{3}$$

(Note that m_n is an integer if $2\Delta_\alpha$ is an integer multiple of Δ_θ.) This redundancy has been utilized for imaging with H-CT data [2]. We will also utilize this redundancy to interpret H-CT data for positive and negative detector angles in terms of periodically nonuniform samples of A-CT data in the axial direction.

B. Helical Computed Tomography

In H-CT with continuous axial target motion and angular gantry motion, the helical (unwrapped) source angle, call it Θ, is related to the target axial location via $\Theta \equiv Cz$, where C is a constant. In this case, the continuous wrapped source angle domain is related to the axial domain via $\theta = \mathrm{R}[\frac{\Theta}{2\pi}] = \mathrm{R}[\frac{Cz}{2\pi}]$ where $\mathrm{R}[\frac{A}{B}]$ denotes the remainder of dividing A by B. We denote the continuous H-CT measurement signal via $s_H(\alpha, z)$. This signal is related to the A-CT measurement signal via

$$s_H(\alpha, z) = s_A(\theta, \alpha, z), \tag{4}$$

where $\theta = \mathrm{R}[\frac{Cz}{2\pi}]$. We denote the distance that the target moves in the axial z domain for one revolution (2π radians) of the fan beam source (or gantry) by $\Delta_z \equiv \frac{2\pi}{C}$. In this case, the H-CT signal can be rewritten via

$$s_H(\alpha, z) = s_A(\theta, \alpha, Z_\ell + \frac{\theta}{C}), \tag{5}$$

where $Z_\ell \equiv \ell\Delta_z$, with $\ell \equiv \mathrm{Q}[\frac{Cz}{2\pi}]$, and $\mathrm{Q}[\frac{A}{B}]$ denotes the quotient of dividing A by B. Note that Z_ℓ takes on discrete values which are integer multiples of Δ_z. Next, we consider the measured discrete H-CT data. Let Δ_θ be the angular sample spacing in the helical source Θ domain, and Δ_α be the sample spacing in the detector α domain. Thus, the sample spacing in the axial z domain is $\delta_z \equiv \frac{\Delta_\theta}{C} = \frac{\Delta_z}{M}$, where M is the number of helical source angles per revolution (2π radians) of the fan beam source. The measured discrete H-CT data in the (α, z) domain (i.e., sampled data of $s_H(\alpha, z)$) translate into following discrete A-CT data:

$$\begin{aligned} s_H(n\Delta_\alpha, k\delta_z) &= s_A(m\Delta_\theta, n\Delta_\alpha, \ell\Delta_z + \frac{m\Delta_\theta}{C}) \\ &= s_A(m\Delta_\theta, n\Delta_\alpha, \ell\Delta_z + m\delta_z) \end{aligned} \tag{6}$$

where $m\Delta_\theta = \mathrm{R}[\frac{Ck\delta_z}{2\pi}]$, or $m = \mathrm{R}[\frac{k}{M}]$, and $\ell = \mathrm{Q}[\frac{Ck\delta_z}{2\pi}]$. For a fixed detector location $n\Delta_\alpha$, this corresponds to a hexagonal sampling in the (θ, z) domain of the A-CT signal $s_A(\theta, n\Delta_\alpha, z)$. An example of this form of data coverage for a fixed detector location is shown in Figure 1 (filled circles).

Consider the redundancy of fan beam data which was identified in (1). For H-CT data, this redundancy yields

$$\begin{aligned} s_H(-n\Delta_\alpha, k\delta_z) &= s_A(m\Delta_\theta, -n\Delta_\alpha, \ell\Delta_z + m\delta_z) \\ &= s_A(m_n\Delta_\theta, n\Delta_\alpha, \ell\Delta_z + m\delta_z) \end{aligned} \tag{7}$$

where $m_n\Delta_\theta \equiv \mathrm{R}[\frac{\Theta_n}{2\pi}]$, and $\Theta_n \equiv m\Delta_\theta + 2n\Delta_\alpha - \frac{M}{2}\Delta_\theta$. An example of the samples which are provided by the right side of (7) for a fixed detector angle $n\Delta_\alpha$ is provided in Figure 1 (unfilled circles).

