BLIND SIGNAL EXTRACTION BASED ON HIGHER-ORDER CYCLOSTATIONARITY PROPERTIES

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ABSTRACT

The paper deals with signal extraction performed by processing data received by an array of sensors. The proposed method is blind, i.e., it does not require any *a priori* knowledge of directional information associated with the signals of interest (SOI's). Such information is obtained directly from the received data by exploiting the higher-order cyclostationarity (HOCS) properties exhibited by most communication signals. The proposed method is inherently tolerant to both possibly non-stationary Gaussian disturbances as well as non-Gaussian interferences not exhibiting the same HOCS properties presented by the SOI's. Simulation results confirm the effectiveness of the method when it operates in severely degraded disturbance environments.

1. INTRODUCTION

Many array processing techniques for narrowband-signal extraction are based on linear spatial filtering performed on the received signal vector [1]. The weight vector of the spatial filter, optimal in several senses, can be evaluated if an accurate knowledge of both the array manifold and the direction-of-arrival (DOA) of each signal of interest (SOI) is available, namely, if any SOI steering vector is known. Indeed, small errors in both array manifold and DOA can severely degrade the performances of such methods, which prevents their application in unknown or time-varying environments and/or in the presence of relative motion between sources and sensors. To overcome this problem, in recent years a number of *blind* (i.e, no knowledge of both the array manifold and DOA's of SOI's is assumed) spatial filtering methods have been proposed. They are usually based on direct estimation of the steering vector of any SOI from the received data. This is, in general, a difficult task, owing to the presence of the interfering signals, unless a certain *a priori* knowledge on the SOI's is available. More precisely, blind methods usually assume that any SOI exhibits some (known) property that is not shared by both the other SOI's and disturbance signals.

A class [2] of blind techniques exploit the higher-order statistics (HOS) of the received signals to discriminate non-Gaussian SOI's from Gaussian interferences. More specifically, the methods are cumulant-based and, therefore, they are ideally immune to Gaussian disturbances. Consequently, they cannot assure satisfactory performances when non-Gaussian interferers are present, which is a common situation as, for example, in multiple access communications.

Another class of blind techniques [3] exploit the second-order cyclostationarity (SOCS) properties, i.e, the cyclic properties of the second-order statistics, exhibited by most man-made communication signals to discriminate the SOI's from interfering and noise signals. Of course, these techniques cannot operate when the SOCS properties of the SOI's are shared by at least one interfering signal or when the SOI's do not exhibit any SOCS property at all (e.g., they have balanced QAM format).

More recently, a higher-order cyclostationarity (HOCS) based blind beamforming technique has been proposed [4]. The method, in the following referred to as the generalized constant modulus (GCM) method, since it extends in some sense the constant modulus technique, is based on the property of cyclostationary signals to generate sine waves with frequencies α (called *cycle frequencies*) when they pass through certain nonlinear transformations. Although the estimation of the weights of the spatial filter is based on higher-order statistics, the GCM method is not immune to Gaussian disturbances since, unlike the methods proposed in [2], the estimation is based on moments rather than cumulants. Moreover, since the cost function is a nonquadratic form of the weight vector, it is not possible to ensure that a single minimum exists. Specifically, the technique suffers from a capture problem similar to that presented by previously proposed methods, whenever the presence of multiple statistically independent signals does not allow to associate one-to-one a spectral line to each SOI. In these cases, the method is able to cancel signals showing the same cyclostationarity properties of the considered SOI only if the algorithm can be forced towards the desired minimum. This can be accomplished by choosing an adequate set of initial conditions, which requires, however, a knowledge, even if inaccurate, of the array manifold and DOA's of the SOI's, rendering so the technique only virtually blind.

Based on the previous considerations, we develop here a new technique that exploits as [4] both the cyclostationarity and higher-order statistical properties of the SOI's although in a more effective way, so as to avoid the intrinsic shortcomings of the GCM method. The key idea of the proposed method is the observation that the nth-order cyclic cumulant [5] of the received signals is proportional to the steering vector associated with the SOI to be estimated, provided that such SOI, assumed non-Gaussian, exhibits nth-order ($n \geq 3$) cyclostationarity with a cycle frequency at which any other SOI or non-Gaussian interference does not exhibit nth-order cyclostationarity. Therefore, the method performs an estimation of the SOI steering vectors on the basis of estimates of the nth-order cyclic cumulants, assuring so inherent tolerance to both Gaussian disturbances as well as non-Gaussian signals not exhibiting nth-order cyclostationarity at the same cycle frequency.

