PREDICTION BASED ON BOOLEAN, FIR–BOOLEAN HYBRID AND STACK FILTERS FOR LOSSLESS IMAGE CODING

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ABSTRACT

This paper proposes the use of MAE–optimal Boolean and stack filters for sequential prediction in lossless grey-level image coding. FIR–Boolean hybrid filters are introduced as variations of Boolean filter structure and shown to be very effective for the prediction task. Different instances of optimal filtering are considered for realizing the prediction stage. First, the use of global–optimal predictors is analyzed, when the global MAE–optimal filter is used as a predictor. Then more refined structures, block–optimal and adaptive–size–block–optimal are considered, where predictors are adapted to local characteristics. These structures prove most suitable when small prediction masks are used. Extensive simulations are carried out for analyzing and comparing the performance of the newly introduced predictors and various other sequential predictors.

1. INTRODUCTION

Boolean and stack filters excel in solving image processing tasks, when the available images are corrupted by impulsive noise. These applications exploit the ability of Boolean filtering structures to cope simultaneously with noise attenuation and detail preservation.

In this paper, we introduce a different class of applications for Boolean filters, namely prediction, when the available data may be perfectly clean. Other attractive features of this class of filters are thus exploited here: (a) the procedure for the optimal design is much simpler than the procedure for linear filter design (at least for small window sizes); (b) the code-length necessary for transmitting the parameters of the filter is smaller for Boolean and stack filters than for linear filters. Due to these properties the predictors can be very well fitted to the images to be transmitted, such as to reduce significantly the entropy of the residuals. Meanwhile, the cost incurred by the transmission of the predictor information is not high, and globally, a lower bitrate than in the fixed predictor case is obtained.

Slight variations of the Boolean filter structure are also considered, in order to costumize it for the prediction application. FIR–Boolean hybrid (FBH) structures are considered, where the first level comprises some fixed parameter linear combiners, whose outputs feed the second level, consisting in a Boolean filter. The optimization process is carried out only for the second level, the weights of the linear combinations being kept fixed.

2. PREDICTION BASED ON OPTIMAL BOOLEAN AND STACK FILTERING

Consider an $n_r \times n_c$ M-valued image, the current pixel being denoted $D(i,j)$. N pixels in the neighborhood $\mathcal{N}_{D(i,j)}$ are selected for predicting $D(i,j)$, and they are arranged into an N–dimensional vector, denoted $\mathbf{D}(i,j)$. The neighborhood $\mathcal{N}_{D(i,j)}$ has to be causal with respect to the scanning order selected for the image (e.g. from left to right and from top to bottom in our experiments). A Boolean filter[2],[6] or a stack filter is used to process the information from $\mathbf{D}(i,j)$ for predicting the value of $D(i,j)$. The predicted value is computed as:

$$\hat{D}(i,j) = \sum_{m=0}^{M-1} f(T_m(\mathbf{D}(i,j))) \tag{1}$$

where T_m denotes the thresholding operator, which is applied componentwise to a vector. When the Boolean function f is positive, the filter is a stack filter and can be computed in the integer domain using min and max operators (acting on the pixels in $\mathcal{N}_{D(i,j)}$).

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One way to asses the predicting performance of the Boolean filters is to use the Sum of Absolute Error criterion, denoted here as J_f (that is sometimes reported as the Mean Absolute Error (MAE) criterion)

$$J_f = \frac{1}{n_r \cdot n_c} \sum_{i=1}^{n_r} \sum_{j=1}^{n_c} |D(i,j) - \hat{D}(i,j)| \qquad (2)$$

In the case of stack filters, the positive Boolean function minimizing the prediction criterion (2) can be found by solving a linear programming problem[1] using e.g. the fast algorithms provided in [3],[6]. On the other hand, for the Boolean filter class, the optimal predictor minimizing criterion (2) can be found extremely easily for the binary case ($M = 1$). For $M > 1$, the problem of finding the global optimal solution has an exponential complexity; however it is easy to find a suboptimal solution w.r.t. criterion (2). This suboptimal solution will minimize an upper bound of J_f, i.e. it will globally minimize the criterion

$$J_f^b = \frac{1}{n_r \cdot n_c} \sum_{i=1}^{n_r} \sum_{j=1}^{n_c} \sum_{m=0}^{M-1} |T_m(D(i,j)) - T_m(\hat{D}(i,j))|$$

In order to exploit the efficiency of linear predictors, a mixed filtering structure is proposed. FIR–Boolean hybrid predictors consist in Boolean filters that include in the processing mask, besides the values of the pixels inside $\mathcal{N}_{D(i,j)}$, some linear combinations of them. The linear combinations are fixed, and they are selected among the linear predictors of JPEG standard. The optimization is carried out only for the Boolean function.

