USING FRACTIONAL CYCLIC MOMENTS OF CPFSK SIGNALS IN BLIND ADAPTIVE BEAMFORMING *

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ABSTRACT

This paper explains how fractional cyclic moments of CPFSK signals can be used in adaptive beamforming. It is shown that CPFSK signals have non-zero fractional cyclic moments because they generate spectral lines when they are raised to the inverse of its modulation index, which is generally a fractional number. To exploit this property, an optimization criterion is proposed to compute the antenna coefficients that maximize the output SINR. The resulting technique is blind because it does not need to know the transmitted symbols: only the carrier frequency, the symbol rate and the modulation index is required.

1. INTRODUCTION

Continuous Phase Frequency Shift Keying (CPFSK) is a nonlinear modulation method with memory widely used in wireless digital communication systems. The complex representation of a CPFSK signal, $x(t)$, is the following [1]

$$x(t) = A \exp\left\{ j \left(2\pi f_c t + 2\pi h \int_{-\infty}^{t} d(\tau) d\tau \right) \right\} \qquad (1)$$

where A is the signal amplitude, f_c is the carrier frequency, h is the modulation index and $d(t)$ is a PAM signal given by

$$d(t) = \sum_k I_k g(t - kT) \qquad (2)$$

where T is the symbol period, $g(t)$ is a rectangular pulse of duration T and amplitude $1/2T$ and the symbols I_k are the amplitudes which result of mapping digits of the information sequence to the amplitude levels $\{\pm 1, \pm 3..., \pm(M-1)\}$.

Similarly to most communication signals, $x(t)$ can be modeled as a cyclostationary random process. This fact can be succesfully used to select the coefficients of an antenna array in order to remove cochannel interferences. Cyclostationarity-based methods are capable of extracting a desired signal from the array output using only the information contained in the fluctuation frequencies of its statistical averages. The advantages of these methods are remarkable: they do not require the *a priori* knowledge of the desired signal steering vector [2], they do not need a reference signal [2] and they do not suffer from capture problems as the Constant Modulus (CM) beamformer [3].

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Existing approaches [4, 5, 6] make use of the periodicities of the desired signal autocorrelation function and, initially, they can be applied to extract CPFSK signals from an array output. However, these approaches are computationally very complex since they are based on solving generalized eigenvalues problems [4] or finding matrix pencils [5]. Even though simpler adaptive implementations have been proposed [4, 6], these latter fail to work when there are interferences with the same cyclostationarity properties as the desired signal and multipath propagation.

In this paper we describe a new technique for adaptive beamforming which uses the cyclostationary nature of CPFSK signals in a different way. The technique utilizes the fact that CPFSK signals generate spectral lines when they are raised to a fractional number which is the inverse of its modulation index. This property arises because certain *fractional* statistical moments of CPFSK signals are periodic functions of time or, equivalently, because CPFSK signals have non-zero *fractional cyclic moments* at certain frequencies. To exploit this property, a new optimization criterion involving Higher Order Statistics (HOS) is proposed to take advantage of the statistical independence between the desired and the unwanted (noise and interferences) signals. In addition, optimum weights can be computed using a simple stochastic gradient algorithm.

This paper is organized in five sections. Section 2 explains the spectral line generation property of CPFSK signals. Section 3 presents the optimization criterion that makes use of this property. In section 4 simulation results are presented. Finally, section 5 is devoted to the conclusions.

2. FRACTIONAL CYCLIC MOMENTS

A continuous-time signal $x(t)$ generates a spectral line with frequency α when it passes through the nonlinearity $(\cdot)^r$ if and only if the *r-th* order cyclic moment defined as

$$\begin{aligned} m_{rx}^{\alpha} &= \left\langle x^r(t) e^{-j2\pi\alpha t} \right\rangle \\ &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^r(t) e^{-j2\pi\alpha t}\, dt \end{aligned} \qquad (3)$$

exists and is nonzero [7]. The operator $\langle\cdot\rangle$ denotes the time average operation and r is a real number not necessarily integer. In this case, the Power Spectral Density (PSD) of $x^r(t)$ contains a spectral line at frequency α with an amplitude $|m_{rx}^{\alpha}|^2$.

