BANDWIDTH COMPRESSION FOR CONTINUOUS AMPLITUDE CHANNELS BASED ON VECTOR APPROXIMATION TO A CONTINUOUS SUBSET OF THE SOURCE SIGNAL SPACE

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ABSTRACT

Two methods for transmission of a continuous amplitude source signal over a continuous amplitude channel with a power constraint are proposed. For both methods, bandwidth reduction is achieved by mapping from a higher dimensional source space to a lower dimensional channel space. In the first system, a source vector is quantized and mapped to a discrete set of points in a multidimensional PAM signal constellation. In the second system the source vector is approximated with a point in a continuous subset of the source space. This operation is followed by mapping the resulting vector to the channel space by a one-to-one continuous mapping resulting in continuous amplitude channel symbols. The proposed methods are evaluated for a memoryless Gaussian source with an additive white Gaussian noise channel, and offer significant gains over previously reported methods. Specifically, in the case of two-dimensional source vectors, and one-dimensional channel vectors, the gap to the optimum performance theoretically attainable is less than 1.0 dB for a wide range of channel signal-to-noise ratios.

1. INTRODUCTION

In this paper, we consider the problem of transmitting a discrete time continuous amplitude source signal over a discrete time, continuous amplitude channel with a power constraint. The channel is assumed to include a modulator, the waveform channel, and a soft decision demodulator with continuous amplitude output. Furthermore, we assume an additive white Gaussian noise (AWGN) channel. For a given channel signal-to-noise ratio (CSNR), the task is to find a mapping from the source space to the channel space such as to minimize the distortion between the original and the received source signal.

A similar problem was considered in [1, 2, 3]. In [1], the optimum *linear* continuous mapping from the source space to the channel space was derived given the source and the channel statistics. In [2], a discrete system was considered, where the encoder mapping consisted of a vector quantizer (VQ) followed by a linear mapping from the VQ centroids to some discrete points in the channel space. At the receiver side, a linear map from the channel space to the source space was used. As pointed out in [2], the resulting mean squared error (mse) of this system is bounded from below by the mse of the system in [1]. Finally, in [3] the channel symbols were restricted to points in a regular multidimensional pulse amplitude modulation (PAM) signaling alphabet, and the encoder consisted of a VQ followed by an index assignment function between the source and the channel space. At the receiver side, the channel output vector was mapped to an index which was used to select a decoded vector from a codebook of reconstruction vectors. This optimization problem can be formulated as a power constrained channel optimized vector quantization (PCCOVQ) problem and is solved by the generalized Lloyd algorithm (GLA).

There are two contributions of this paper. First, the PCCOVQ system in [3] is improved by choosing a good initial codebook for the GLA-based PCCOVQ training and by optimizing the scaling of the signal constellation within the GLA. Second, the performance is improved further by allowing for continuous amplitude channel symbols. This is achieved by replacing the discrete set represented by the PCCOVQ reconstruction vectors with a continuous subset of the source signal space. Thus, the source vector is approximated by a vector in the continuous subset. The resulting vector is then mapped to the channel space by a one-to-one continuous mapping.

2. THEORY

2.1. General Problem Formulation

Assume that the source signal to be encoded is a real-valued, discrete time, and stationary random process X_i, with zero mean, variance per symbol σ_x^2, and symbol rate f_s. The source signal is to be transmitted over an AWGN channel with a power constraint P_{max}, using real-valued discrete time symbols with symbol rate $f_c \leq f_s$. Let $\mathbf{x}$ be an L-dimensional vector derived from X_i according to $\mathbf{x} = (x_{nL}, x_{nL+1}, ..., x_{nL+L-1})$. The source vector $\mathbf{x}$ is mapped to a K-dimensional vector $\mathbf{y}$ of channel symbols by an encoder function $\alpha : \mathbb{R}^L \rightarrow \mathbb{R}^K$, where the relationship between L and K is given by $L/K = f_s/f_c \geq 1$.

When transmitted over the channel, the vector $\mathbf{y}$ is corrupted by an additive noise vector $\mathbf{n}$, generated by an independent and identically distributed (iid) Gaussian process with zero mean and variance σ_n^2 per sample. Finally, the received vector $\hat{\mathbf{y}} = \mathbf{y} + \mathbf{n}$ is processed by the decoder function $\beta : \mathbb{R}^K \rightarrow \mathbb{R}^L$ to produce the reconstructed vector $\hat{\mathbf{x}}$. This communication system is illustrated in Figure 1.

