SURVEY OF ODD AND EVEN LENGTH FILTERS IN TREE-STRUCTURED FILTER BANKS FOR SUBBAND IMAGE COMPRESSION

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ABSTRACT

The performance of subband image coders depends on proper choice of filter banks. Although odd length filters in the filter banks produce waveform type artifacts, this can be alleviated by enforcing a smooth interpolation property to the synthesis lowpass filter. Evaluation of even, odd, and combinations of even and odd length filters in tree-structured filter banks, where the filter coefficients are obtained by optimizing for coding gain at *each stage*, is done for image coding purposes. Favorable results are obtained when a combination of odd and even length filters are used.

1. INTRODUCTION

Subband coding of highly correlated sources like speech and image signals have been studied extensively in the past. For image compression, decomposition by nonunitary filter banks with nonuniform frequency separation is better than unitary or nonunitary uniform filter banks. One possible candidate for nonuniform filter banks is a tree-structured system. The discrete wavelet transform constitutes a subclass of this construction.

Tree-structured filter banks, if correctly optimized, can alleviate some of the typical artifacts experienced in subband coding, notably *ringing* when the filters' unit sample responses are long, and *blocking* in the case of short responses. High frequency resolution at low frequencies and lower frequency resolution at higher frequencies represent a good compromise in terms of the coders' ability to cope both with large areas of constant spectral contents, i.e., visually stable areas, and transients. If odd length filters are used to construct tree-structured filter banks, waveform type artifacts are often observed in areas of constant spectral contents. Hence, appropriate filters in tree-structured filter banks for image compression must be carefully selected and optimized.

In this paper, we propose a smooth interpolation criterion for odd length synthesis lowpass filters, and investigate all possible combinations of both odd and even length filters upto a certain length in tree-structured filter banks for subband image compression.

2. TWO-CHANNEL FILTER BANKS

Two-channel nonunitary uniform filter banks are used to construct tree-structured filter banks. To guarantee perfect reconstruction through the analysis-synthesis system, constraints among the filter coefficients have to be enforced. The system is then completely free of aliasing, amplitude, and phase distortions. Polyphase representation, invented by Bellanger et al. [1] in multirate systems, introduced an efficient method of implementing filters, especially filter banks. The perfect reconstruction property of a decimated filter bank can be guaranteed by using the polyphase matrices [2]. Assume that $\mathbf{P}(z)$ and $\mathbf{Q}(z)$ denote decimated analysis and synthesis polyphase matrices, respectively. Perfect reconstruction is obtained if

$$\mathbf{Q}(z) = cz^{-k}\mathbf{P}^{-1}(z), \qquad (1)$$

where c is an arbitrary constant and k is an integer. Then, the reconstructed signal becomes equal to a scaled and delayed version of the input signal. By using FIR analysis filters, FIR synthesis filters are obtained by setting appropriate terms to zero in the determinant of $\mathbf{P}(z)$. For the two-channel case, the filter relationships are: $G_{LP}(z) = H_{HP}(-z)$ and $G_{HP}(z) = -H_{LP}(-z)$ where $\mathbf{H}(z)$ and $\mathbf{G}(z)$ denote analysis and synthesis filters, respectively.

Linear phase perfect reconstruction two-channel filter banks consist of either even or odd length filters. The synthesis lowpass filter, $G_{LP}(z)$, known as the interpolation filter, interpolates the lowpass samples from the analysis lowpass filter to produce areas containing slowly varying signals. Hence, the l_1-norms of the even and the odd polyphase components of the synthesis lowpass filter need to be equal in order to alleviate waveform type artifacts. The l_1-norms of the even and the odd polyphase components are equal in even length filters. However, this is not true in the case of odd length filters. Therefore, in odd length filters an additional constraint among the filter coefficients needs to be enforced so that the even and the odd polyphase components have equal l_1-norm. This additional constraint forces the analysis highpass filter to have at least one of its zeros at 0, i.e., the synthesis lowpass filter has at least one of its zeros at π. This attributes to better dc leakage suppression. Good peak signal-to-noise ratio (PSNR) is observed when similar constraints are also imposed to the analysis lowpass or the synthesis highpass filter.

In this survey, we restrict the sum of the filter lengths in the two-channel case to be less than or equal to 20 where the maximum length of a channel cannot exceed 12. Table 1 gives the possible combinations for both even and odd length filter banks where the sum of the lowpass and the highpass filter lengths is a multiple of 4, if linear phase and perfect reconstruction properties are required. Including the trivial combination of 2/2, the number of possible distinct filter banks in this case amounts to 29.

3. TREE-STRUCTURED FILTER BANKS

Perfect reconstruction octave-band two- and three-stage tree-structured filter banks (OTFB) are constructed by two- and three sets of two-channel filter banks in cascade form,

Table 3. Building blocks of OTFBs.