The existence of these cyclic moments is a consequence of the cyclostationary nature of $x(t)$. It can be demonstrated that, under certain conditions, m_{rx}^{α} correspond to

the Fourier series coefficients of the statistical fractional moment

$$m_{rx}(t) = E[x^r(t)] = \int_{-\infty}^{\infty} x^r(t) f(x,t)\, dx \qquad (4)$$

where $E[\cdot]$ denotes the statistical average operation and $f(x,t)$ is the first order density function of $x(t)$.

Next, we are going to show that a CPFSK signal contains nonzero fractional cyclic moments of order $r = 1/h$ where h is the modulation index. This is equivalent to show that a CPFSK signal generates spectral lines when it is raised to the fractional number $r = 1/h$. Let us start noting that the PSD of a M-ary CPFSK signal contains M spectral lines when its modulation index is equal to one [1]. In general, a CPFSK signal, $x(t)$, will have an amplitude A, a carrier frequency f_c and a modulation index h different from one. However, note that $x^{1/h}(t)$ is also a CPFSK signal but with an amplitude $A^{1/h}$, a carrier frequency f_c/h and a modulation index equal to one. Therefore, the PSD of $x^{1/h}(t)$ contains M spectral lines. From the PSD calculations presented in [1], it is straightforward to show that the frequency of these spectral lines is given by

$$f_i = \frac{f_c}{h} + \frac{M-1-2i}{2T} \qquad i = 0, \cdots, M-1 \qquad (5)$$

Moreover, the amplitude of these spectral lines can also be computed from the results in [1]. These are the values of the fractional cyclic moments

$$|m_{1/h\ x}^{\alpha=f_i}|^2 = \left(\frac{1}{M} A^{1/h}\right)^2 \qquad i = 0, \cdots, M-1 \qquad (6)$$

Figure 1 and 2 show the PSD of a quaternary CPFSK signal $x(t)$ before and after being raised to $1/h$. Observe that four spectral lines appear in the PSD of $x^{1/h}(t)$.

Fractional moments are not the only statistical averages that are periodic functions of time for a CPFSK signal. It is well-known that the autocorrelation function is also a periodic function of time with a fundamental period equal to the symbol period, T. This means that the PSD of $z(t) = x(t)x^*(t+\tau)$ contains spectral lines at integer multiples of the symbol rate $f_s = 1/T$. However, it is important to note that these spectral lines are weaker than those in $x^{1/h}(t)$ as can be seen in figure 2.

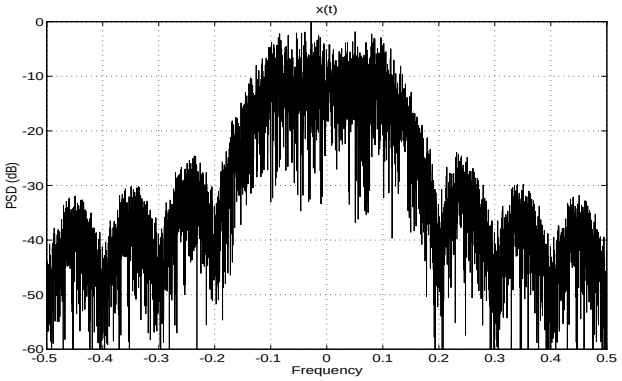

Figure 1. PSD of a quaternary CPFSK signal, $x(t)$, with $h = 0.75$)