With the mean squared distortion measure, the general optimization problem can be formulated as follows. Given the source probability density function (pdf) $p_{\mathbf{X}}(\mathbf{x})$ and the values of L, K, and σ_n^2, choose the encoder mapping α and the decoder mapping β such as to minimize the distortion

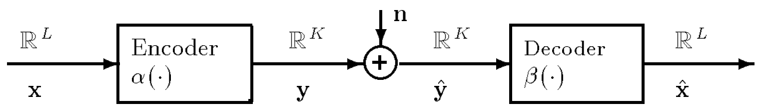

Figure 1. General communication system.

$D = E[\|\mathbf{x} - \hat{\mathbf{x}}\|^2]/L$ subject to a power constraint $P = E[\|\mathbf{y}\|^2]/K \le P_{max}$. The expectations are taken over both the source and the noise pdfs.

The system performance will be evaluated in terms of the source signal-to-noise ratio (SNR)

$$SNR = 10\log_{10}(\sigma_x^2/E[\|x - \hat{x}\|^2]/L) \tag{1}$$

versus the channel signal-to-noise ratio (CSNR)

$$CSNR = 10\log_{10}(P/\sigma_n^2). \tag{2}$$

2.2. Discrete Amplitude System

First, we consider the case of discrete amplitude channel symbols. In particular, the channel symbols are restricted to belong to the Cartesian product of a regular K-dimensional M-ary PAM signaling alphabet. The discrete amplitude system is illustrated in Figure 2.

The source encoder maps the vector $\mathbf{x}$ to an index i in an index set $I = \{0, 1, ..., N-1\}$. The source encoder is specified by a partition $\mathcal{P} = \{\Omega_0, \Omega_1, ..., \Omega_{N-1}\}$ of $\mathbb{R}^L$, and the index i is selected whenever $\mathbf{x} \in \Omega_i$. Next, the signal selection unit maps the index i to the corresponding channel symbol $\mathbf{y} = \Delta \cdot \mathrm{s_i} \in \mathcal{S}$ where the signal set $\mathcal{S} \subset \mathbb{R}^K$ is the K-fold Cartesian product of an M-ary PAM signaling alphabet with minimum distance equal to one, and Δ is a scale factor. At the receiver side, the signal detection unit chooses an index j such as to minimize $\|\hat{\mathbf{y}} - \Delta \cdot \mathrm{s}_j\|^2$, $j \in I$. Finally, the source decoder maps the index j to the corresponding reconstruction vector $\hat{\mathbf{x}} = \mathbf{c}_j$ belonging to a finite subset $\mathcal{C} = \{\mathbf{c}_0, \mathbf{c}_1, ..., \mathbf{c}_{N-1}\}$ where $\mathbf{c}_i \in \mathbb{R}^L$. We will refer to the subset $\mathcal{C}$ as the reconstruction codebook. For this system, the optimization problem can be expressed as that of minimizing

$$F(\mathcal{P}, \Delta, \mathcal{C}) = \sum_{i=0}^{N-1} \int_{\Omega_i} p(\mathbf{x}) d_i(\mathbf{x}) \mathbf{dx}, \tag{3}$$

where

$$d_i(\mathbf{x}) = \sum_{k=0}^{N-1} P(k|i)\, \|\mathbf{x} - \mathbf{c}_k\|^2 + \lambda\Delta^2\|\mathrm{s}_i\|^2 \tag{4}$$

with respect to $\mathcal{P}$, Δ, $\mathcal{C}$. Here, $P(k|i)$ is the probability of receiving index k given that index i was transmitted, and λ is the Lagrange multiplier chosen to satisfy the power constraint.

For a given set of transition probabilities and reconstruction codebook, it was shown in [3] that the optimum partition $\mathcal{P}$ is given as

$$\Omega_i = \{\mathbf{x} : d_i(\mathbf{x}) \le d_j(\mathbf{x}) \;\; \forall j \ne i\}, \;\; i = 0, ..., N{-}1. \tag{5}$$

Furthermore, for a given partition and transition probabilities, the reconstruction vectors $\mathbf{c}_i, i = 0, 1, ..., N-1$ can be expressed as [3]

$$\mathbf{c}_i = \frac{\sum_{k=0}^{N-1} P(i|k) \int_{\Omega_k} \mathbf{x} p_X(\mathbf{x}) d\mathbf{x}}{\sum_{k=0}^{N-1} P(i|k) \int_{\Omega_k} p_X(\mathbf{x}) d\mathbf{x}}, \quad i = 0, ..., N{-}1. \tag{6}$$

We will refer to the encoder/decoder structure of the discrete amplitude system as power constrained channel optimized vector quantization (PCCOVQ). Note that for an optimization problem without a power constraint, Equations 5 and 6 correspond to those of conventional COVQ [4] ($\lambda = 0$). If, in addition, the channel is noiseless, the resulting encoder/decoder structure is a conventional VQ.