2. BACKGROUND ON HOCS

Higher-order cyclostationarity is concerned with the generation of finite-strength additive sine waves from multiple time-series by nth-order ($n \geq 3$) nonlinear transformations.

Given M complex time series $z_1(t), z_2(t), \ldots, z_M(t)$, let us consider the *nth-order lag product*

$$L_{\mathbf{z}}(t, \tau)_n \triangleq \prod_{j=1}^{n} z_{i_j}^{(*)_j}(t + \tau_j), \tag{1}$$

where $\tau = [\tau_1, \tau_2, \ldots, \tau_n]$, the superscript $(*)_j$ denotes optional conjugation, $\mathbf{z}(t) = [z_{i_1}^{(*)_1}(t), z_{i_2}^{(*)_2}(t), \ldots, z_{i_n}^{(*)_n}(t)]$, and i_j belongs to the index set $\{1, 2, \ldots, M\}$. It can be shown [5] that generation of sine waves from $\mathbf{z}(t)$ through arbitrary homogeneous nth-order nonlinear transformations is completely characterized by the function

$$R_{\mathbf{z}}^{\alpha}(\tau)_n \triangleq \langle L_{\mathbf{z}}(t, \tau)_n e^{-j2\pi\alpha t} \rangle, \tag{2}$$

which represents the magnitude and phase of the sine wave with frequency α contained in the nth-order lag-product. For $\alpha \neq 0$, the function (2) is called the *nth-order cyclic temporal moment function* (CTMF) [5]. Accordingly, the components of the time series vector $\mathbf{z}(t)$ are said to exhibit *joint nth-order (wide-sense) cyclostationarity* [5] with cycle frequency $\alpha \neq 0$ if $R_{\mathbf{z}}^{\alpha}(\tau)_n$ is not identically zero as a function of the lag delay vector τ.

If the components of the time series vector $\mathbf{z}(t)$ exhibit also joint lower-order cyclostationarity, namely, there exist factors of the nth-order lag product (1) containing sine waves, then the sine wave characterized by (2) can contain sinusoidal components originated from multiplication of sine waves associated with lower-order cyclostationarities. In such a case, the nth-order sine wave is called *impure* [5]. *Pure* sine waves, therefore, can be obtained by subtracting from the CTMF all contributions from lower-order sine waves. It is shown in [5] that, after removal, the magnitude and phase of the pure sine wave with frequency β contained in (1) is given by the *nth-order cyclic temporal cumulant function* (CTCF) [5]

$$C_{\mathbf{z}}^{\beta}(\tau) \triangleq \sum_{P_n} \left[(-1)^{p-1}(p-1)! \sum_{\alpha^T \mathbf{1} = \beta} \prod_{j=1}^{p} R_{\mathbf{z}}^{\alpha_j}(\tau_{\nu_j})_{n_j} \right]. \tag{3}$$

In (3), $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_p]^T$ and $\mathbf{1} = [1, 1, \ldots, 1]^T$ are p-column vectors, the sum is over all distinct partitions P_n, such as $\{\nu_k\}_{k=1}^{p}$, of the index set $\{1, 2, \ldots, n\}$, p is the number of elements in a partition ($1 \leq p \leq n$), and, finally, n_j is the cardinality of ν_j.

Motivation for using the CTCF rather than the CTMF is twofold: first, the CTCF of a sum of independent time series is the sum of individual CTCF's; second, the CTCF of a Gaussian time series for $n \geq 3$ is identically zero. As a consequence, signal processing methods based on CTCF exhibit inherent *signal selectivity* not only with respect to non-Gaussian signals with different cycle frequencies, but also with respect to Gaussian signals possibly having the same cycle frequencies.

For a thorough discussion of theory and applications of HOCS, the reader is referred to [5, 6].

3. THE PROPOSED METHOD

Consider a passive array consisting of M sensors with arbitrary sensor response characteristics and locations. Assume that D non-Gaussian independent SOI's impinge on the array in the presence of J possibly non-Gaussian interfering signals, independent of the SOI's. The radiating sources are located in the far-field of the array so that a planar wavefront approximation is possible. Moreover, sensors and sources are assumed coplanar, so that the position of each source is described by a single parameter, i.e., the DOA of the planewave.