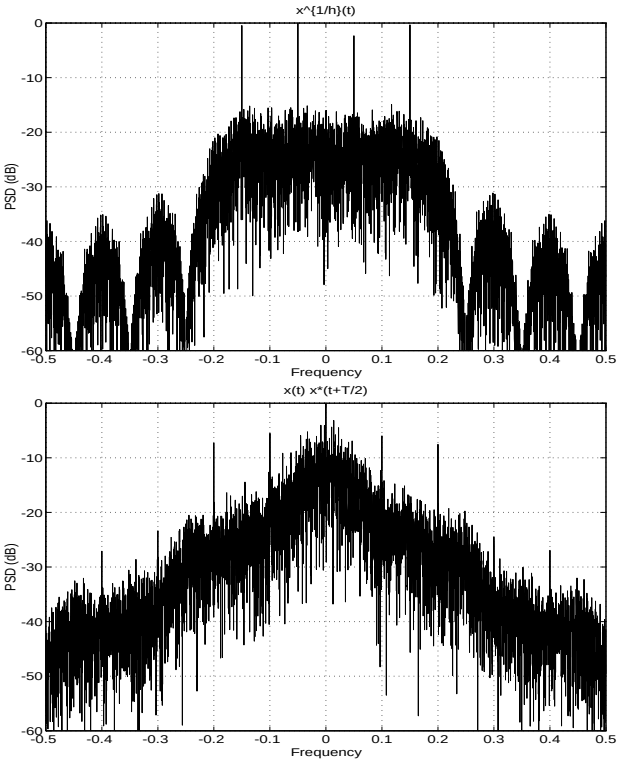

Figure 2. PSD of $x^{1/h}(t)$ and $z(t) = x(t)x^*(t+T/2)$

3. OPTIMIZATION CRITERION

To exploit the above property in adaptive beamforming, we propose that the coefficients $\mathbf{w}$ of an antenna array be selected in order to minimize the following cost function

$$J = < |e^{j2\pi\alpha t} - y^r(t)|^2 > \qquad (7)$$

where $y(t) = \mathbf{w}^H \mathbf{x}(t)$ is the array output, r is a fractional number equal to the inverse of the desired signal modulation index and α is one of the frequencies f_i where the desired signal contains a non-zero r-th order cyclic moment. This cost funcion is the Mean Squared Error (MSE) between a complex exponential and the array output after the non-linearity $(\cdot)^r$. This optimization problem is similar to that proposed in [8] for linear digital modulations (ASK, PSK and QAM) where integer values of r were used. In [8] it was demonstrated that the beamformer performance at the minima when $r = 2$ correspond to the optimal extraction of the desired signal in the sense that the Signal to Interference and Noise Ratio (SINR) is maximized. Simulations presented in the following section show that this result also holds true for fractional values of r.

Another advantage of our approach is that optimum weights can be computed using a simple adaptive algorithm. A reasonable way to compute the optimum coefficients $\mathbf{w}$ is the steepest descent method

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu \nabla_{\mathbf{w}} J \qquad (8)$$

where μ is the algorithm step size and $\nabla_{\mathbf{w}} J$ represents the complex gradient of J with respect to $\mathbf{w}$. In our case $\nabla_{\mathbf{w}} J$ is

$$\nabla_{\mathbf{w}} J = -\frac{1}{h} < \mathrm{e}^*(n) y^{(\frac{1}{h}-1)}(n) \mathbf{x}(n) > \qquad (9)$$

where $\mathbf{x}(n)$ is the array input and $\mathrm{e}(n) = e^{j2\pi\alpha n} - y^{1/h}(n)$ is the error signal whose variance we want to minimize. Substituing the time average in (9) by its instantaneous estimate we obtain the following stochastic gradient algorithm

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu}{h}\mathrm{e}^*(n)y^{\frac{1}{h}-1}(n)\mathbf{x}(n) \qquad (10)$$

It is interesting to note that implementation of this algorithm only requires the modulation index h and the frequency α. Since from (5) the value of α depends on h, f_c and T, only these three parameters are required to extract the desired signal. Note that this information is always available at the receiver since it is necessary to perform demodulation. Therefore, this algorithm can be considered blind because the knowledge of the transmitted symbols is not required.

Implementation of (10) typically requires raising a complex number to a fractional exponent. This operation is carried out as follows

$$z = \rho(z)e^{jarg(z)} \Rightarrow z^r = \rho^r(z)e^{j\ arg(z)\ r} \qquad (11)$$

where r is any real number. It should be mentioned that when computing $arg(z)$ we cannot use a conventional arctangent subroutine. This type of subroutines give us the principal value of a complex number, $ARG(z)$, which is within the interval $(-\pi, \pi]$. The relationship between the true value of the phase, $arg(z)$, and its principal value is

$$arg(z) = ARG(z) + 2\pi k \qquad (12)$$

where k is an integer. However note that for an arbitrary real number r

$$e^{j\ arg(z)\ r} \neq e^{j\ ARG(z)\ r}\quad e^{j2\pi k\ r} \qquad (13)$$

Therefore, the principal value of the phase cannot be used. To compute $arg(z)$ it is necessary an unwrapping phase algorithm such as the one described in [9].

