LINEAR CONSTRAINTS IN PRE-DOPPLER STAP PROCESSING

Lloyd J. Griffiths

University of Colorado
Department of Electrical and Computer Engineering
Campus Box 425
Boulder, CO 80309-0425

ABSTRACT

This paper addresses the issue of employing space-time adaptive processing (STAP) prior to doppler filtering in radar systems. When STAP beamformer processing is applied to spatial/temporal samples that include successive radar pulses, the adaptive weights can cause modulation (spreading) of the desired target doppler. In this paper, a linearly constrained adaptive beamformer is proposed that ensures pre-doppler adaptive processing will not degrade desired signal coherence. A formulation of the processor is presented and its properties described. This formulation is first given for the case of slow-time STAP and is then extended to include processors which use both fast and slow-time samples. Examples showing the application of the proposed structure to recorded data are used to illustrate its performance. Comparisons are made with adaptive systems that do not employ constraints to illustrate the advantages of the proposed system. An extension to the full STAP system which employ time taps in both range and pulse number is described.

1. INTRODUCTION

The data recorded at the antenna array element outputs (after demodulation) during one coherent dwell in a radar system can be represented by a data cube containing $N_I = KNM$ samples. In this representation, K is the number of antenna receiving elements, M is the number of radar pulses transmitted during the coherent dwell, and N represents the number of range samples recorded during each pulse. In conventional (non-adaptive) radar processing, a total of $\tilde{K} \geq K$ output beams are formed by procedes by applying $\tilde{K}$ different steering vectors, $V(\theta_k)$, to the element outputs. Target Doppler information is obtained by performing a $\tilde{M}$-point FFT operation across the M pules. Thus, the output (processed) radar data cube containts $N_O = \tilde{K}N\tilde{M}$ samples.

In adaptive spatial-only processing, K-dimensional adaptive weights $W(\theta_k)$ are used in place of the fixed steering vectors $V(\theta_k)$ to form the output beams. The data covariance matrix employed in computing the adaptive weights has dimension K. Space-Time Adaptive Processing (STAP), utilizes additional weights which are distributed in either the range time dimension (fast time), the radar pulse dimension (slow time), or both. Figure 1 illustrates the three possible STAP implementations with fast-time STAP proceding in the horizontal direction and slow-time adaptation downward from the input data cube. Full STAP, with samples in both time dimensions, is illustrated by the grey diagonal line in this figure. Note that an FFT operation is also required as part of this processing.

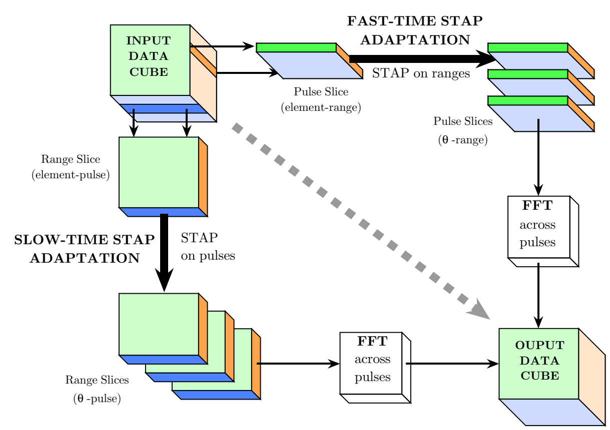

Figure 1. Fast- and slow-time STAP processing implementations.

Jim Ward [1] has provided an excellent description of STAP processing methodolgies, including procedures for computing the required covariance matrices. His work includes a discussion stating that application of adaptive processing prior to the doppler FFT operation results in modulation of the desired target signal (see [1], page 105). This modulation has been termed Doppler spreading and can lead to a performance degredation of the slow-time STAP system. The purpose of the present paper is to illustrate that linear constraints can be employed with slow-time STAP to ensure that Doppler spreading does not occur. In a previous publication, [2], the author described a procedure for using multiple linear constraints with fast-time STAP to ensure that the temporal properties of the desired target waveform are preserved.