To evaluate the efficiency of each predictor we use the entropy of prediction errors, which is a lower bound on the required codelength per pixel obtained with independent source coding techniques (Huffman, arithmetic coding). Denoting the prediction errors for each pixel $e(i,j) = D(i,j) - \hat{D}(i,j)$, the entropy is:

$$H(p) = - \sum_{e=-(M-1)}^{M-1} p(e) \log_2 p(e) \qquad (3)$$

where $p(e)$ is the relative frequency of occurrence of the prediction error value e, $e \in \{-(M-1), -(M-2), ..., 0, ..., (M-1)\}$. Denoting the histogram of the prediction errors by $h(e)$

$$h(e) = Card\{(i,j) | e(i,j) = e\} \qquad (4)$$

and the total number of pixels by $T = n_r \cdot n_c$, the relative frequency of error occurrence will be $p(e) = h(e)/T$ and hence

$$H(h) = \log_2 T - \frac{1}{T} \sum_{e=-(M-1)}^{M-1} h(e) \log_2 h_(e) \qquad (5)$$

3. PREDICTION USING BLOCK–OPTIMAL BOOLEAN AND FIR–BOOLEAN HYBRID FILTERS

3.1. Fixed size block–optimal prediction

Locally adaptive (block–optimal) Boolean filters have been introduced in [5] for image restoration applications, where they were shown to outperform the one-block optimal Boolean filters. Similar improvement is obtained in the present prediction application, but here it is very important to take into account the high increase in predictor complexity.

The image is subdivided into small blocks and one MAE–optimal Boolean predictor is fitted for each block. All the predictors must be transmitted altogether with the prediction errors, in order to recover the image at the decoder.

The prediction efficiency is measured using a cumulative global criterion, $\mathcal{R}$, which includes both the entropy of the residual image and the additional bitrate needed to code each local predictor.

For the case of partitioning the image into n_b blocks the cumulative criterion must be computed as follows

$$\mathcal{R} = H + 2^N \cdot n_b / T \qquad (6)$$

where H is the entropy of the errors (computed using the *whole error image*), N denotes the prediction window size and 2^N is the number of bits necessary for coding each predictor.

3.2. Adaptive–size–block–optimal–prediction

There are various possibilities of performing an adaptive block size partition. We chose the following strategy: starting from one block and checking whether splitting would yield improvement in performance.

The main goal of the proposed procedure is to decide if a block should be divided or not into four equally sized subblocks in order to design an optimal filter for each of them separately. The decision to split one block is taken if this improves the global criterion $\mathcal{R}$. The most important term in $\mathcal{R}$ is the entropy H, which depends on the histogram of the overall error image. During the splitting process, the modification of the entropy H, due to block splitting, is highly dependent on how the rest of the image is already partitioned, and hence the splitting decision may prove only suboptimal.

Quadtree procedure for subdividing images for prediction

The top–down splitting is started having the image already partioned into blocks of equal size, $L \times L$. Then the image is scanned left-right, top-down, selecting one $L \times L$ block each time. A recursive procedure decides

Image		$\circ\ \circ$ $\circ\ \bullet$				$\circ$ $\circ\ \circ\ \circ$ $\circ\ \circ\ \bullet$			$\circ\ \circ\ \circ\ \circ$ $\circ\ \circ\ \circ\ \circ$ $\circ\ \circ\ \bullet$	
		Boolean	Stack	FBH(3,7)	JPEG	Boolean	Stack	FBH(6,8)	Boolean	Stack
Balloon	7.346	3.191	3.331	3.104	3.172	3.146	3.225	3.024	3.100	3.225
Barb1	7.555	5.343	5.343	5.203	5.302	5.283	5.343	5.061	5.007	5.337
Barb2	7.484	5.190	5.190	5.124	5.236	5.190	5.190	5.047	5.056	5.183
Boats	7.088	4.313	4.673	4.313	4.469	4.324	4.600	4.256	4.293	4.600
Girl	7.288	4.496	4.496	4.180	4.225	4.400	4 435	4.075	4.108	4.435
Gold	7.530	4.717	5.089	4.717	4.875	4.753	4.992	4.690	4.708	4.978
Hotel	7.546	4.735	5.122	4.735	4.943	4.697	4.985	4.649	4.637	4.985
Zelda	7.334	4.161	4.161	4.062	4.179	4.061	4.067	3.968	3.964	4.066
Average	**7.396**	**4.518**	**4.676**	**4.430**	**4.550**	**4.482**	**4.604**	**4.346**	**4.359**	**4.602**

