LOCALIZATION OF MULTIPLE SOURCES WITH MOVING ARRAYS

Jacob Sheinvald and Mati Wax
RAFAEL 83, P.O.B. 2250, Haifa, ISRAEL 31021
Tel:4-8794687 Fax:4-8792170

Anthony J. Weiss
Electrical Eng. Dept.
Tel-aviv University
Tel-aviv, ISRAEL 69978

Abstract

We consider the problem of localizing multiple narrow-band stationary signals using an arbitrary time-varying array such as an array mounted on a moving platform. We assume a Gaussian stochastic model for the received signals and employ the Generalized Least Squares (GLS) estimator to get an asymptotically-efficient estimation of the model parameters. In case the signals are a-priori known to be uncorrelated, this estimator allows to exploit this prior knowledge to its benefit. For the important case of translational motion of a rigid array, a computationally-efficient spatial-smoothing method is presented. Simulation results confirming the theoretical results are included.

1 Introduction

Most of the work on the problem of direction-finding by sensor arrays addressed the case of time-invariant arrays, namely, arrays whose elements are fixed in space. In contrast, in this paper we address the case where the array is time-varying, i.e., its elements move in space in some arbitrary but known way. A typical example is an array mounted on a moving platform.

Time-varying arrays have recently been discussed by several authors. In [1], the Doppler effect was used to decorrelate coherent signals. In [2], a sonar system is analyzed consisting of a fixed non-moving linear subarray and an additional towed subarray. In [3], a computationally-efficient Maximum Likelihood Estimator (MLE) is derived for the single-source case, and the capability of spatially-sparse time-varying arrays to cope with ambiguity errors is demonstrated. In [4], two computationally-efficient estimation techniques are suggested for multiple sources, based on array interpolation and on focusing matrices. In [5], a deterministic-signals model is employed and its corresponding MLE analyzed.

In this paper we present an *asymptotically-efficient* estimator based on the Generalized Least Squares (GLS) criterion. This estimator approximates the MLE for the large-sample case but is computationally much simpler. In case the signals are a-priori known to be uncorrelated, it allows to exploit this prior knowledge and get better performance. Also, for the special case of a translational motion of a rigid array we present a spatial-smoothing method that is a generalization of the method presented in [6], and is similar to the method used in [1], which allows application of computationally-efficient eigenstructure algorithms such as MUSIC [7], to the time-varying case. Both techniques can also handle the important case of coherent signals arising, for instance, in specular multipath propagation. It should also be remarked that the number of sources that can be handled by the GLS estimator is not necessarily limited by the number of sensors.

2 Problem Formulation

Consider q wave-fronts impinging from locations $\theta_1, \ldots, \theta_q$ on a time-varying array consisting of

p sensors. For simplicity assume that the sensors and the sources are all located on the same plane and that the sources are in the far-field of the array, so that $\{\theta_k\}$ represent the Directions-Of-Arrival (DOAs). Assume also that the sources emit narrow-band signals all centered around a common frequency. Let $s_k(\zeta)$ denote the complex envelope of the k-th source signal measured at time ζ at some fixed reference point, and let $\mathbf{x}(\zeta) = (x_1(\zeta), x_2(\zeta), \ldots, x_p(\zeta))^T$ denote the vector of complex envelopes formed from the signals received by the sensors, with T denoting transposition. In the presence of additive noise, this received vector can be expressed as:

$$\mathbf{x}(\zeta) = \sum_{k=1}^{q} \mathbf{a}(\zeta, \theta_k) s_k(\zeta) + \mathbf{n}(\zeta) \qquad (1)$$

where $\mathbf{n}(\zeta)$ is the complex envelope of the noise, and where $\mathbf{a}(\zeta, \theta)$ is the array's steering vector expressing its complex response at time ζ to a planar wavefront arriving from direction θ. This expression can be written more compactly as:

$$\mathbf{x}(\zeta) = \mathbf{A}(\zeta, \boldsymbol{\theta})\mathbf{s}(\zeta) + \mathbf{n}(\zeta) \qquad (2)$$