FB	Analysis filter bank							Synthesis filter bank						
	$\frac{No.of\ LP\ taps}{No.of\ HP\ taps}$			Total no. of taps				Total no. of taps				$\frac{No.of\ LP\ taps}{No.of\ HP\ taps}$		
	S-1	S-2	S-3	HP	BP2	BP1	LP	LP	BP1	BP2	HP	S-3	S-2	S-1
Symmetrical/asymmetrical even length OTFBs														
FB 60_44_16_10	2/10	12/8	10/6	10	16	44	60	44	60	32	2	6/10	8/12	10/2
FB 52_36_24_10	6/10	2/10	12/8	10	24	36	52	56	72	12	6	8/12	10/2	10/6
Asymmetrical odd length OTFBs														
FB 25_21_7	9/7	9/7	-	7	21	-	25	19	-	23	9	-	7/9	7/9
FB 7_11_5	3/5	3/5	-	5	11	-	7	13	-	9	3	-	5/3	5/3
FB 59_35_15_9	11/9	3/9	9/11	9	15	35	59	65	57	13	11	11/9	9/3	9/11
FB 57_49_21_7	9/7	9/7	9/7	7	21	49	57	43	51	23	9	7/9	7/9	7/9
FB 47_55_27_9	11/9	3/9	9/11	9	27	55	47	65	57	13	11	11/9	9/3	9/11
FB 33_25_21_7	5/7	7/9	5/3	7	21	25	33	31	39	19	5	3/5	9/7	7/5
Asymmetrical even/odd length OTFBs														
FB 24_12_12	8/12	9/3	-	12	12	-	24	16	-	28	8	-	3/9	12/8
FB 56_40_24_12	8/12	3/9	12/8	12	24	40	56	56	72	16	8	8/12	9/3	12/8
FB 54_38_22_10	6/10	3/9	12/8	10	22	38	54	54	70	14	6	8/12	9/3	10/6

respectively, where the lowpass channel in *each stage* is split into two subchannels. The main objective here is to construct OTFB allowing both odd and even length filters. In order to avoid *ringing* and *blocking* artifacts, proper length of filters are needed. All possible distinct two-stage, three-stage, and four-stage filter banks are given in Table 2. The total number of filter banks at *each stage* is given by 29^L, where L is the number of stages. Generally, the total number of filter banks at *any stage* can be found by M^L, where M is the total number of destinct two-channel filter banks, and L is the number of stages.

Table 1. Possible combinations of two-channel filter banks.

No.of LP taps/No.of HP taps			
12/8	12/4	10/10	10/6
10/2	8/8	8/4	6/6
6/2	4/4	11/9	11/5
9/7	9/3	7/5	5/3

Table 2. Number of possible filter banks.

Type	2-stage	3-stage	4-stage
Even length FBs	289	4913	83521
Odd length FBs	144	1728	20736
Even/odd length FBs	408	17748	603024
Total	841	24389	707281

4. OPTIMIZATION OF THE FILTER COEFFICIENTS

Construction of linear phase perfect reconstruction OTFBs by using two-channel filter banks are presented in the previous section. To guarantee linear phase and perfect reconstruction in two-channel filter banks, a number of relations amoung the parameters are established. The remaining degrees of freedom are used to obtain the filter coefficients by optimizing for subband coding gain at *each stage*. The coding gain is a common used measure for data compression ability [3]. A compact formula to evaluate the generalized subband coding gain for nonunitary nonuniform filter banks can be found in [4]. From our experimental results, the underlying statistics of an image can be approximated as an AR(1) process with nearest sample correlation $\rho = 0.95$. Hence, the maximum one-dimensional theoretical coding gain equals 10.11 dB [3]. However, if the filters are only optimized in the *first stage* and then employed in *other stages*, the *average* nearest sample correlation of the input signal can be approximated to 0.80. Hence, in this survey, we have optimized the filter coefficients in two-channel filter banks for $\rho = 0.80$. Fine tuning of the coefficients at *each stage* is done for an input process with $\rho = 0.95$ after selecting the "best" filter bank according to its practical coding gain (PSNR). The "optimization toolbox" in MATLAB is used to maximize the coding gain to obtain the one-dimensional filter coefficients. Assuming separability, the two-dimensional filter bank is constructed from a one-dimensional filter bank.

5. CODING SCHEME

Our objective here is to compare different types of filter banks for image coding purposes. Hence, the quantization and the coding scheme should be identical for all types of filter banks in order to do a fair comparison test.

A good and fast approach to code the subband image by Said et. al [5] has received much attention lately. The method is an extension of embedded zero tree wavelet (EZW) coding introduced by Shapiro [6]. The main ingredients of the coding scheme, as described in [5], can be summarized into three concepts as follows: 1) the partial ordering of the subband coefficients by magnitude with a set partitioning sorting algorithm, 2) the ordered bit plane transmission of refinement bits, and 3) the exploitation of self-similarity of the subband image across different bands.