Under the narrowband assumption, the received analytic signal at the ith sensor can be expressed as

$$x_i(t) \approx \sum_{k=1}^{D} a_i(\theta_{s_k})\, s_k(t) + \sum_{m=1}^{J} a_i(\theta_{u_m})\, u_m(t) + n_i(t), \tag{4}$$

where $s_k(t)$ is the kth zero-mean analytic SOI impinging from DOA θ_{s_k}, $u_m(t)$ is the mth zero-mean analytic interference signal impinging from DOA θ_{u_m}, $a_i(\cdot)$ is the ith component of the steering vector $\mathbf{a}(\cdot)$, and $n_i(t)$ is the zero-mean analytic Gaussian noise at the ith sensor.

The most common algorithms for signal extraction are based on linear spatial filtering performed on the received signal vector $\mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_M(t)]^T$:

$$\widehat{s}_k(t) = \mathbf{w}_k^H\, \mathbf{x}(t), \tag{5}$$

where $\mathbf{w}_k$ is a complex weight vector aimed at enhancing the kth SOI while attempting to minimize the contribution due to the other $k-1$ SOI's, interfering signals, and noise. In (5), the superscript H denotes Hermitian (conjugate transpose) operation.

A solution for $\mathbf{w}_k$ that is optimal in several senses (e.g., maximum signal-to-interference-plus-noise-ratio (SINR) and minimum mean-square error) is

$$\mathbf{w}_k = \gamma\, \mathbf{R}_{\mathbf{XX}}^{-1}\, \mathbf{a}(\theta_{s_k}) \tag{6}$$

where $\mathbf{R}_{\mathbf{XX}} \triangleq \langle \mathbf{x}(t)\, \mathbf{x}^H(t) \rangle$ and γ is an appropriate scale factor. Methods based on (6) require accurate knowledge of the steering vector of the kth SOI.

The proposed method performs estimation of $\mathbf{a}(\theta_{s_k})$ on the basis of the nth-order cyclic cumulant of the received signals evaluated assuming a fixed sensor (e.g., the first) as a reference. More precisely, for a given conjugation configuration, consider the M n–dimensional vectors $\mathbf{x}_i(t) = [x_1^{(*)_1}(t), x_1^{(*)_2}(t), \ldots, x_1^{(*)_{n-1}}(t), x_i^{(*)_n}(t)]$ ($i = 1, 2, \ldots, M$) and let $C_{\mathbf{X}_i}^{\beta}(\tau)_n$ denote their nth-order CTCF's. Consider, then, the M–column vector $\mathbf{C}_{\mathbf{X}}^{\beta}(\tau)_n$ whose ith element is the CTCF $C_{\mathbf{X}_i}^{\beta}(\tau)_n$.

Under the assumption that the kth SOI exhibits nth-order ($n \geq 3$) cyclostationarity with cycle frequency β whereas the other SOI's and non-Gaussian interferences do not, by taking into account (4) $\mathbf{C}^{\beta}_{\mathbf{X}}(\tau)_n$ can be expressed as

$$\mathbf{C}^{\beta}_{\mathbf{X}}(\tau)_n = \left[C^{\beta}_{\mathbf{s}_k}(\tau)_n \prod_{j=1}^{n-1} a_1^{(*)_j}(\theta_{s_k}) \right] \mathbf{a}(\theta_{s_k}) \, , \qquad (7)$$

where $C^{\beta}_{\mathbf{s}_k}(\tau)_n$ is the nth-order CTCF associated with the kth SOI, i.e., it is obtained by (3) with $\mathrm{s}_k(t) = [s_k^{(*)_1}(t), s_k^{(*)_2}(t), \ldots, s_k^{(*)_n}(t)]$. Note that possibly cyclostationary Gaussian disturbances do not contribute to (7) regardless of the value of β as long as $n \geq 3$.

Equation (7) shows that $\mathbf{C}^{\beta}_{\mathbf{X}}(\tau)_n$ turns out to be proportional to the steering vector $\mathbf{a}(\theta_{s_k})$, provided that the optional conjugations $(*)_j$ and the value of τ are properly selected according to the modulation format of the kth SOI to assure that the CTCF $C^{\beta}_{\mathbf{s}_k}(\tau)_n$ be nonzero. Hence, $\mathbf{C}^{\beta}_{\mathbf{X}}(\tau)_n$ can be used instead of $\mathbf{a}(\theta_{s_k})$ in (6) with the scale factor γ adjusted in order to meet the prespecified criterion. In practice, estimates of $\mathbf{R_{XX}}$ and $\mathbf{C}^{\beta}_{\mathbf{X}}(\tau)_n$ obtained from a finite number of snapshots are available, which can assure nearly optimal performances provided that the sample size is sufficiently large. It is worthwhile to note that it is always desirable to exploit the smallest possible order n of cyclostationarity since, for a fixed data length, both computational complexity and inaccuracy of the estimate of the CTCF increases with n.