where $\boldsymbol{\theta} \stackrel{\text{def}}{=} (\theta_1 \ldots \theta_q)^T$, $\mathbf{A}(\zeta, \boldsymbol{\theta}) \stackrel{\text{def}}{=} [\mathbf{a}(\zeta, \theta_1), \ldots, \mathbf{a}(\zeta, \theta_q)]$ is a $p \times q$ steering-matrix, and $\mathbf{s}(\zeta) \stackrel{\text{def}}{=} (s_1(\zeta), \ldots, s_q(\zeta))^T$ is a vector formed from the emitted signals. Let the array be sampled sequentially at $t = 1, \ldots, K$ different mutually-exclusive time-slots, and assume that the steering-matrix can be regarded as quasi-static during each slot, and denote by $\mathbf{A}_t(\boldsymbol{\theta})$ the steering-matrix corresponding to the t-th time-slot, i.e.

$$\mathbf{A}_t(\boldsymbol{\theta}) \stackrel{\text{def}}{=} \mathbf{A}(\zeta, \boldsymbol{\theta}) \quad \text{forall } \zeta \in t\text{-th time-slot}$$

Let $\mathbf{X}_t \stackrel{\text{def}}{=} [\mathbf{x}_t(\zeta_1^t), \ldots, \mathbf{x}_t(\zeta_{m_t}^t)]$ denote the t-th time-slot batch of samples, with m_t denoting the number of samples taken at this time-slot, and $\zeta_1^t, \ldots, \zeta_{m_t}^t$ denoting the sampling instants. Now, the problem is stated as follows: Given the K batches of samples $\mathbf{X} \stackrel{\text{def}}{=} \{\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_K\}$ – estimate the directions $\boldsymbol{\theta}$.

To solve this problem, we make the following assumptions:

$A1$. The steering vectors $\{\mathbf{a}(\zeta, \theta)\}$ are known for all ζ and all $\theta \in \boldsymbol{\Theta}$, where $\boldsymbol{\Theta}$ denotes the field-of-view.

$A2$. The sources are far enough so that $\boldsymbol{\theta}$ can be considered constant during the whole observation time.

$A3$. The noise-vector $\mathbf{n}(\zeta)$ of the whole array is a zero-mean complex-Gaussian wide-sense stationary process with a covariance matrix $\sigma^2\mathbf{I}$, where σ^2 is an unknown positive scalar and $\mathbf{I}$ is the identity matrix, and $\{\mathbf{n}(\zeta_i^t)\}$ are uncorrelated $\forall t, i$.

$A4$. The signal-vector $\mathbf{s}(\zeta)$ is a zero-mean complex-Gaussian wide-sense stationary process uncorrelated with the noise-vector and having an unknown *Hermitian* covariance matrix $\mathbf{P}$. The signal samples $\{\mathbf{s}(\zeta_i^t)\}$ are uncorrelated $\forall t, i$.

Based on these assumptions and using (2), the covariance matrix corresponding to the t-th batch is given by

$$\mathbf{R}_t(\phi) = \mathbf{A}_t(\boldsymbol{\theta})\mathbf{P}\mathbf{A}_t^H(\boldsymbol{\theta}) + \sigma^2\mathbf{I} \qquad (3)$$

where $()^H$ denotes complex-conjugate transposition, and where ϕ denotes a vector composed of all the real free parameters:

$$\phi \stackrel{\text{def}}{=} (\boldsymbol{\theta}^T, \bar{\mathbf{P}}^T, \sigma^2)^T \qquad (4)$$

where $\bar{\mathbf{P}}$ is a real vector formed from the free real parameters of the Hermitian matrix $\mathbf{P}$ in some way. In case the signals are a-priori known to be uncorrelated, $\bar{\mathbf{P}} = \text{diag}(\mathbf{P})$, a vector containing the diagonal entries only.

3 The GLS Estimator

The basic idea behind our approach is to select those parameters $\hat{\phi}$ that give the "best fit" between the sample-covariances $\{\hat{\mathbf{R}}_t\}$ and the model-covariances $\{\mathbf{R}_t(\phi)\}$. A reasonable goodness-of-fit criterion is the sum of squares of the entries of the difference matrices $\{\mathbf{E}_t \stackrel{\text{def}}{=} \hat{\mathbf{R}}_t - \mathbf{R}_t(\phi)\}$:

$$L_{LS}(\phi) \stackrel{\text{def}}{=} \sum_t m_t \|\hat{\mathbf{R}}_t - \mathbf{R}_t(\phi)\|_F^2$$