Table 4. Coding gains for AR(1), $\rho = 0.95$.

FB	Coding gain (dB)
FB 60_44_16_10	9.444
FB 52_36_24_10	9.492
FB 25_21_7	8.698
WL[1] 25_21_7	8.469
FB 7_11_5	6.372
FB 59_35_15_9	9.506
FB 57_49_21_7	9.527
WL[1] 57_49_21_7	9.459
FB 47_55_27_9	9.462
FB 33_25_21_7	9.418
FB 24_12_12	8.757
FB 56_40_24_12	9.455
FB 54_38_22_10	9.425

6. CODING RESULTS

Our aim is to find the "best" combination of filters in octave-band three-stage filter banks. This means that all possible 24389 combinations must be examined for some given, hopefully representative, images and choose the "best" performer. However, this requires a vast amount of computer resources and seems to be an impossible task within a short period of time. Hence, we first approach this problem by examining all those 841 possible filter banks for a two-stage system, and select the "best" 40 filter banks. Then, the chosen filter banks are extended from two-stage to three-stage. In this way, we reduced the number of combinations to 1160 (340-even length, 240-odd length and 580-even/odd length) filter banks.

Two 512 × 512 images "Lenna" and "Barbara" (source: ftp.eedsp.gatech.edu) are used to characterize the coding performance of the filter banks. To determine the performances of the filter banks, we compared them with the well known 9/7 wavelet filter bank, given in [7].

Table 3 lists the most interesting combinations found from our simulation test based on PSNR. Table 4 provides the corresponding theoretical subband coding gain. Figure 1 depicts the advantage of enforcing the smoothness property to an odd length filter bank, FB 33_25_21_7. (W(O)SI - with(out) smoothness property.) It is clear that this property enhances both the subjective and the objective coding performances. The "best" coding performance was obtained for a two-stage system by FB 24_12_12, and the results are shown in Figures 2(a) and (b). The filter bank performs better than wavelet filter bank WL[1] 25_21_7. To validate our claim that filters in filter banks need to have appropriate lengths, the performance curve for the "worst" performer, FB 7_11_5, is also shown in Figure 2.

Based on our a priori knowledge by assessing the coding results for two-stage filter banks, 1160 different filter banks are selected in a three-stage system. Table 5 provides the "best" coding performances for "Lenna" and "Barbara". We observe the following from this table: the 9/7 gain optimized, at *each stage*, filter bank (FB 57_49_21_7) performs better than its counterpart namely the 9/7 wavelet filter bank. Both have the same number of filter coefficients at *each stage*, however, the filter coefficients differ owing to different optimization criteria. Filter banks combining odd and even length filters tend to perform equally well.

[1]Coding gain for the case of wavelet filter bank is found by using the filter coefficients in an AR(1) model where $\rho = 0.95$.

7. DISCUSSION AND CONCLUSIONS

By testing all possible combinations (841) for two-stage filter banks does not necessarily mean that the filter bank reported here is the "optimum". The performance of filter banks for image compression is based on the following concepts: 1) the filter combinations at *each stage*, 2) the optimization criteria of the filters, and 3) the coding scheme. A good way of finding the filter coefficients is to optimize to the statistics of the image at hand. However, the AR(1) model tends to be a good compromise. Here, for three-stage filter banks, only 1160 out of 24389 were tested. And from our simulation results for "Lenna" and "Barbara", we conclude that the "optimum" for this system was none of those in 1160. This is due to the fact that the 9/7 combination was not amoung the "best" found in the test. Hence, one has to conduct an exhaustive search of all 24389 possible combinations to find the "best" filter bank. This will give us an idea of finding the "best" combination for the four-stage system. Examining all 707281 filter banks seems as an impossible task if a priori knowledge from two-and three-stage systems are not available.

According to [8] even length filter banks are better at preserving location, shape, and intensity of impulses than odd length filter banks. However, from our simulation results based on PSNR and subjective comparisons, odd length filters having the smooth interpolation property perform as good as even length filters. Hence, they should be considered in constructing filter banks for image coding purposes. Furthermore, combinations of both odd and even length filter banks are attractive and should be considered in future systems.

