

MAXIMUM-LIKELIHOOD BLIND FIR MULTI-CHANNEL ESTIMATION WITH GAUSSIAN PRIOR FOR THE SYMBOLS[†]

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ABSTRACT

We present two approaches to stochastic Maximum Likelihood identification of multiple FIR channels, where the input symbols are assumed Gaussian and the channel deterministic. These methods allow semi-blind identification, as they accommodate a priori knowledge in the form of a (short) training sequence and appears to be more relevant in practice than purely blind techniques. The two approaches are parameterized both in terms of channel coefficients and in terms of prediction filter coefficients. Corresponding methods are presented and some are simulated. Furthermore, Cramer-Rao Bounds for semi-blind ML are presented: a significant improvement of the performance for a moderate number of known symbols can be noticed.

1. INTRODUCTION

Consider a sequence of symbols $a(k)$ received through m channels $\mathbf{y}(k) = \sum_{i=0}^{N-1} \mathbf{h}(i)a(k-i) + \mathbf{v}(k) = \mathbf{H}_N A_N(k) + \mathbf{v}(k)$, $\mathbf{y}(k) = [y_1^H(k) \cdots y_m^H(k)]^H$, $\mathbf{H}_N = [\mathbf{h}(N-1) \cdots \mathbf{h}(0)]$, $A_N(k) = [a(k-N+1)^H \cdots a(k)^H]^H$, where superscript H denotes Hermitian transpose. Let $\mathbf{H}(z) = \sum_{i=0}^{N-1} \mathbf{h}(i)z^{-i} = [\mathbf{H}_1^H(z) \cdots \mathbf{H}_m^H(z)]^H$ be the SIMO channel transfer function, and $\mathbf{h} = [\mathbf{h}^H(N-1) \cdots \mathbf{h}^H(0)]^H$. Consider additive independent white Gaussian noise $\mathbf{v}(k)$ with $r\mathbf{v}\mathbf{v}^H(k-i) = \mathbf{E}\mathbf{v}(k)\mathbf{v}(i)^H = \sigma_v^2 I_m \delta_{ki}$. Assume we receive M samples:

$$\mathbf{Y}_M(k) = \mathcal{T}_M(\mathbf{H}_N) A_{M+N-1}(k) + \mathbf{V}_M(k) \quad (1)$$

where $\mathbf{Y}_M(k) = [\mathbf{y}^H(k-M+1) \cdots \mathbf{y}^H(k)]^H$ and similarly for $\mathbf{V}_M(k)$, and $\mathcal{T}_M(\mathbf{H}_N)$ is a block Toeplitz matrix with M block rows and $[\mathbf{H}_N \ 0_{m \times (M-1)}]$ as first block row. We shall simplify the notation in (1) with $k = M-1$ to

$$\mathbf{Y} = \mathcal{T}(\mathbf{H}) A + \mathbf{V} . \quad (2)$$

We assume that $mM > M+N-1$ in which case the channel convolution matrix $\mathcal{T}(\mathbf{H})$ has more rows than columns. If the $\mathbf{H}_i(z)$, $i = 1, \dots, m$ have no zeros in common, then $\mathcal{T}(\mathbf{H})$ has full column rank (which we will henceforth assume). For obvious reasons, the column space of $\mathcal{T}(\mathbf{H})$ is called the signal subspace and its orthogonal complement the noise subspace.

We propose two stochastic Maximum Likelihood (ML) approaches in which the symbols are assumed Gaussian. These methods can easily accommodate a priori knowledge

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in the form of a (short) training sequence: we call the corresponding equalization methods semi-blind. It does not appear to be easy to incorporate such a priori information in other methods such as subspace fitting techniques. Considering actual mobile communication standards and on the basis of robustness considerations, semiblind equalization appears to be more relevant in practice than purely blind equalization. The robustness improvements of the semi-blind approach will be illustrated via an analysis of the Cramer-Rao bound (CRB) and simulations.