The proposed method can be considered as the extension of the CUM method [2] to the case of cyclostationary SOI's and, hence, it will be referred to as the *cyclic CUM* (C-CUM) method. Unlike the CUM method, the C-CUM method can reject not only Gaussian signals, but also non-Gaussian signals that do not share the same HOCS properties of the SOI under consideration.

Note that the assumption of independent SOI's, even though adopted in the previous derivations for the sake of simplicity, is not strictly needed to assure the effectiveness of the proposed method. Indeed, model (4) holds also in the case of multipath propagation, with each SOI steering vector substistuted by a *generalized* steering vector obtained as a linear combination of steering vectors associated with each propagation path. In this case, optimal filtering (6) for extraction of the kth SOI must rely on this generalized steering vector, whose estimation can be reliably performed by means of the C-CUM method.

Finally, note that modifications of the C-CUM method to improve its reliability and robustness as well as to accomodate wideband SOI's and, moreover, to allow for efficient adaptive implementations are currently under examination.

4. SIMULATION RESULTS

In this section we present various experiments to assess the performance of the proposed method and to compare them with those of the most competitive blind methods mentioned in the Introduction. More specifically, the methods considered are the SOCS-based LS-SCORE algorithm [3] (in the following, simply referred to as SCORE), the HOS-based CUM method [2], and the GCM technique [4]. In all

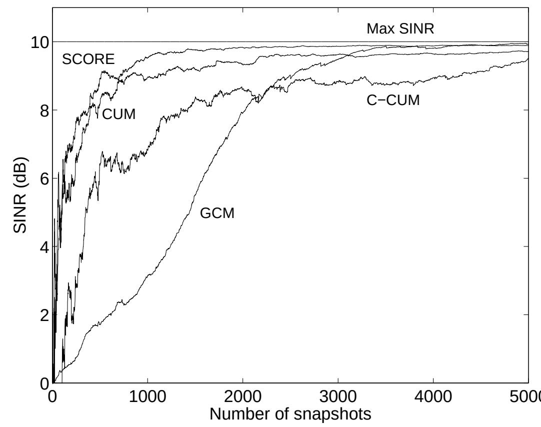

Figure 1. SINR versus number of snapshots in the first experiment (extraction of a BPSK SOI corrupted by Gaussian noise).

of the experiments a 10-element linearly equispaced aperture whose spacing is half wavelength is considered. All the sensors are assumed to be ideal and having omnidirectional radiation pattern. The array output is converted from the center frequency f_0 of the receiver band to baseband and quadrature sampled with a rate f_S.

The aim of the first experiment is to test the considered methods in operative conditions where all of them are expected to work satisfactorily. To this end, a BPSK SOI arriving from DOA 0^o and corrupted only by spatially and temporally white Gaussian noise is considered. The BPSK signal has a rectangular keying envelope, $0.2 f_S$ baud-rate, $f_c = 0.1 f_S$ carrier offset (relative to frequency f_0), and 0-dB signal-to-noise-ratio (SNR). In this environment, the SCORE method exploits second-order conjugate cyclostationarity of the SOI with cycle frequency $2f_c$, whereas the GCM technique works with cycle frequency $4f_c$ and parameters $p = 4$ and $\mu = 8.0 \cdot 10^{-4}$ (see [4]). Moreover, all the components of the GCM initial weight vector are set to zero except for the first set at 0.2. Finally, the proposed C-CUM method considers the nth-order CTCF vector (7) with $n = 4$, $\tau = \mathbf{0}$, $\beta = 4f_c$, and no conjugation in the lag products.

Figure 1 reports results in terms of output SINR versus the number of snapshots for all the considered blind methods. For comparison purposes, the results of the non-blind optimal (maximum SINR) method are also reported in the figure and, moreover, values of SINR in correspondence of the maximum considered value (i.e., 5000) of the sample size are summarized in Table 1.

Max SINR	SCORE	GCM	CUM	C-CUM
10.00	9.75	9.83	9.37	8.94

Table 1. Values of SINR (dB) for 5000 snapshots in the first experiment.