For a given value of λ, the proposed system is designed using a variation of the GLA as follows:

- Select an initial value of Δ, and an initial reconstruction codebook $\mathcal{C}$.
- Update the partition $\mathcal{P}$ according to Equation 5.
- Update the reconstruction codebook $\mathcal{C}$ according to 6.
- Update Δ using an iterative search routine to minimize the expression of Equation 3.
- Compare the current distortion (Equation 3) with the distortion of the previous iteration to check for convergence. If not, continue from the second step above, otherwise stop.

Note that this method for optimizing the PCCOVQ system differs from the method in [3] since in our work, Δ is optimized within the iterative algorithm. In [3], Δ was chosen to depend on P_{max}, but not necessarily in an optimal way.

Another important issue is the index assignment which is the matching of the reconstruction vectors $\mathbf{c}_i$ with the signal set vectors s_i. As pointed out in [4], the index assignment is a by-product of the COVQ design algorithm. Thus, if a global optimum is reached, no further gain can be achieved by rearranging the reconstruction vectors $\mathbf{c}_i$. However, the iterative algorithm guarantees only a local minimum which might be dependent on the index assignment chosen for the initial set of reconstruction vectors.

In this work, this problem is addressed by use of the noisy channel relaxation method, originally developed for a BSC [5, 6]. Adapting this technique to the problem at hand, the system is initially optimized for a very low CSNR value, with the following choice of initial codebook vectors

$$c_i(m) = \begin{cases} k s_i(m) & \text{if } 0 \le m \le K{-}1 \\ k s_i(K{-}1) & \text{if } K \le m \le L{-}1 \end{cases} \tag{7}$$

where $c_i(m)$ and $s_i(m)$ are the m'th components of the vectors $\mathbf{c}_i$ and s_i respectively, and k is a scale factor. Thus, the initial reconstruction vectors lie in a K-dimensional subspace of $\mathbb{R}^L$. Next, the CSNR is increased in small steps towards the target CSNR ($= P_{max}/\sigma_n^2$). For each intermediate CSNR value, a full iterative algorithm is performed using the reconstruction codebook from the previous CSNR as the initial codebook for the current CSNR.

2.3. Continuous Amplitude System

The continuous amplitude system is illustrated in Figure 3. In this case, the source vector $\mathbf{x}$ is approximated by a vector $\tilde{\mathbf{x}}$ in a continuous subset of $\mathbb{R}^L$. As described below, the continuous subset is specified by the N reconstruction vectors of the discrete amplitude system. Next, the vector $\tilde{\mathbf{x}}$ is

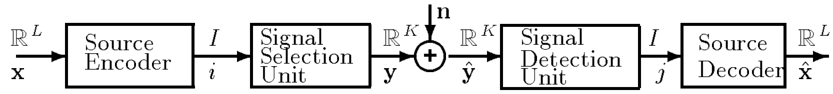

Figure 2. Discrete amplitude system.

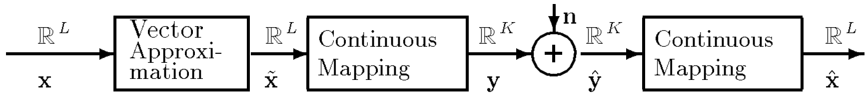

Figure 3. Continuous amplitude system.

mapped by a one-to-one continuous mapping to the channel symbol $\mathbf{y}$. At the receiver side, the decoder maps the received vector $\hat{\mathbf{y}}$ to the reconstructed vector $\hat{\mathbf{x}}$ by a continuous mapping from $\mathbb{R}^K$ to $\mathbb{R}^L$. In this paper, we consider the cases $K = 1$ and $K = 2$ only.