2. DETERMINISTIC ML

In the Deterministic (or conditional) Maximum Likelihood (DML) method, both the channel \mathbf{H} and the symbols A are considered as deterministic quantities. We introduced the DML method for blind channel estimation in [1] for $m = 2$ channels and we extended it to arbitrary m in [2]. An extension to the multi-user case was introduced in [3],[4]. The key to a computationally attractive solution of the DML problem is a linear parameterization of the noise subspace since the DML criterion boils down to a projection on the noise subspace. Such a parameterization has been proposed in [2], based on cyclic prediction filters for the noise-free received signal. Further discussions can be found in [5] and [6].

3. CHANNEL-BASED STOCHASTIC ML.

In stochastic ML, the input symbols are no longer deterministic, but Gaussian quantities. ML estimation with a Gaussian prior for the symbols has been introduced in [5]. The Gaussian hypothesis would have effect to robustify DML. Two stochastic ML approaches can be pursued.

3.1. Two Gaussian Prior Approaches

We consider a general problem where the received signal is $\mathbf{Y} = \mathcal{T}(\mathbf{H}) A + \mathbf{V}$: $\mathbf{V} \sim \mathcal{N}(0, R_{VV})$ is independent of $A \sim \mathcal{N}(A^\circ, R_{AA})$ (complex normal variables are assumed to be circular). A° , the prior mean for the symbols, represents a prior knowledge on the symbols, which will be specialized below to the semi-blind case.

In a first approach, which we call Gaussian ML (GML), only the channel is being estimated, not the symbols, as well as R_{VV} . It corresponds to the classical approach in Stochastic Direction of Arrival Estimation. In the ML problem, the quantity to be maximized is $f(\mathbf{Y}/\theta)$, the complex probability density function of \mathbf{Y} given $\theta = [h \ R_{AA} \ R_{VV}]$:

$$f(\mathbf{Y}/\theta) = \frac{1}{\pi^M \det R_{YY}} \exp [-(\mathbf{Y} - \mathcal{T}A^\circ)^H R_{YY}^{-1} (\mathbf{Y} - \mathcal{T}A^\circ)] \quad (3)$$

where: $R_{YY} = \mathcal{T}R_{AA}\mathcal{T}^H + R_{VV}$. The corresponding log-likelihood function to be minimized is:

$$\ln(\det R_{YY}) + (\mathbf{Y} - \mathcal{T}A^\circ)^H R_{YY}^{-1} (\mathbf{Y} - \mathcal{T}A^\circ) \quad (4)$$

In a second approach, both h and A are being estimated as well as R_{VV} : maximum a posteriori (MAP) for A and ML for h and R_{VV} . We call this method GMAPML. The quantity to be maximized is:

$$f(\mathbf{Y}, A/h, R_{VV}) = \frac{1}{\pi^{(m+1)M+N-1} \det R} \exp \left[-(\mathbf{Y}' - \mathcal{T}'A)^H R^{-1} (\mathbf{Y}' - \mathcal{T}'A) \right] \quad (5)$$

with: $R = \begin{bmatrix} R_{VV} & 0 \\ 0 & R_{AA} \end{bmatrix}$, $\mathbf{Y}' = \begin{bmatrix} \mathbf{Y}^H & A^{\circ H} \end{bmatrix}^H$ and $\mathcal{T}' = \begin{bmatrix} \mathcal{T}^H & I^H \end{bmatrix}^H$. The corresponding log-likelihood function to be minimized is:

$$\ln(\det R) + (\mathbf{Y}' - \mathcal{T}'A)^H R^{-1} (\mathbf{Y}' - \mathcal{T}'A) \quad (6)$$

3.2. Relationship between the two Approaches.

The minimization of (6), being separable in A and h , is usually done by minimizing w.r.t. A first:

$$A = (\mathcal{T}^H R_{VV}^{-1} \mathcal{T} + R_{AA}^{-1})^{-1} (\mathcal{T}^H R_{VV}^{-1} \mathbf{Y} + R_{AA}^{-1} A^{\circ}) \quad (7)$$

which is of the form of a MMSE non-causal decision-feedback equalizer (NCDFE), being a MMSE linear estimator in terms of the received data \mathbf{Y} and the uncertain preliminary symbols A° with uncertainty reflected in R_{AA} . Note that when $A^{\circ} = 0$ (blind case), the estimate for the symbols corresponds to the output of a MMSE linear equalizer. This illustrates the superiority of the GMAPML method over the DML method, for which the symbol estimates are given by the output of a MMSE ZF linear equalizer.