2. MATHEMATICAL FRAMEWORK

The snapshot vector observed at the antenna element outputs during the m^{th} pulse and at the n^{th} range sample is denoted by $X_m(n)$. A total of M pulses are collected during each coherent radar dwell. For the case of slow-time STAP, the total snapshot vector which serves as input to the adaptive processor is $\mathbf{X}_l(n)$ consisting of $L \leq M$ successive pulse samples,

$$\mathbf{X}_l^\dagger(n) \;=\; \left[X_l^\dagger(n) X_{l-1}^\dagger(n) \cdots X_{l-L+1}^\dagger(n)\right] \;. \tag{1}$$

This vector has KL total samples and the corresponding output sample for beam direction θ_k and the l^{th} radar pulse, $y_l(n, \theta_k)$, is computed as the inner product of the weight vector $\mathbf{W}_l(\theta_k)$ and the data vector $\mathbf{X}_l(n)$,

$$y_l(n, \theta_k) \;=\; \mathbf{W}_l^\dagger(\theta_k)\mathbf{X}_l(n) \;. \tag{2}$$

Weight vectors covering a complete set of steering angles,$\theta_k = [\theta_1, \theta_2, \cdots]$, are employed in the radar processor to obtain signal estimates over the desired angular region. Figure 2 illustrates an example of the slow-time STAP processor for the case of data collected under the Mountaintop experimental radar program [3]. In these data, the radar dwell consisted of $M = 16$

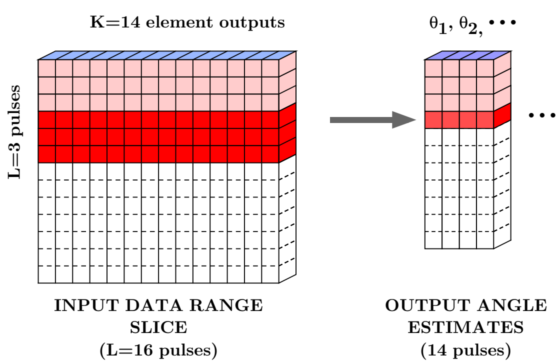

Figure 2. Slow-time STAP processing example.

pulses and the array had $K = 14$ elements. The STAP is applied across $L = 3$ successive input pulses and thus the number of output (processed) pulses is $M - L + 1 = 14$. Additional relevant results for the use of fast-time STAP on Mountaintop data have been presented by in [2] and by Seliktar, *et al* [4]. The example shown in Fig. 2 illustrates the processing to produce outputs for a single range bin and for steering angles $\theta_1, \theta_2, \cdots$.

3. OPTIMAL WEIGHT COMPUTATION

The $K-$dimensional array steering vector $V(\theta_s)$ represents the set of phase delays for a signal incident from direction θ_s. Non-STAP optimal element weights for this steering direction are given by,

$$W_{opt} \;=\; \frac{R_{XX}^{-1} V(\theta_s)}{V^\dagger(\theta_s) R_{XX}^{-1} V(\theta_s)} \;, \tag{3}$$

where R_{XX} is the element data covariance matrix,

$$R_{XX} \;=\; \mathbf{E}\left[X_m(n) X_m^\dagger(n)\right] \;, \tag{4}$$

and the scalar normalization in the denominator ensures unity gain of the processor in direction θ_s. It can readily be shown [5] that (3) is identical to the weight vector which minimizes the beamformed output power, $\mathbf{E}\left[|W^\dagger X_m(n)|^2\right]$, under a single linear constraint,

$$V^\dagger(\theta_s) W \;=\; 1 \;. \tag{5}$$

When adjacent time and/or pulse samples are added to the processing system, i.e., when STAP is employed, additional weights are required in the weight vector. The number of optimal weights in the vector W_{opt} is equal to the number of elements in the array. With slow-time STAP, the data snapshot is that given by (1) which contains KL elements. The appropriate weight vector $\mathbf{W}$ is then given by,

$$\mathbf{W} \;=\; \begin{bmatrix} W_0 \\ W_1 \\ \vdots \\ W_{L-1} \end{bmatrix} \;, \tag{6}$$

where W_m is the $K-$dimensional weights which are applied to the m^{th} pulse, X_{l-m}, in (1).

