JOINT SOURCE-CHANNEL SUBBAND CODING OF IMAGES

Murari Srinivasan and Rama Chellappa

Center for Automation Research and
Dept. of Electrical Engineering
University of Maryland at College Park
College Park, MD 20740

ABSTRACT

This paper proposes a new adaptive source-channel coding scheme in the context of subband coding. We first express the total mean-squared distortion suffered by the source in terms of source and channel distortions of the subbands. We then minimize this total distortion by an appropriate choice of source and channel coding rates for the subbands. This corresponds to casting the conventional bit allocation problem in a joint source-channel coding context. The choice of rates depends on the state of the physical channel, modeled by a binary symmetric channel (BSC). We then use a finite state Markov model for a fading channel to generalize results obtained for the BSC. This results in a joint source-channel coding scheme that is optimized to the current state of a fading channel.

1. INTRODUCTION

Shannon's separation principle establishes the optimality of separate design of source and channel coders, and states that total distortion is essentially limited to the source coding distortion as long as the rate of the source coder is less than channel capacity. This however, is an asymptotic result, and real-world systems benefit through joint design of the source and channel coders, given knowledge of the channel. The aim of a joint source-channel coding approach is to optimally allocate bits between the source and channel coders to minimize total distortion, while satisfying a constraint on the total rate.

Several approaches for joint source-channel coding have been proposed in the literature. Modestino et. al illustrated the advantages of source-channel coding using the DCT in [1] extending their earlier work using DPCM coding. Joint source-channel coding of synthetic sources using vector quantization combined with Rate Compatible Punctured Convolutional codes is addressed in [2]. A joint source-channel decoding approach in the form of MAP decoding for exploiting residual redundancy in images transmitted over channels with memory has been recently addressed in [3]. More recently, we have become aware of an approach towards joint source-channel subband coding of scalable video [4] that is similar in spirit to the work presented in this paper. Ruf and Modestino [5] use bit-sensitivity analysis to compute operational rate-distortion curves and also provide information-theoretic bounds on performance.

The problem of rate allocation i.e, quantizing subbands at different rates in order to minimize overall source coding distortion, given a constraint on the source rate has been studied extensively [7, 8]. When a subband coded source is transmitted over a channel, it is also logical to protect the subbands differently against channel errors since they have varying importance in the reconstruction of the signal at the receiver. One way to accomplish this unequal error protection (UEP) is by allocating different channel coding rates to the subbands. In this formulation presented in section 2, each of the subbands is coded and protected independently, constrained however, by an overall bit-rate. We illustrate the optimal allocation of source and channel coding rates for the different subbands that minimizes the overall distortion.

In section 3,we propose a method that chooses these rates to minimize the overall distortion, while obeying an overall rate constraint when the channel is a BSC. We show that this problem is analogous to the classical bit allocation problem, applied now on the total rate-distortion (both source and channel distortions) curves. The allocation naturally depends on the bit error rate (BER) of the channel, whose knowledge is assumed to be known at the transmitter. A wireless channel is characterized by temporal variations that manifest themselves in the form of fading. This is due to multiple transmission paths arriving at the re-

ceiver, with random phase differences. A fading channel can be modeled using a finite-state Markov model [10] where each state is modeled by a BSC with a different BER. In Section 4, we extend our method to a general fading channel by choosing an optimal rate allocation for each state of the channel. Advantages of this adaptive scheme are demonstrated through simulation experiments in Section 5.

2. PROBLEM FORMULATION

The problem of jointly choosing the source and channel coding rates for a source has been addressed from various perspectives. We are motivated by [2] where the authors propose an optimal source-channel coding scheme for a vector quantized source protected using convolutional codes and transmitted over a fading channel. In this paper, we solve the rate allocation problem in a subband coding framework.

Consider a source that is decomposed into M subbands. When the subband filters are orthogonal, the total distortion in the source can be expressed as the sum of the distortions in the individual subbands [7]. (FIR linear-phase filters also satisfy this property approximately.)

$$D = \sum_{i=1}^{M} D_i \qquad (1)$$

Each subband suffers distortion due to quantization at the source coder, and due to errors introduced by the channel in the transmitted codewords. Therefore, the total distortion is a function of the source coding rate as well as the channel coding rate (degree of protection).

$$D_i = D_i(R_{s,i}, R_{c,i}) \qquad (2)$$

$R_{s,i}$ and $R_{c,i}$ denote the source and channel coding rates for the ith subband respectively. Under the conditions that the source codewords satisfy the centroid condition and that the channel errors are independent of the source codewords, the distortion may be decomposed as [2, 6]

$$D_i(R_{s,i}, R_{c,i}) = D_{s,i}(R_{s,i}) + D_{c,i}(R_{c,i}) \qquad (3)$$

where $D_{s,i}(R_{s,i})$ and $D_{c,i}(R_{c,i})$ are distortions in the ith subband due to the source coder and channel, respectively. Therefore, the total distortion suffered by the source under these assumptions is

$$D = \sum_{i=1}^{M} D_{s,i}(R_{s,i}) + \sum_{i=1}^{M} D_{c,i}(R_{c,i}) \qquad (4)$$

The objective is to choose $\{(R_{s,i}, R_{c,i}), i = 1, ..., M\}$ such that

$$D \text{ is minimized, and } \frac{1}{M} \sum_{i=1}^{M} \frac{R_{s,i}}{R_{c,i}} \leq R \qquad (5)$$

.