Table 1: Experiment 1. Entropy of errors for MAE–optimal global predictors for different prediction masks. The entropy when no prediction is used is contained in the first column. The best JPEG predictor is used for comparison.

whether or not to split the current block. If the decision is to split, the procedure calls itself for all four resulting subblocks.

A summary description of the recursive splitting procedure is given in the following. Denote by $\mathcal{B}$ the block which is currently being checked whether to split it or not. Denote by h_{out} the histogram of the errors at pixel locations outside block $\mathcal{B}$.

1. Compute the optimal predictor $f_{\mathcal{B}}$ for the overall block and $f_{\mathcal{B}_1}, \ldots, f_{\mathcal{B}_4}$, the optimal predictors for the subblocks $\mathcal{B}_1, \ldots, \mathcal{B}_4$.

2. Compute the prediction errors and the histograms $h_{\mathcal{B}}$, $h_{\mathcal{B}_1}, \ldots, h_{\mathcal{B}_4}$

3. Compute the entropy for one block, $H_{\text{no-split}} = H(h_{out} + h_{\mathcal{B}})$, and for the four blocks $H_{\text{ split}} = H(h_{out} + h_{\mathcal{B}_1} + \ldots + h_{\mathcal{B}_4})$.

4. The cost for extra-coding the predictors for the four subblocks is 3×2^N and that for transmitting the code for the quadtree branching is 4 bits.

5. Decide to split the block if
$$H_{\text{no-split}} > H_{\text{split}} + \frac{1}{T}(3 \times 2^N + 4) \qquad (7)$$

6. If the block is split, and if it is not at the deepest level accepted for splitting (4×4 in our experiments), call the present procedure for each of the resulting subblocks.

One additional bit, b, is allocated for each subblock, stating whether the subblock is split, $b = 1$, or not, $b = 0$.

The $L \times L$ blocks are scanned in left-right, top-bottom order, and the quadtree is coded in top-down mode.

4. EXPERIMENTAL RESULTS

Several experiments were performed using eight 576×720 images (Balloon, Barb1, Barb2, Boats, Girl, Gold, Hotel and Zelda) from the JPEG test image set. Only the Y (luminance) component, represented with 8-bits/ pixel, of each image was used. The entropy H of the errors is computed using (3) or (5).

Experiment 1. Global prediction
MAE–optimal Boolean, stack and FIR Boolean hybrid predictors were designed for different prediction masks. The entropies of prediction errors are presented in Table 1. For comparison, the entropy of the unpredicted image and the entropy of errors for the best performing JPEG predictor are also included. FBH(3,7), FBH(6,8) denote FIR–Boolean hybrid predictors with 3 and 6 pixels respectively in the prediction mask, and 7, 8 inputs respectively in the Boolean filter. The additional inputs are linear combinations of the pixel values, selected among JPEG linear predictors. The lowest entropy for the residuals is obtained for the FBH predictors, and for Boolean predictors with large prediction masks.

Experiment 2. Block–optimal prediction
In this experiment both fixed and adaptive–size blocks were considered. Boolean and FIR–Boolean filters were used for prediction. The codelength needed to encode the predictor themselves is taken into account, and the performance are evaluated using the global criterion $\mathcal{R}$ (6). In Table 2 the average over the test image set of the global criterion $\mathcal{R}$ is presented for both fixed block size and adaptive block size procedures. Figure 1 illustrates the partition of one image from JPEG test image set, which is internally build by the adaptive–size–block–optimal algorithm to take the maximum advantage of image nonstationarity.