Among the blind techniques, the SCORE method, as expected, assures the best overall performance in terms of convergence speed and estimation accuracy, since it exploits the statistics of lowest order (second order). The proposed method, as well as the other techniques exploiting (cyclic or

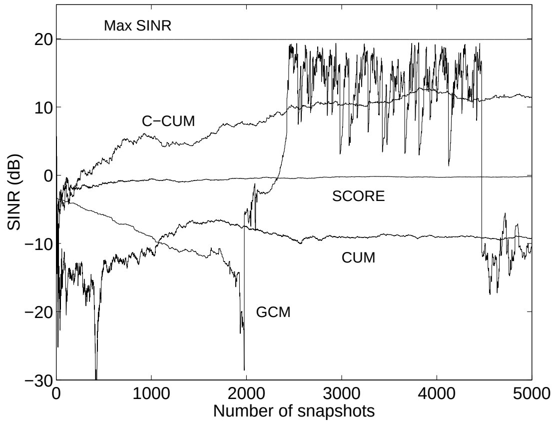

Figure 2. SINR versus number of snapshots in the second experiment (extraction of a QPSK SOI corrupted by both Gaussian and non-Gaussian disturbances).

not) higher-order statistics, is not competitive with second-order-based methods, mainly because the operative environment does not require the adoption of higher-order properties to discriminate the SOI. Finally, note that, although the results of the GCM method seem quite satisfactory at least in terms of estimation accuracy, such a method performed poorly in a wide range of simulations carried out with different choices of the initialization weight vector.

The second experiment is aimed at showing the effectiveness of the C-CUM method in operative conditions where the other techniques cannot operate satisfactorily. In this case, a balanced QPSK SOI corrupted by the presence of two interfering signals plus temporally and spatially white Gaussian background noise is considered. The QPSK SOI has a raised-cosine keying envelope, $0.2f_S$ baud-rate, $f_c = 0.05f_S$ carrier offset, 10-dB SNR, and DOA 0^o. The first interferer is a Gaussian AM signal with $0.05f_S$ carrier offset, 10-dB SNR, and DOA 40^o. The second interferer is a BPSK signal with raised-cosine keying envelope, $0.2f_S$ baud-rate, $0.09f_S$ carrier offset, 10-dB SNR, and DOA -30^o.

As regards the SCORE method, note that the considered QPSK SOI, due to its balanced modulation format, exhibits SOCS properties related only to the baud rate. Hence, an appropriate choice of the cycle frequency for this method is the SOI baud rate $0.2f_S$. Instead, since, as it can be easily shown, the SOI exhibits fourth-order cyclostationarity properties related to the carrier frequency, the GCM technique works with cycle frequency $4f_c$, parameters $p = 4$ and $\mu = 4.0 \cdot 10^{-4}$, and an initial weight vector whose elements are all zero except for the first set at 0.015, whereas C-CUM operates with $n = 4$, $\tau = \mathbf{0}$, $\beta = 4f_c$, and no conjugation in the lag products.

Figure 2 clearly shows that only the proposed method assures satisfactory performances in this environment. Indeed, as expected, the SCORE technique performs very poorly, according with the fact that the SOI and the BPSK interferer share the same SOCS properties. Moreover, the CUM method does not provide an accurate estimate of the SOI, since the steering vector estimate is affected by

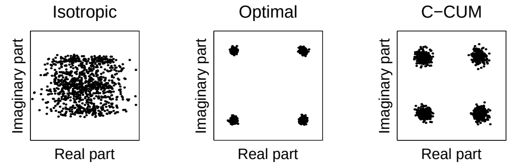

Figure 3. Symbol constellation diagrams in the second experiment (extraction of a QPSK SOI corrupted by both Gaussian and non-Gaussian disturbances).

the presence of the non-Gaussian (i.e., the BPSK) interferer. As regards the GCM method, in the reported experiment as well as in a wide range of simulations carried out with different choices of the initialization, it has not assured reliable convergence to the maximum SINR solution, being dominated by the interference capture effect. Finally, note that, although the proposed method clearly outperforms all the blind considered techniques, its SINR performances remain significantly far from the upper bound given by the maximum-SINR processor. Such performance degradation is due to the fact that, in this severely degraded environment, the considered values of sample size are not large enough to assure very accurate estimates of the HOCS statistics required by the method. However, in spite of this significant difference between the optimal and actual SINR performances, the effectiveness of the proposed method is corroborated by results of Fig.3, which reports output-signal constellation diagrams obtained using the spatially isotropic filter, the optimal spatial filter, and the C-CUM-based spatial filter. It is clear, then, that the proposed method can restore the QPSK constellation of the SOI nearly as well as the optimal filter.

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