The obvious extension of the single constraint weight solution in (3) for the extended weights $\mathbf{W}$ is to replace the $K-$dimensional steering vector $V(\theta_s)$ with an extended $KL-$dimensional vector $\mathbf{V}(\theta_s)$ that consists of sets of zero vectors of length K and a single non-zero vector equal to $V(\theta_s)$. This embedded non-zero steering vector is generally placed near the midpoint of the extended vector, corresponding to one-half of the pulse time-delay span of the STAP weights. The result of this widely used approach is to maintain a single-constraint solution through use of an "extended" steering vector that contains a large number of zeros.

The $KL-$dimensional optimal weights $\mathbf{W}_{opt}$ are then determined in a manner similar to that shown in Eq. (3) - (5) but with the covariance matrix $\mathbf{R_{XX}}$ corresponding to the expanded dimension of the STAP data vector $\mathbf{X}_l(n)$. This approach, however, may result in distortion of the desired signal Doppler spectrum, even when this signal perfectly matches the

$K-$dimensional steering vector $V(\theta_s)$. To illustrate, consider a sinusoidal, single-frequency Doppler signal in which the temporal frequency is ω_0 and the Doppler frequency is ω_d. The vector signal component $S(n)$ of the l^{th} $K-$dimensional pulse is then,

$$S(n) \;=\; e^{j\omega_d l} e^{j\omega_0 n} V(\theta_s) \;, \qquad (7)$$

and the signal output $y_s(n)$ component corresponding to combining the first L pulses using (6) is,

$$\begin{aligned} y_s(n) &= \sum_{l=0}^{L-1} e^{j\omega_d l} W_l^\dagger V(\theta_s) \;, \\ &= \sum_{l=0}^{L-1} \alpha_l e^{j\omega_d l} \;, \end{aligned} \qquad (8)$$

The α_l coefficients effectively act as a set of FIR filter coefficients that will modify the magnitude and phase of the Doppler component at frequency ω_d. The problem is further complicated by the fact that the W_l components depend upon the data covariance matrix. Variations in the estimation of this matrix within the data cube can readily occur due to the presence of strong clutter components. In addition, the desired signal may consist of a Doppler spectrum rather than a single Doppler frequency component. These factors combine to produce spreading and distortion of the desired signal Doppler spectrum in many practical applications. For this reason, Doppler processing is often applied *prior* to adaptive processing [1].

The problems cited above with respect to desired signal Doppler distortion can be avoided with the use of multiple constraints [5]. The approach suggested here is to constrain each of the W_l components in $\mathbf{W}$,

$$W_l^\dagger V(\theta_s) \;=\; \delta(l_0 - l) \;. \qquad (9)$$

These L constraints ensure that only one of the α_l terms in (8), that corresponding to $l = l_0$, is non zero. With this set of constraints, the FIR linear filter applied to the Doppler spectrum is *all-pass.* If a specific filtering function is desired, the L constraints can be of the more general form,

$$W_l^\dagger V(\theta_s) \;=\; \beta_l \;. \qquad (10)$$

The β_l values are then selected to provide a desired Doppler frequency filtering for the desired signal such as in the Moving Target Indicator (MTI) case where,

$$\sum_{l=0}^{L-1} \beta_l \;=\; 0 \;. \qquad (11)$$

In effect, the zero DC response of this operator blocks any signals that are incident on the array from the steering direction θ_s and which have zero Doppler frequency. For the airborne radar environment, [1], this corresponds to steering in the broadside direction where the clutter Doppler frequency is zero. For the three-pulse STAP system, one set of such coefficients are,

$$[\beta_0 \; \beta_1 \; \beta_2] \;=\; [-0.5 \; 1.0 \; -0.5] \;, \qquad (12)$$

Nulls at other frequencies, corresponding to airborne mainbeam directions other than broadside, are readily generated in a similar manner.