III. HELICAL CT RECONSTRUCTION

A common procedure for volumetric imaging from H-CT data is to convert this database to A-CT data of the three-dimensional target region. As we mentioned earlier, for a fixed detector location $n\Delta_\alpha$, the H-CT data corresponds to a hexagonal sampling in the (θ, z) domain; i.e.,

$$s_H(n\Delta_\alpha, k\delta_z) = s_A(m\Delta_\theta, n\Delta_\alpha, \ell\Delta_z + m\delta_z). \tag{8}$$

The hexagonal sampled data in the (θ, z) domain can be converted to rectangular sampling in the (θ, z) domain via shifting the data in the axial domain by $-m\delta_z$ for a given source angle $m\Delta_\theta$. Using the shifting property of Fourier transform, this can be achieved via

$$s_A(m\Delta_\theta, n\Delta_\alpha, \ell\Delta_z) = \mathcal{F}^{-1}_{(\imath\Delta_{k_z})}\Big[\ \mathcal{F}_{(\ell\Delta_z)}\big[s_H(n\Delta_\alpha, k\delta_z)\big] \exp(-jm\delta_z\imath\Delta_{k_z})\ \Big] \tag{9}$$

A similar procedure was suggested in [1] (sinc interpolation to shift the data in the axial domain) to achieve the H-CT to A-CT data conversion in (9) for volumetric helical imaging with parallel beam sources. Ref. [1] utilizes Yen's interpolation, which was originally introduced for "unevenly-spaced" data by Yen, for interpolation with "evenly-spaced" hexagonal data of H-CT. Yen's interpolation, which is based on a matrix inversion, is unnecessary in the H-CT problem since the inverse of the matrix for "evenly-spaced" data is well-known: it is the discrete sinc function. The method is optimal if the target function $f(x,y,z)$ is bandlimited in the axial domain within the band $|k_z| < \frac{\pi}{\Delta_z}$. The finest axial resolution which can be achieved in these imaging systems is the axial thickness of the collimated (pencil) X-ray beam. Due to system limitations, most volumetric CT systems possess a Δ_z value which is larger than the thickness of the collimated X-ray beam. Thus, the hexagonal H-CT data are aliased in the axial direction. This results in blurring of the targets which are smaller than Δ_z in the axial direction.

To reduce the axial aliasing errors, the use of redundant fan beam data has been suggested [2]. For this purpose, extrapolative/interpolative methods are introduced which utilize positive-negative detector H-CT data (see Figure 1) in either one or two revolutions of the source. However, one can view the H-CT database in Figure 1 as periodically nonuniform hexagonal sampled data. There are interpolation methods for reconstructing a one-dimensional signal from its periodically nonuniform sampled data. One can also develop a two-dimensional interpolation method for the H-CT data in Figure 1. However, by a proper selection of the imaging system parameters or digital preprocessing of the H-CT data, the interpolation problem can be converted to a one-dimensional one. This is described next.

In Figure 1, the value of $2\Delta_\alpha$ (twice the detector sample spacing) is not an integer multiple of Δ_θ (the source sample spacing). For the processing which we will use, we require $2\Delta_\alpha$ to be an integer multiple of Δ_θ which results in the H-CT samples of Figure 2; this will be shown. This can be achieved in the manner the CT system's hardware is set up. If that is not feasible, the user can perform sampling rate conversion in the α domain in the software; this is not discussed here. Consider the H-CT database in which $2\Delta_\alpha$ is an integer multiple of Δ_θ; thus, we have $2\Delta_\alpha = m_0\Delta_\theta$, where m_0 is an integer. Then, the redundant H-CT data in (7) can be rewritten via

$$s_H(-n\Delta_\alpha, k\delta_z) = s_A(m_n\Delta_\theta, n\Delta_\alpha, \ell\Delta_z + m\delta_z) \tag{10}$$

where $m_n\Delta_\theta \equiv \mathrm{R}[\frac{\Theta_n}{2\pi}]$, and $\Theta_n \equiv (m + m_0 n - \frac{M}{2})\ \Delta_\theta$.