Substituting (7) in (6) results in the minimization of:

$$\ln(\det R) + (\mathbf{Y} - \mathcal{T}A^{\circ})^H R_{YY}^{-1} (\mathbf{Y} - \mathcal{T}A^{\circ}) \quad (8)$$

If we suppose R known, GMAPML appears to be a simplified version of GML from which the term $\ln(\det R_{YY})$ has been removed.

3.3. Semi-Blind Estimation

We specialize now this general formulation to the case of semi-blind equalization in which we use a partial training sequence and a Gaussian prior for the remaining unknown symbols. Let $A = \mathcal{P} [A_1^{\circ H} A_2^{\circ H}]^H$ where A_1° are the training symbols and \mathcal{P} is a permutation matrix to account for the fact that the training sequence does not necessarily occur at the beginning. Hence we have

$$A^{\circ} = \mathcal{P} \begin{bmatrix} A_1^{\circ} \\ 0 \end{bmatrix}, \quad R_{AA} = \sigma_a^2 \mathcal{P} \begin{bmatrix} \epsilon I & 0 \\ 0 & I \end{bmatrix} \mathcal{P}^H \quad (9)$$

where we shall let $\epsilon \xrightarrow{\Delta} 0$. Also, as mentioned earlier, we take $R_{VV} = \sigma_v^2 I$.

3.4. Cramer-Rao Bounds

GML estimates reach asymptotically their Cramer-Rao Bound (CRB). In GMAPML, however, the input symbols estimates cannot be consistent because their number is asymptotically infinite. Hence, joint estimates of A and h cannot be consistent and do not reach their CRB.

A comparison in terms of CRB between both methods is not meaningful. GML appears however to give better performance. Whereas DML cannot estimate the module of the channel, the introduction of the Gaussian assumption allows this estimation in GML, but not in GMAPML. For a channel of length $N = 1$, GMAPML gives an estimate of the normalized channel in the same direction as GML, but with a infinite module. We will now consider the complex CRBs for GML and GMAPML.

3.4.1. GML

The Fisher Information Matrix (FIM) for deterministic complex parameters θ is defined as:

$$J(\theta) = -E_Y \frac{\partial}{\partial \theta^*} \left(\frac{\partial \ln f(Y/\theta)}{\partial \theta^*} \right)^H. \quad (10)$$

$J(\theta)^{-1}$ is the complex CRB: $C_{\hat{\theta}\hat{\theta}} \geq J(\theta)^{-1}$, where $C_{\hat{\theta}\hat{\theta}} = E\hat{\theta}\hat{\theta}^H$, $\hat{\theta} = \theta - \hat{\theta}$, $E\hat{\theta} = 0$. The parameters are $\theta = \begin{bmatrix} h^H & \sigma_v^2 \end{bmatrix}^H$.

$$J(\theta)(i, j) = \left(\mathcal{A}^{\circ H} R_{YY}^{-1} \mathcal{A}^{\circ} \right)_{i, j} + \text{trace} \left\{ R_{YY}^{-1} \left(\frac{\partial R_{YY}}{\partial \theta_i^*} \right) R_{YY}^{-1} \left(\frac{\partial R_{YY}}{\partial \theta_j^*} \right)^H \right\}, \quad (11)$$

$\mathcal{T}(\mathbf{H})A^{\circ} = \mathcal{A}^{\circ} h$, \mathcal{A}° is a structured matrix filled with the elements of A° .