The cost function that we are minimizing is not a quadratic form of $\mathbf{w}$. This raises the question of whether there are undesirable stationary points that may impair the convergence of the adaptive algorithm. The analysis in [8] shows that for $r = 2$ the cost function (7) is free of undesirable minima for the environments typically found in communications. Analysis for fractional values of r turns out to be rather involved and has not been performed yet. However, simulations only showed the existence of undesirable minima in multipath scenarios.

4. SIMULATION RESULTS

Computer simulations were carried out to illustrate the performance of the proposed method. We considered a 10-elements uniform linear array whose spacing is half wavelength. The array input signals are sampled at a rate five times faster than the symbol rate of the desired signal.

In the first computer experiment we considered a simple environment with one binary CPFSK signal and Gaussian noise. The input SNR is 3 dB and the Direction Of Arrival (DOA) is $0°$. The algorithm step-size is 10^{-4}. Figure 3 plots the time evolution of the output SNR for two different values of the modulation index. It can be seen that the algorithm converges to the maximum SNR solution in less than 100 symbols. Figure 3 also plots the SNR for the adaptive implementation of the Cross-SCORE method described in [4]. A delay $\tau = T$ was considered since this

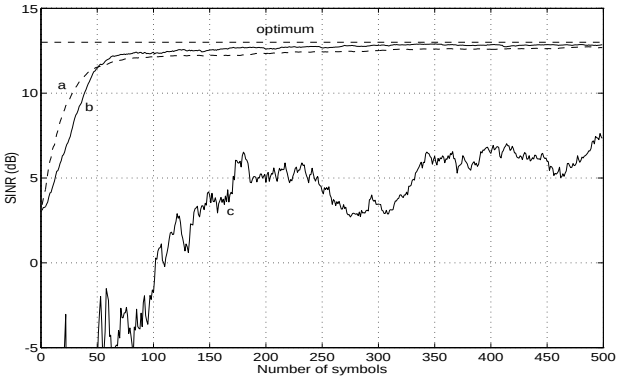

Figure 3. Time evolution of output SNR: a) Proposed approach with $h = 2/3$, b) Proposed approach with $h = 2/5$ c) Cross-SCORE.

Signal	Symbol Rate	Input SNR	Direction
Desired	1/5	5 dB	$0°$
Interf # 1	1/8	10 dB	$30°$
Interf # 2	1/10	15 dB	$-30°$

Table 1. Interference environment parameters

corresponds to the maximum amplitude of the spectral lines in $x(t)x^*(t+T)$. Rate of convergence is slower because spectral lines in $x(t)x^*(t+T)$ are weaker than in $x^{1/h}(t)$.

In the second computer experiment we considered three binary CPFSK signals with the same modulation index ($h = 2/3$) arriving at the antenna. The parameters of the signals can be seen in table 1. Figures 4 and 5 plot the output SINR for the proposed algorithm ($\mu = 3 \times 10^{-6}$) and Cross-SCORE. In figure 4 the symbol rates are those in table 1 and interferences have cyclostationary properties different from the desired signal. Again, the maximum SINR is achieved in less than 100 symbols while convergence of the Cross-SCORE is slower. In figure 5 the three CPFSK signals have the same symbol rate and, as a consequence, interferences generate spectral lines at the same frequencies as the desired signal. In this case, the proposed approach is still able to extract the desired signal whereas Cross-SCORE fail to work. This is because our approach makes use of Higher Order Statistics and is able to exploit the statistical independence between the desired signal and the interferences.

In the third simulation experiment a multipath environment was considered. The desired signal has an input SNR of 30 dB and arrives at the antenna through four different paths whose features are indicated in table 2. In this environment performance is more adequately evaluated in terms of the MSE between the transmitted signal and the array output. Figure 6 plots a comparison between the MSE achieved with the proposed approach and the conventional LMS algorithm. It can be seen that our method is not capable of achieving the minimum MSE and therefore it does not combine the multipath signals in an optimal way.