where $\|\cdot\|_F$ denotes the Frobenius norm. However, the resulting parameter estimates are not asymptotically efficient. A better criterion results if instead of taking the sum of squares of $\{\mathbf{E}_t\}$, we take the sum of squares of their transformed version

$$L_{gls}(\phi) \stackrel{\text{def}}{=} \sum_t \|\mathbf{T}_t(\hat{\mathbf{R}}_t - \mathbf{R}_t(\phi))\mathbf{T}_t^H\|_F^2 \qquad (5)$$

where the transformation matrix $\mathbf{T}_t$ is given by $\mathbf{T}_t = \sqrt[4]{m_t}\,\hat{\mathbf{R}}_t^{-1/2}$. The elements of the transformed error-matrices $\{\tilde{\mathbf{E}}_t \stackrel{\text{def}}{=} \mathbf{T}_t(\hat{\mathbf{R}}_t - \mathbf{R}_t(\phi))\mathbf{T}_t^H\}$ get white asymptotically, i.e., it is guaranteed that for $m_t \to \infty; \forall t$, the cross-correlation between any two elements of $\tilde{\mathbf{E}}_t$ is zero, while the variances of all elements are identical [8]. The estimator we therefore propose is given by

$$(\hat{\boldsymbol{\theta}}, \hat{\mathbf{P}}, \hat{\sigma}^2) = \arg \min_{\boldsymbol{\theta}, \mathbf{P}, \sigma^2} \{L_{gls}(\phi)\} \qquad (6)$$

$$L_{gls}(\phi) = \sum_t \|\mathbf{T}_t[\hat{\mathbf{R}}_t - \mathbf{A}_t(\boldsymbol{\theta})\mathbf{P}\mathbf{A}_t^H(\boldsymbol{\theta}) - \sigma^2\mathbf{I}]\mathbf{T}_t^H\|_F^2$$

where (3) was used to replace $\mathbf{R}_t(\phi)$ in (5). This estimator can be regarded as a variant of what in the statistical literature [9] [10] is known as the Generalized Least Squares (GLS) estimator. We prove in [8] that this estimator is asymptotically efficient, i.e., it achieves the Cramer-Rao bound as $m_t \to \infty; \forall t$.

To solve the minimization problem we first minimize with respect to $\bar{\mathbf{P}}$ and σ^2 while holding $\boldsymbol{\theta}$ fixed. Then, we substitute the minimizing values, $\hat{\mathbf{P}}(\boldsymbol{\theta})$ and $\hat{\sigma}^2(\boldsymbol{\theta})$, back into the cost function and get a reduced cost function that is a function of $\boldsymbol{\theta}$ only, thus making the minimization problem much simpler. The result is [8]

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \|\mathbf{r} - \mathcal{A}(\boldsymbol{\theta})(\bar{\mathcal{A}}^{\mathbf{T}}(\boldsymbol{\theta})\bar{\mathcal{A}}(\boldsymbol{\theta}))^{-1}\mathcal{A}_R^T(\boldsymbol{\theta})\mathbf{r}\|^2$$

where $\mathbf{r}$ is a $Kp^2 \times 1$ vector formed from the sample-covariances $\{\hat{\mathbf{R}}_t\}$, and where $\mathcal{A}(\boldsymbol{\theta})$, $\bar{\mathcal{A}}(\boldsymbol{\theta})$ are $Kp^2 \times (q^2+1)$, $2Kp^2 \times (q^2+1)$ matrices formed from the transformed steering-matrices $\{\mathbf{T}_t\mathbf{A}_t(\boldsymbol{\theta})\}$.