Blocks size	∘ ∘ ∘ • Boolean	FBH(3,4)	∘ ∘ ∘ ∘ • Boolean	∘ ∘ ∘ ∘ ∘ • Boolean
128×128	4.518	4.393	4.483	4.445
64×64	4.493	4.371	4.448	4.411
32×32	4.460	4.346	4.406	4.380
16×16	4.435	4.348	4.386	4.395
Adaptive size	4.404	4.330	4.356	4.353

Table 2: Experiment 2. Average global criterion $\mathcal{R}$ for the test image set, in block–optimal prediction case.

5. CONCLUSIONS

The prediction performances of optimal Boolean, stack and FIR–Boolean hybrid predictors for lossless image compression have been investigated in this paper.

New optimal algorithmic structures were proposed, fitted as much as possible to the image to be transmitted, and shown that the cost of transmitting the predictor parameters is low enough to obtain an overall competitive lossless coding scheme.

The experiments we performed on images from the JPEG test image set give a clear picture of the performances obtained for different predictors, parameters and prediction approaches.

The most important conclusions derived from the extensive experiments are the following:

The best performance is obtained with block–optimal FIR–Boolean hybrid filters for small prediction masks of 3 to 6 pixels. The FBH(3,4) predictor yields the average cumulative criterion $\mathcal{R} = 4.330$, ranking the first among all our predictors and other predictors we found in the literature. The Gradient adjusted Predictor, used by CALIC system [4] gives an average entropy of the prediction errors for the test image set of 4.363.

For Boolean and FIR–Boolean hybrid predictors and small prediction masks the adaptive–size block–optimal prediction slightly outperforms regular block–optimal prediction.

The best performance of one-block sequential prediction is obtained by Boolean predictors with 10 pixels prediction mask.

6. REFERENCES

[1] E.J. Coyle and J.-H. Lin. Stack filters and the mean absolute error criterion. *IEEE Transactions on Acoustics, Speech and Signal Processing*, ASSP-36(8):1244–1254, Aug. 1988.

[2] K.D. Lee and Y.H. Lee. Threshold Boolean filters. *IEEE Transactions on Signal Processing*, SP-42:2022–2036, Aug. 1994.

[3] J.-H. Lin and Y.-T. Kim. Fast algorithms for training stack filters. *IEEE Transactions on Signal Processing*, SP-42:772–781, April 1994.

[4] N.D. Memon and K. Sayood. A comparison of prediction schemes proposed for a new lossless image compression standard. In *Proc. IEEE Int. Symposium on Circuits and Systems*, volume II, pages 309–312, Atlanta, U.S.A., 1996.

[5] D. Petrescu, I. Tăbuş, and M. Gabbouj. Locally adaptive techniques for stack filtering. In *Proc. Eusipco-96, VIII European Signal Processing Conference*, pages 587–590, Trieste, Italy, Sept. 1996.

[6] I. Tăbuş, D. Petrescu, and M. Gabbouj. A training framework for Boolean and stack filtering. Optimal design and robustness case studies. *IEEE Transactions on Image Processing Special Issue on Nonlinear Image Processing*, IP-5:809–826, June 1996.

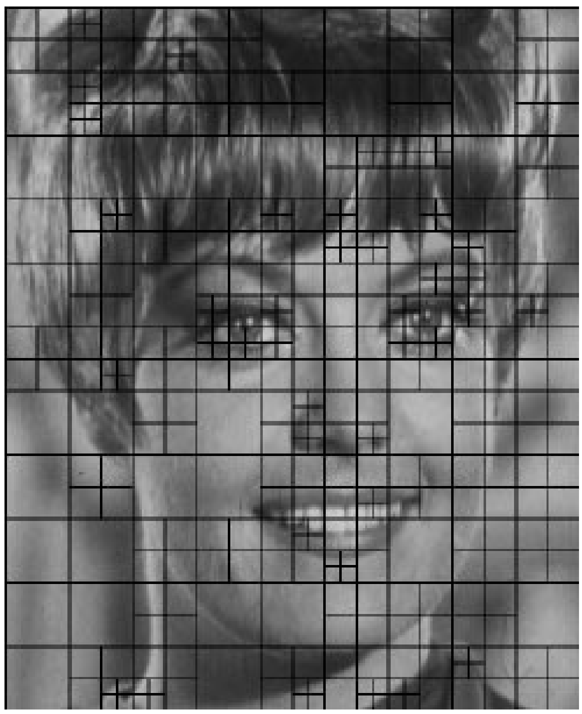

Figure 1: Partition of image Zelda using the adaptive–size–block–optimal Boolean prediction with 4-pixels mask