3. SOLUTION FOR A BSC

Consider the ith subband. The rate-distortion curve for this subband may be computed if the subband is modeled appropriately. In practice, an operational rate-distortion curve may be obtained which links the source coding rate and the resulting distortion. The operational R-D curve differs from the theoretical R-D curve for the source because practical quantizers operate on short lengths of data. Using this curve, we can compute $D_{s,i}(R_{s,i})$. In order to compute the distortion introduced by the channel in the ith subband, we have to evaluate

$$
\begin{aligned}
D_{c,i}(R_{c,i}) &= \sum_u \sum_v p(u)p(v/u)D_i(u,v) \\
& u, j \in \{1, 2, ., \mathcal{C}\} \qquad (6)
\end{aligned}
$$

where $\mathcal{C}$ is the cardinality of the codebook and $D_i(u,v)$ is the per sample distortion between the source samples corresponding to code vectors with indices u and v. The probability $p(u)$ may be computed from the source statistics. The transition probability $p(v/u)$ may be computed as

$$p(v/u) = \epsilon^{W_H(u,v)}(1 - \epsilon^{n - W_H(u,v)}) \qquad (7)$$

where $W_H(u,v)$ is the Hamming distance between the codewords corresponding to indices u and v. The parameter ϵ is a function of the channel BER (ϵ_{ch}) as well as the coding rate $r_{c,i}$. It may be viewed as the BER of the equivalent channel that comprises the channel encoder, the physical channel and the channel decoder since the bit errors after channel decoding are essentially independent for block codes. The operational channel rate-distortion curve, $D_{c,i}(R_{c,i})$, is computed using (6) and (7).

Using $D_{s,i}(R_{s,i})$ and $D_{c,i}(R_{c,i})$ along with (3), we can derive an (operational) rate-distortion curve for the ith subband that takes both source and channel distortions into consideration. We denote this curve by $D_i(R_i)$. This curve determines the best allocation for source and channel coding rates for the ith subband, under the constraint that the total rate for the ith subband is R_i.

In order to optimally allocate the source and channel coding rates for the subbands, we consider the ensemble of composite source/channel (operational) rate-distortion curves of all the subbands, and choose an operating point $\{R_1, R_2, ..., R_M\}$ that results in minimum $\sum_{i=1}^{M} D_i$. This problem of choosing the optimal operating point from the ensemble of rate-distortion curves is identical to the classical problem of bit allocation [?]. This optimal solution has to be derived from the operational rate-distortion curves, which may not necessarily be convex or even monotone decreasing. The algorithm presented in [9] may be used under these conditions. The simpler, greedy algorithm described in [8] may also be used, although the allocation is, in general, suboptimal. The optimal solution naturally depends on the BER of the physical channel.

4. EXTENSION TO A FADING CHANNEL

A fading wireless channel may be modeled by a finite-state Markov model [10]. In this model, each state corresponds to a BSC with a certain BER. The channel makes transitions between the states based on an underlying Markov model. This is done by partitioning the range of received signal-to-noise ratios at the receiver into a finite number of states, and computing the state and transition probabilities. Since we have an optimal solution for a BSC whose BER is known, we can obtain a table of optimal solution with each solution corresponding to a different state of the physical channel. An estimate of the channel state is obtained by the transmitter through feedback from the receiver. The transmitter then chooses the optimal operating point for that state $\{(R_{s,i}, R_{c,i}), i = 1, 2, .., M\}$.

We make a change in notation for the channel-induced distortions in order to make the dependence on the channel explicit. We use $D_{c,i}(R_{c,i}, \epsilon_k)$ to denote channel distortion when the channel is in the kth state (ϵ_k denotes the BER in this state.) Therefore, the total distortion in (1) is now denoted by

$$D(\epsilon_k) = \sum_{i=1}^{M} D_i(R_i, \epsilon_k) \qquad (8)$$

The average total distortion may then be expressed as

$$D = \sum_{k=1}^{L} D(\epsilon_k) p(k) \qquad (9)$$

where $p(k)$ denotes the probability of the kth state.

5. SIMULATION RESULTS

The optimal rate allocation scheme is applied to the transmission of images over noisy channels. The image is first decomposed into several subbands. A vector quantizer (of dimension 4) is used to code the coefficients in the subbands. This is followed by a Reed-Solomon (block) channel coder. The modulation scheme used is BPSK. We chose a channel model that has three states with corresponding BERs 0.0, 0.01 and 0.05. The state with BER = 0.0 corresponds to a clean channel.The probabilities of these states are 0.8, 0.15 and 0.05 respectively.