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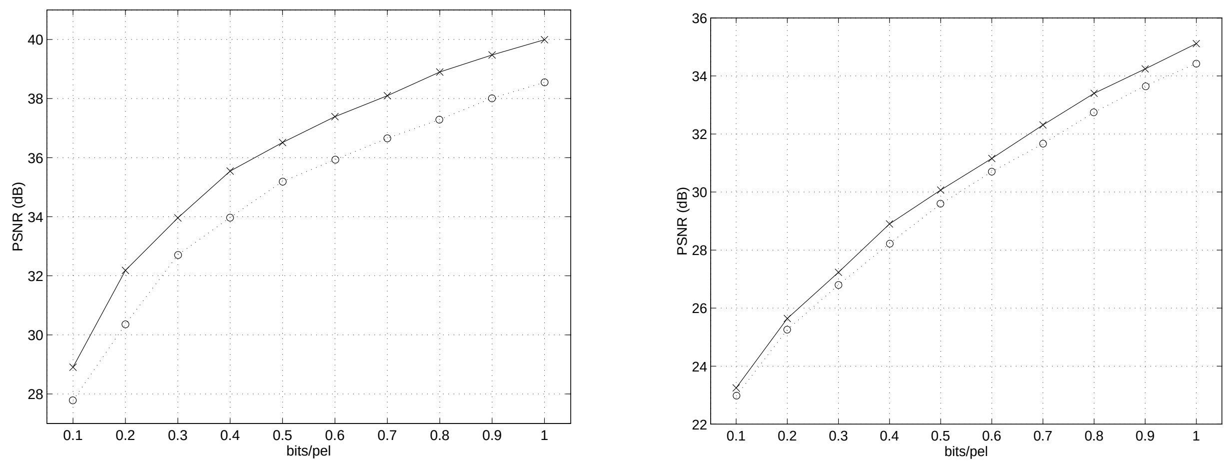
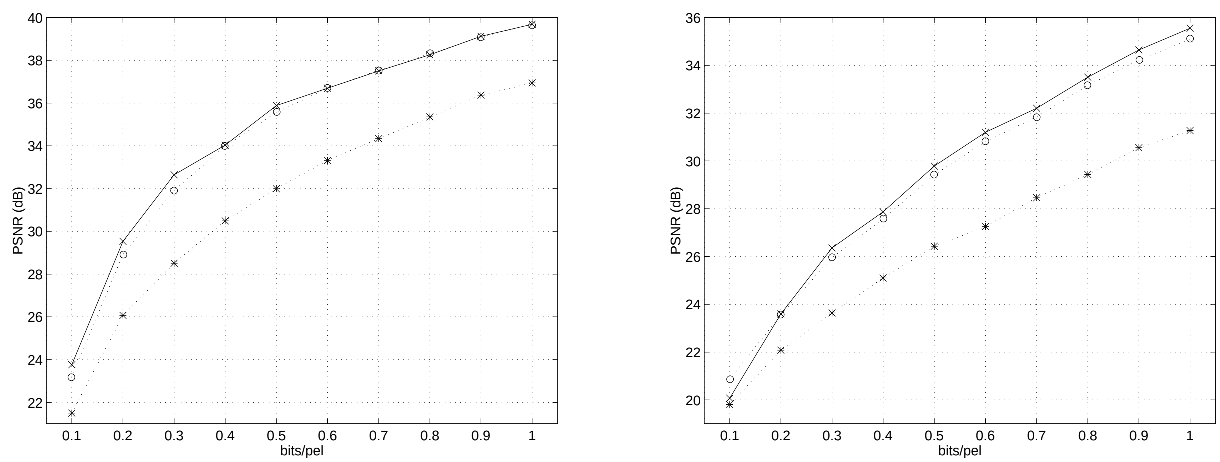
Table 5. **Coding results for *three-stage* filter banks.**

FB	"Lenna"					"Barbara"				
	Bpp vs. PSNR in dB					Bpp vs. PSNR in dB				
	0.10	0.30	0.50	0.70	1.00	0.10	0.30	0.50	0.70	1.00
FB 60_44_16_10	29.13	34.32	36.73	38.29	40.11	-	-	-	-	-
FB 52_36_24_10	-	-	-	-	-	23.44	27.63	30.76	33.13	36.02
FB 59_35_15_9	28.83	34.21	36.78	38.31	40.17	-	-	-	-	-
FB 47_55_27_9	-	-	-	-	-	23.53	27.63	30.62	32.96	35.74
FB 57_49_21_7	29.07	34.54	37.00	38.56	40.33	23.48	27.92	31.12	33.50	36.25
WL 57_49_21_7	28.80	34.45	36.93	38.54	40.28	23.49	27.78	30.94	33.32	36.09
FB 56_40_24_12	28.85	34.31	36.79	38.33	40.16	-	-	-	-	-
FB 54_38_22_10	-	-	-	-	-	23.52	27.88	31.15	33.46	36.30



(a) (b)

Figure 1. FB 33_25_21_7 WSI (×), and FB 33_25_21_7 WOSI (∘). (a) "Lenna" and (b) "Barbara".



(a) (b)

Figure 2. FB 24_12_12 (×), WL 25_21_7 (∘), and FB 7_11_5 (∗). (a) "Lenna" and (b) "Barbara".