A general formulation of the multiple-constraint processor which based on the Generalized Sidelobe Canceller is given in Ref. [5]. The multiply-constrained optimal weight vector $\mathbf{W}_{opt}$ is,

$$\begin{aligned} \mathbf{C}^\dagger \mathbf{W} &= \mathbf{f} \;, & (13) \\ \mathbf{W}_{opt} &= \mathbf{R_{XX}}^{-1} \mathbf{C} \left[\mathbf{C}^\dagger \mathbf{R_{XX}}^{-1} \mathbf{C} \right] \mathbf{f} \;. & (14) \end{aligned}$$

The first of these equations defines a set of L constraints comparable to Eq. (9). The constraint matrix $\mathbf{C}$ is zero except for L K-dimensional steering vectors $\mathbf{V}(\theta_s)$ along the diagonal. The constraint vector $\mathbf{f}$ is a column vector of zeros except for a single 1 in position l_0. Equation (14) specifies the set of optimal weights, i.e. those which minimize the output power, $\mathbf{E}\left[|y_s(n)|^2\right]$, subject to the constraints in (13).

4. EXTENSION TO FULL STAP CASE

The previous section has provided a description of the slow-time STAP processor which preserves the Doppler spectrum of any signal that arrives from the desired steer direction θ_s. Reference [2] presents the comparable approach for the fast-time STAP processor in which weights are applied to succesive range samples. In this case, adaptation is generally applied *after* Doppler processing. With proper choice of linear constraints, however, fast-time STAP can also be used prior to Doppler processing. If L_2 successive range samples are employed, the extended KL_2-dimensional data vector comparable to that in Eq. (3) is $\mathbf{X}_k(n)$ given by,

$$\mathbf{X}_l^\dagger(n) \;=\; \left[X_l^\dagger(n), \; \cdots \; X_l^\dagger(n - L_2 + 1) \right] \;, \qquad (15)$$

and the subscript l denotes the fact that adaptation takes place on the l^{th} radar pulse.

In full-STAP adaptation, weights are applied to both successive time samples in range and to successive radar pulses. Assuming that this process extends over L_1 radar pulses and L_2 range samples implies that the weight vector consists of KL_1L_2 elements. The extended data vector can then be expressed using either (1) or (15). For example, using (1), the extended vector $\tilde{\mathbf{X}}_l(n)$ contains L_2 sub vectors $\mathbf{X}_l(n-k)$, each of

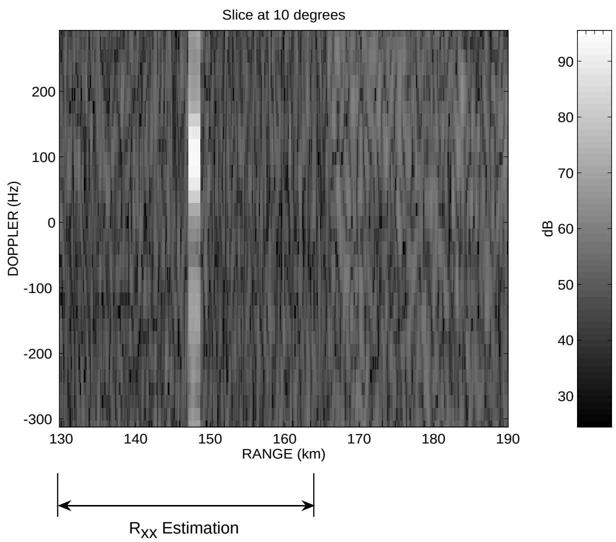

Figure 3. Range-Doppler plot obtained with single-constraint processing.