The images used are frames from the "salesman" sequence. The training for the VQs and the computation of the operational RD curves is accomplished using subband coefficients from several frames of the sequence. We use intraband VQ since the notion of rate allocation among the subbands is natural when coefficients belonging to the same subband are grouped together to form the source vectors. This simulation differs from conventional approaches in that the lowest frequency subband is also encoded using VQ without any DPCM coding. Most coding techniques use DPCM or transform coding strategies on the coefficients of this subband, exploiting the high correlation between the coefficients. However, we do not do this in order to retain a uniform analytical approach for the computation of the channel distortion where a transmitted source codeword index may be corrupted due to the channel.

An image is coded as follows. Given the target rate and the state of the channel, bit allocation is performed on the composite RD curves specific to this state of the channel using the integer programming approach outlined in [9]. Once the total rate for each of the subbands is known, it may be decomposed into a source coding rate and a channel coding rate. The subbands are then quantized and channel coding is accomplished using Reed-Solomon codes of the appropriate rates. After Reed-Solomon decoding, the received codeword indices are mapped into the corresponding centroids, and the image reconstructed from the received subbands. The distortion between the original and received images is computed. The results for this experiment are outlined in Table 1 for the two noisy states. The original image as well as images transmitted over a channel with BER = 0.01 and a channel with BER = 0.05 are shown in figures 1 through 3 respectively.

6. CONCLUSIONS

The paper describes an optimal source-channel coding scheme for a subband coder. This is a new approach to jointly selecting source and channel coding rates optimized to the current channel conditions. The scheme outlined is general enough to apply to any class of source and channel coders. The simulation results demonstrate the advantage of an adaptive coding strategy that is optimal given the knowledge of the channel state.

7. REFERENCES

[1] J.W.Modestino, D.G.Daut and A.L.Vickers, "Combined Source-Channel Coding of Images Using the Block Cosine Transform," IEEE Transactions on Communications, Vol.29, pp.1261-1274, Sep. 1981,.

[2] H.Jafarkhani, P.Ligdas and N.Farvardin, "Adaptive Rate Allocation in a Joint Source/Channel Coding Framework for Wireless Channels," Proc. of the IEEE Vehicular Technology Conference, Atlanta, GA, pp.492-496, April 1996.

[3] F.Alajaji, P.Burlina and R.Chellappa, "MAP decoding of Gray-Level Images over Binary Channels with Memory", in Proc. Int. Conf. Image Proc.,Lausanne, Switzerland, Sep. 1996.

[4] G.Cheung and A.Zakhor, "Joint Source/Channel Coding of Scalable Video Over Noisy Channels," in Proc. Int. Conf. Image Proc.,Lausanne, Switzerland, vol. III, pp. 767-770, Sep. 1996.

[5] M.Ruf and J.W.Modestino, "Rate-distortion Performance for Joint Source and Channel Coding of Images," Technical Report LIDS-P-2344, LIDS, Massachussetts Institute of Technology, June 1996.

[6] N.Farvardin, "A study of Vector Quantization for Noisy Channels," IEEE Trans. on Info. Theory, Vol.36, pp.799-809, July 1990.

[7] M.Vetterli and J. Kovacevic, "Wavelets and Subband Coding," Prentice Hall, 1995.

[8] A.Gersho and R.Gray, "Vector Quantization and Signal Compression", Kluwer Academic Publishers, 1992.

[9] Y.Shoham and A.Gersho, "Efficient Bit Allocation for an Arbitrary Set of Quantizers", IEEE Trans. on Acoust. Speech and Sig. Proc., Vol.36, pp.1445-1453, Sep.1988.

[10] H.S.Wang and N.Moayeri, "Finite-State Markov Channel - A Useful Model for Radio Communication Channels," IEEE Trans. on Vehicular Tech., Vol.44, pp.163-171, February 1995.

Table 1 : Simulation results for images

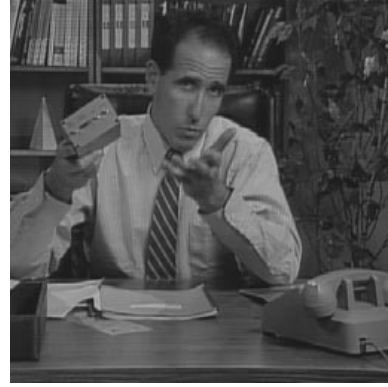| (BER)	0.01	0.05
Rate (bpp)	2.50	3.00
Dstn (dB PSNR)	30.12	27.06

Figure 1: Original Image

Figure 2: Channel BER = 0.01, R_{tot} = 2.5 bpp

Figure 3: Channel BER = 0.05, R_{tot} = 3.0 bpp