It is important to note that approaches to blind adaptive beamforming similar to ours, like the CM beamformer [10], perform like a *spatial equalization* in multipath scenarios and achieve the Minimum MSE solution [8, 10]. However, these works always assume linear modulations without memory where samples of the desired signal taken at

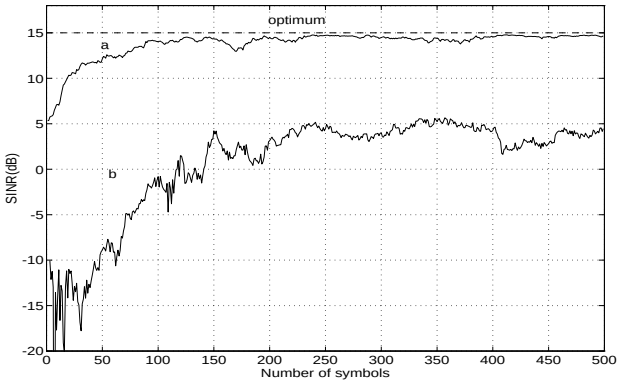

Figure 4. Interferences with different symbol rate: a) Proposed approach, b) Cross-SCORE.

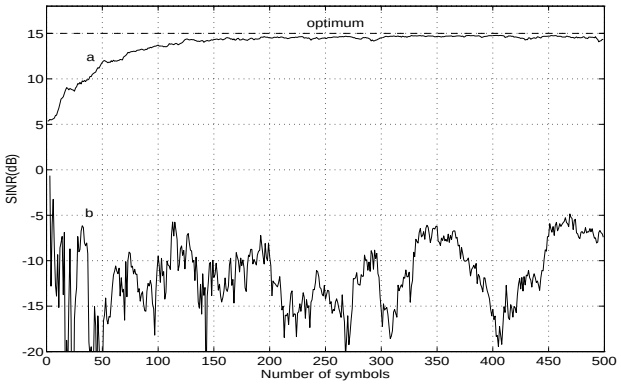

Figure 5. Interferences with the same symbol rate as the desired signal: a) Proposed approach with $h = 2/3$, b) Cross-SCORE.

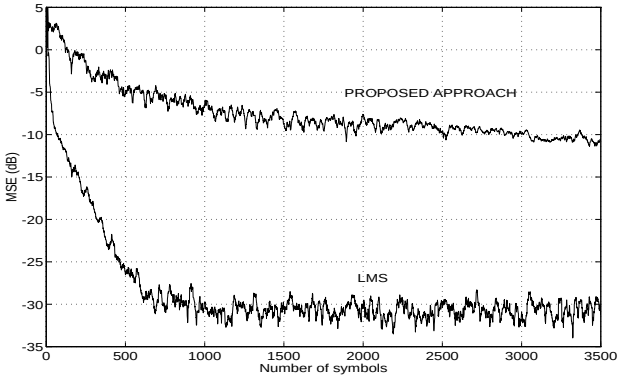

Figure 6. Time evolution of the MSE in the multipath scenario.

Multipath	Direction	Delay	Attenuation
# 1	0°	0	1
# 2	20°	$4T$	$1.1e^{-j0.56\pi}$
# 3	-40°	T	$0.40e^{j0.42\pi}$
# 4	-20°	$4T$	$0.86e^{j0.11\pi}$

Table 2. Multipath environment parameters

the symbol rate are statistically independent. This hypothesis is not satisfied in our case since CPFSK is a modulation method with memory. We conjecture that this is the reason why our method presents undesirable performance in multipath scenarios. In fact, simulations show that the CM beamformer does not achieve the Minimum MSE for CPFSK signals in this type of environments.

5. CONCLUSIONS

In this paper we have investigated how CPFSK signals generate spectral lines when they are raised to the inverse of its modulation index. This is because CPFSK signals contain non-zero cyclic moments. We have also proposed an optimization criterion to successfully use this property in adaptive beamforming. The analysis of the stationary points of the proposed cost function is very involved and simulation results were presented instead. Simulations revealed that the algorithm maximizes the output SINR in scenarios where Gaussian noise and statistical independent interferences are present even when there are interferences with the same cyclostationary properties as the desired signal. No undesirable minima were observed in these environments. Unfortunately, simulations showed that the method does not work adequately in multipath scenarios since undesirable minima exist in this case.

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