An example of the encoder and decoder operations are illustrated in Figure 4 for $L = 2$, $K = 1$, and $N = 4$. Assuming that the N reconstruction vectors from the discrete amplitude system are given, the approximated source vector $\tilde{\mathbf{x}}$ is found by mapping $\mathbf{x}$ onto one of the line segments connecting two codebook vectors that correspond to neighbors in the channel space. Next, the channel symbol $\mathbf{y}$ is found by mapping to the corresponding line segment in the channel space. At the decoder side, the reconstructed vector $\hat{\mathbf{x}}$ is derived from the channel output vector $\hat{\mathbf{y}}$ by performing the inverse mapping from the channel space to the source space. Similarly, the case $L = 3$, $K = 2$, and $N = 16$ is illustrated in Figure 5. In this case, the one-to-one mapping is defined by mapping a triangle in the source space to the corresponding triangle in the channel space.

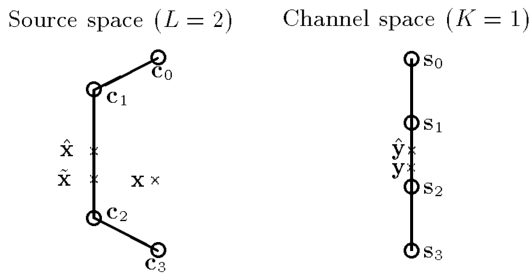

Figure 4. Mapping for continuous amplitude system, $L = 2$, $N = 4$, and $K = 1$.

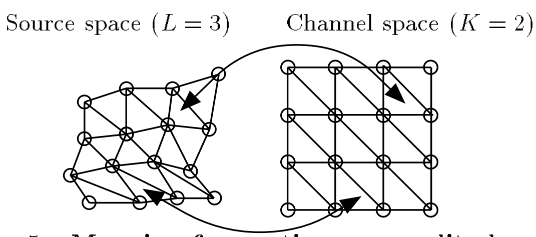

Figure 5. Mapping for continuous amplitude system, $L = 3$, $N = 16$, and $K = 2$.

3. RESULTS

In Figure 6 typical PCCOVQ codebook structures for a zero-mean unit-variance iid Gaussian source are shown for different CSNR values. The codebook structures possess two properties that are crucial for power efficient and robust transmission. First, points that are close in the channel space correspond to points that are close in the source space. Second, the reconstruction vectors having the highest probability correspond to channel symbols having the smallest amplitudes. Note that there is nothing in the GLA-based optimization algorithm that explicitly specifies the spiral shape of the mappings. In Figure 7, the performance is shown for $L = 2$, $K = 1$, and for various values of N. The results are compared against the optimum performance theoretically attainable (OPTA) which is determined by evaluating the distortion-rate function at the channel capacity. Note that for a given CSNR, increasing the number of reconstruction vectors always results in increased performance. In Figure 8 the performance with channel mismatch is illustrated. Here, the actual CSNR is different from the design CSNR (CSNR_d). The performance compared to other methods is illustrated in Figure 9. Finally, in Figures 10 and 11, the performances of the discrete and the continuous amplitude systems are compared. As can be seen from the results, the continuous amplitude system offers a significant gain over the discrete amplitude system, at high CSNR values.

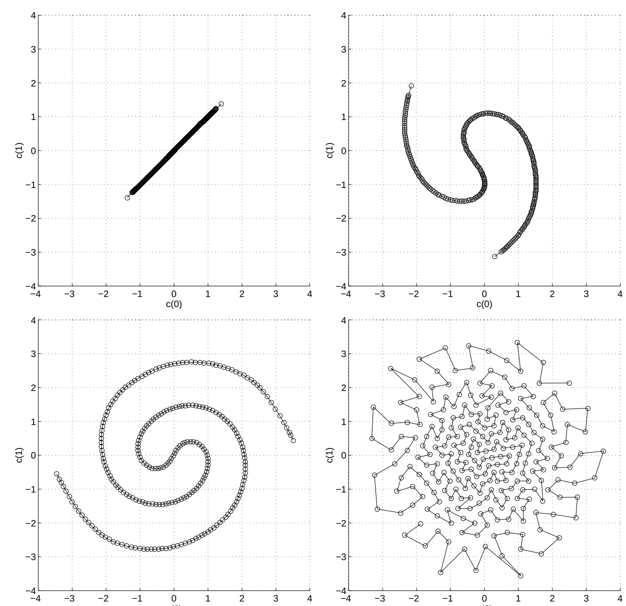

Figure 6. Reconstruction codebook vectors for $L = 2$, $K = 1$, and $N = 256$. The line segments connect two codebook vectors (o) which are mapped to neighbor points in the channel space. Upper left: CSNR = 0 dB, upper right: CSNR = 10 dB, lower left: CSNR = 20 dB, lower right: CSNR = 50 dB.