The database in (10) may also be viewed in the following way using the inverse of m_n transformation in the axial k index domain. For a given source angle $m\Delta_\theta$ and detector angle $n\Delta_\alpha$, A-CT data are not only available at the axial value

$$k\delta_z = \ell\Delta_z + m\delta_z, \tag{11}$$

but also at the axial value

$$k_{mn}\delta_z \equiv \ell\Delta_z + (m - m_0 n + \frac{M}{2})\ \delta_z \tag{12}$$

An example of the resultant samples are shown in Figure 2. We refer to $m\delta_z$ in (11) and $(m - m_0 n + \frac{M}{2})\ \delta_z$ in (12) as the offset axial locations of, respectively, positive and negative detector samples from $\ell\Delta_z$. Note that these offset axial values are invariant of the axial index ℓ; however, they depend on the detector index n. Next, we examine the problem of reconstruction from the H-CT data in Figure 2. For a fixed source angle $m\Delta_\theta$ and detector angle $n\Delta_\alpha$, the H-CT data of Figure 2 correspond to periodically nonuniform data in the axial domain. The following steps are used to interpolate from this database. We first obtain the discrete Fourier transforms of positive and negative detector H-CT data with respect to $\ell\Delta_z$ for a fixed source angle $m\Delta_\theta$, and multiply them with a phase function which depends on their corresponding offset axial locations from $\ell\Delta_z$'s (see (11) and (12)):

$$S_1(m\Delta_\theta, n\Delta_\alpha, \imath\Delta_{k_z}) \equiv \mathcal{F}_{(\ell\Delta_z)}\big[\ s_H(n\Delta_\alpha, k\delta_z)\ \big] \exp(-jm\delta_z\imath\Delta_{k_z})$$
$$S_2(m\Delta_\theta, n\Delta_\alpha, \imath\Delta_{k_z}) \equiv \mathcal{F}_{(\ell\Delta_z)}\big[\ s_H(-n\Delta_\alpha, k_{mn}\delta_z)\ \big] \exp[-j(m - m_0 n + \frac{M}{2})\delta_z\imath\Delta_{k_z}] \tag{13}$$

In the digital implementation of the method, the discrete Fourier transforms in (13) are performed on an array whose length is twice the size of the array which represents s_H. The odd samples of this new array are the samples of s_H; the even samples of the new array are set to zero. We call the resultant the *augmented* s_H array. The axial sample spacing of the augmented array is $\frac{\Delta_z}{2}$. Thus, discrete Fourier transform yields the samples of S_1 and S_2 within the band $|k_z| \le \frac{2\pi}{\Delta_z}$ which is twice the size of the bandwidth of the original s_H array.

We define the following function:

$$W_n \equiv \exp[j(\frac{M}{2} - m_0 n)\frac{\pi\delta_z}{\Delta_z}] \tag{14}$$

We also define

$$E_n(\imath\Delta_{k_z}) \equiv \begin{cases} W_n, & \text{for } \imath\Delta_{k_z} \ge 0; \\ W_n^*, & \text{otherwise} \end{cases} \tag{15}$$

where W_n^* is the complex conjugate of W_n. Next, we form

$$S(m\Delta_\theta, n\Delta_\alpha, \imath\Delta_{k_z}) \equiv \big[S_1(m\Delta_\theta, n\Delta_\alpha, \imath\Delta_{k_z}) - E_n(\imath\Delta_{k_z})\ S_2(m\Delta_\theta, n\Delta_\alpha, \imath\Delta_{k_z})\big]\ \big[1 - E_n(\imath\Delta_{k_z})\big]^{-1} \tag{16}$$

Provided that the target function is bandlimited within $|k_z| \le \frac{2\pi}{\Delta_z}$ in the axial domain, then one can show that (16) yields the unaliased samples of A-CT data (within a known constant); i.e.,

$$S(m\Delta_\theta, n\Delta_\alpha, \imath\Delta_{k_z}) = S_A(m\Delta_\theta, n\Delta_\alpha, \imath\Delta_{k_z}). \qquad (17)$$

In the image reconstructed from the data in (16), the targets which are smaller than $\frac{\Delta_z}{2}$ in the axial domain appear blurred (aliased). The above procedure suffers from the wrap around ringing (boundary) errors of DFT (sampled data truncation). To reduce the wrap around errors, we use a version of discrete cosine/sine transforms which provide continuity at the boundaries. (The implementation is not described here.)

References

[1] Y. Bresler and C. Skrabacz, "Optimal interpolation in helical scan computed tomography," *Proc. ICASSP-89*, vol. 3, pp. 1472-1475.

[2] C. Crawford and K. King, "Computed tomography scanning with simultaneous patient translation," *Med. Phys.*, vol. 17, pp. 967-982, 1990.

[3] J. Goodman, *Introduction to Fourier Optics*, McGraw-Hill, 1968.

[4] M. Soumekh, *Fourier Array Imaging,* Englewood Cliffs, NJ: Prentice Hall, 1994.