4 A spatial-smoothing algorithm for translation of rigid arrays

In case the array is rigid (i.e., it is not time-varying) and its motion is translational (i.e., without rotation), the steering-vector of the array can be written as $\mathbf{a}_t(\theta) = e^{j\gamma_t(\theta)}\mathbf{a}(\theta)$, where $\mathbf{a}(\theta)$ represents the array steering-vector with respect to a local reference point attached to the array (say, the array's center), and where $\gamma_t(\theta)$ is a motion-induced phase difference expressing the phase difference between the local array's reference point and the global reference point. Therefore, the steering-matrix can be represented by

$$\mathbf{A}_t(\boldsymbol{\theta}) = \mathbf{A}(\boldsymbol{\theta})e^{j\boldsymbol{\Gamma}_t(\boldsymbol{\theta})}$$

where $\mathbf{A}(\theta)$ is the array steering-matrix, and where $\boldsymbol{\Gamma}_t(\boldsymbol{\theta}) \stackrel{\text{def}}{=} \text{diag}(\gamma_t(\theta_1), \ldots, \gamma_t(\theta_q))$. A computationally-efficient eigen-structure-based algorithm, based on the spatial-smoothing method described in [6] and can be considered as its generalization, can be obtained by combining the time-varying covariances to get a "spatially smoothed" covariance matrix

$$\mathbf{R}(\phi) = \frac{1}{K}\sum_{t=1}^{K}\mathbf{R}_t(\phi) = \mathbf{A}(\boldsymbol{\theta})\mathcal{P}\mathbf{A}^H(\boldsymbol{\theta}) + \sigma^2\mathbf{I}$$

where $\mathcal{P} \stackrel{\text{def}}{=} \frac{1}{K}\sum_{t=1}^{K}\left\{e^{j\boldsymbol{\Gamma}_t(\boldsymbol{\theta})}\mathbf{P}e^{-j\boldsymbol{\Gamma}_t(\boldsymbol{\theta})}\right\}$. The spatially smoothed covariance matrix $\mathbf{R}(\phi)$ has a structure corresponding to the static case, with $\mathbf{A}(\boldsymbol{\theta})$ being the static array's steering-matrix and $\mathcal{P}$ being a modified covariance matrix of the signals. Therefore, eigenstructure methods, such as MUSIC [7] can be applied to estimate $\boldsymbol{\theta}$.

5 Simulation results

To demonstrate the performance of the proposed algorithms we simulated a 4-omnidirectionalelement Uniform Linear Array (ULA) with a spacing of 0.4λ. Two coherent Gaussian sources were located at $\theta_1 = 5^\circ$ and $\theta_2 = -5^\circ$ relative to boresight, with Signal to Noise Ratios (SNR) of 0db and -3db, respectively. The array was moving along the line connecting its elements, and it was sampled at $K = 5$ equispaced points along its route, with one λ spacing between these sampling places. The number of samples taken at each point was identical, i.e., $m_t = m_1; \forall t$. The phase-difference between the signals at the route's center was $\varphi = \pi/2$. A set of 100 Monte-Carlo runs was carried out for each value of the number-of-samples m_t. The DOAs were estimated in each run and the RMS DOA error of the first source was computed from the whole set. The results obtained by emloying the GLS estimator and the Spatial Smoothing (SS) method (using MUSIC), compared to the CRB [11], are shown in Figure 1. The GLS estimator is clearly superior to the SS estimator and its asymptotical-efficiency is evident.

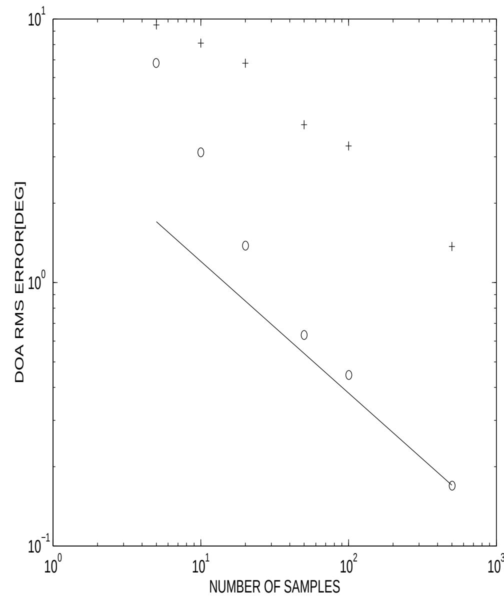

Figure 1: Two coherent signals at $\theta_1 = 5^\circ$, $\theta_2 = -5^\circ$, $\text{SNR}_1 = 0\text{db}$, $\text{SNR}_2 = -3\text{db}$. Four-element ULA, spacing=0.4λ. The solid line displays the CRB for the DOA error of the first source. The results obtained by GLS and SS are displayed by o and + , respectively. Distance traveled is 5λ.

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