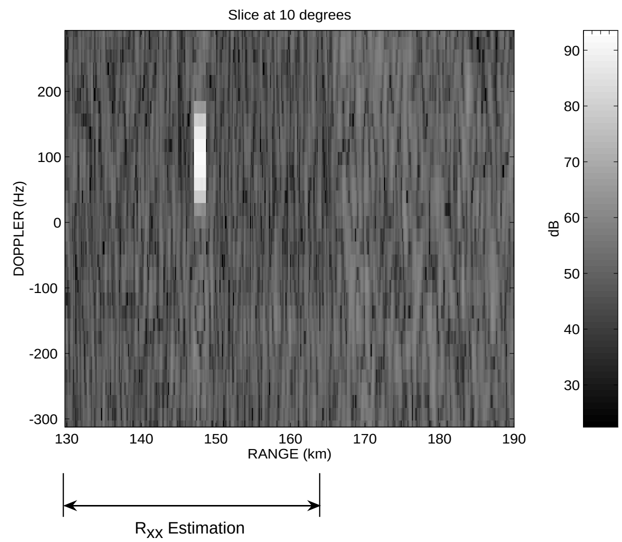

Figure 4. Range-Doppler plot obtained with multiple-constraint processing.

dimension L_1 and spanning L_1 successive radar pulses,

$$\tilde{\mathbf{X}}_l^\dagger \;=\; \left[\mathbf{X}_l^\dagger(n), \cdots, \mathbf{X}_l^\dagger(n-L_2+1)\right] \;. \qquad (16)$$

The weights $\tilde{\mathbf{W}}$ applied to these data must then be constrained to ensure that each sub-component has the desired properties by using L_1 constraints for each of the $\mathbf{X}_l^\dagger(n-k)$ vectors. Application of an additional L_2 constraints across the fast-time dimension completes the process.

5. NUMERICAL EXAMPLES

In order to illustrate the effect of multiple constraints on Doppler response, field-recorded Mountaintop data [3] were employed. The array contains 14 elements and 16 radar pulses were collected in the CPI. A total of three consecutive pulses were used in the slow-time STAP processor. The three constraints in $\mathbf{f}$ were set to values of $[010]^\dagger$ to ensure an all-pass, non-distorting Doppler response. A large target signal located at arrival angle 10^o, Doppler frequency 100 Hz, and centered at a range of 148Km was added to the data to illustrate the advantages of the multiple-constraint approach. The signal extended over 20 range samples. Figures 3 and 4 illustrate results obtained from file T38-03v1 (CPI 6).

As shown by the smearing near the target at 148 Km. in Fig. 3, significant sidelobes were observed for the single-constraint case in the recorded-data environment. These sidelobes both mask the data structure over the range extent of the signal and distort the mainlobe Doppler response. It is therefore concluded that with appropriate selection of the constraints, adaptation can be applied to pre-Doppler data without inducing modification of the desired signal Doppler spectrum. Adaptation with a single constraint, however, may produce masking and distortion in regions close to the desired signal. Finally, it should be noted that the use of additional constraints will necessarily increase the beamformed output power over that observed with a single constraint. Evidence suggests, however, that this "price paid" for the additional protection of signal preservation does not significantly degrade the output.

REFERENCES

[1] J. Ward, *Space-Time Adaptive Processing for Airborne Radar,* Lincoln Laboratory Technical Report 1015, Lexington, MA, December 1994.

[2] L.J. Griffiths, "Linear constraints in hot clutter cancellation," *Proc. ICASSP-96*, Vol. 2, pp. 1181-1184, May 1996.

[3] G. Titi, and D. Marshall, "The ARPA/Navy Mountaintop program - adaptive signal processing for early warning radar," *Proc. ICASSP-96*, Vol. 2, pp. 1165-1168, May 1996.

[4] Y. Seliktar, D. Williams, and J. McClellan, "Evaluation of partially adaptive STAP algorithms on the Mountain Top data set," *Proc. ICASSP-96*, Vol. 2, pp. 1169-1173, May 1996.

[5] L. J. Griffiths and C.W. Jim "An alternative aproach to linearly constrained adaptive beamforming," *IEEE Trans. Ant. Prop.*, Vol. AP-30, pp. 27-34, July 1982.