4. CONCLUSIONS

Two different methods for bandwidth compression of continuous amplitude source signals have been proposed. Both methods offer a significant gain compared to previously reported methods. The proposed methods perform close to

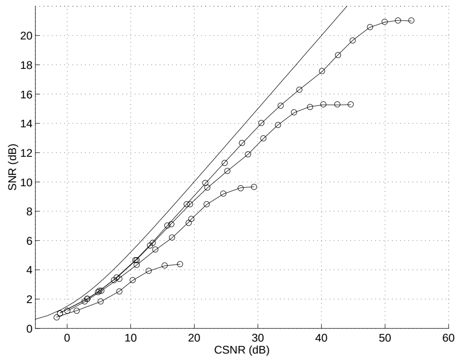

Figure 7. SNR vs. CSNR for $L = 2$, $K = 1$, and for various values of N. From top to bottom: OPTA, $N = 256$, $N = 64$, $N = 16$, and $N = 4$.

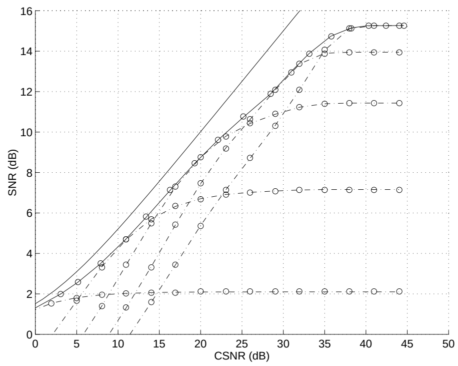

Figure 8. SNR vs. CSNR for PCCOVQ with channel mismatch, L=2, N=64, K=1. From top to bottom at CSNR=40 dB: OPTA, system optimized for true CSNR, CSNR$_d$=0 dB, CSNR$_d$=10 dB, CSNR$_d$=20 dB, CSNR$_d$=30 dB, and CSNR$_d$=40 dB.

the OPTA bound, and offer graceful degradation for channel mismatch situations. Thus, the proposed techniques are well suited for transmission of sound and video signals in mobile communication and broadcasting systems.

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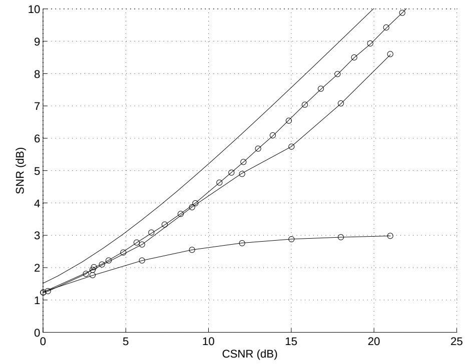

Figure 9. SNR vs. CSNR for $L = 2$, $N = 256$, $K = 1$. From top to bottom: OPTA, discrete amplitude system (PCCOVQ with optimization of Δ), PCCOVQ from [3], linear system [1] (performance results from [3]).

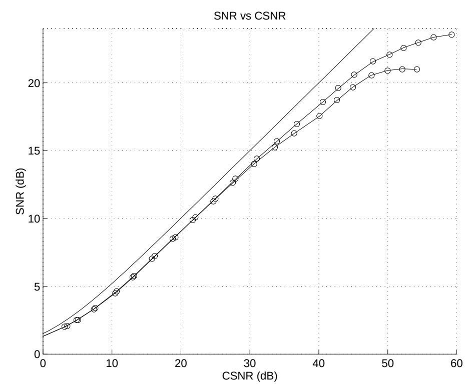

Figure 10. Comparison of discrete and continuous amplitude systems, $L = 2$, $N = 256$, $K = 1$. From top to bottom: OPTA, continuous amplitude system, discrete amplitude system (PCCOVQ).

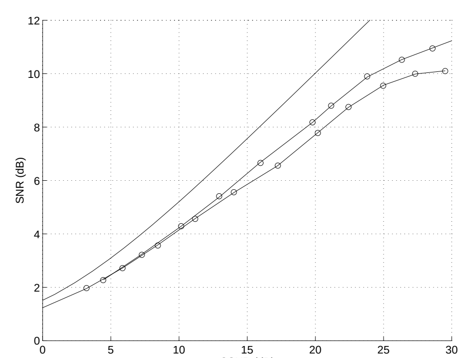

Figure 11. Comparison of discrete and continuous amplitude systems, $L = 4$, $N = 256$, $K = 2$. From top to bottom: OPTA, continuous amplitude system, discrete amplitude system (PCCOVQ).