$$\frac{\partial R_{YY}}{\partial h_i^*} = \mathcal{T}(\mathbf{H}) R_{AA} \mathcal{T}^H \left(\frac{\partial \mathbf{H}^*}{\partial h_i^*} \right) \quad \frac{\partial R_{YY}}{\partial \sigma_v^2} = \sigma_v^2 I. \quad (12)$$

Note that those expressions apply to the general case of sections 3.1. and 3.2.. The real FIM corresponding to the estimation of the real and imaginary part of the channel is singular: indeed the phase of the channel cannot be estimated. The complex FIM is not singular: the complex CRB gives a lower bound for a channel estimate for which the correct phase factor is known.

3.4.2. GMAPML

The estimate parameters are $\theta = \begin{bmatrix} A^H & h^H & \sigma_v^2 \end{bmatrix}^H$. The FIM for stochastic parameters is:

$$J(\theta) = -E_{Y, \theta} \frac{\partial}{\partial \theta^*} \left(\frac{\partial \ln f(Y, \theta)}{\partial \theta^*} \right)^H. \quad (13)$$

Applied to our problem where A is stochastic and h, σ_v^2 deterministic, the FIM is:

$$J(\theta) = -E_{Y, A} \frac{\partial}{\partial \theta^*} \left(\frac{\partial \ln f(Y, A/h, \sigma_v^2)}{\partial \theta^*} \right)^H. \quad (14)$$

The CRB for h is:

$$\sigma_v^2 \left[M \sigma_a^2 I - \mathcal{A}^{\circ H} \mathcal{T}_2 \left(\mathcal{T}_2^H \mathcal{T}_2 + \frac{\sigma_v^2}{\sigma_a^2} I \right)^{-1} \mathcal{T}_2^H \mathcal{A}^{\circ} \right]^{-1} \quad (15)$$

where $\mathcal{T}A = \mathcal{T}_1 A_1^{\circ} + \mathcal{T}_2 A_2^{\circ}$. In the blind case, the CRB is $\frac{\sigma_v^2}{M \sigma_a^2} I$. It corresponds asymptotically to the CRB when all input symbols are known. This CRB for GMAPML appears to be much too optimistic.

3.4.3. Simulations

We present some simulations to illustrate the effect of the number of known symbols on the CRB for GML. We plot the trace of the CRB normalized w.r.t. the channel module and the number of channel coefficients. The SNR, defined as $\frac{\|\mathbf{H}\|^2 \sigma_a^2}{m \sigma_v^2}$ is of 10dB, $M=50$. The channel is:

$$\begin{bmatrix} 1.0000 & 0.8000 \\ -1.5000 & 1.4000 \end{bmatrix}.$$

We notice the significant improvement of semi-blind equalization as more and more symbols are known, especially with few known symbols. Furthermore, the estimation of σ_v^2 , which is also done in GML, does not influence much the performance of channel estimation itself.

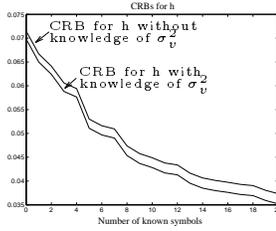


Figure 1. CRBs for GML w.r.t to the number of known symbols

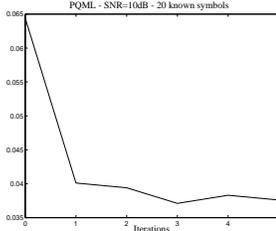


Figure 2. Normalized estimation errors for h (averaged over 100 noise realizations) for PQML, N=1

3.5. Methods

3.5.1. Pseudo-Quadratic ML (PQML)

The principle of this method has been applied to sinusoids in noise estimation [7] as well as to channel-based DML in equalization [8]. In this last method, the gradient of the cost function $C(h)$ to be minimized may be written as $B(h)h$, where $B(h)$ is positive semi-definite. In a first step $B(h)$ is considered as constant: we have the equivalent to a quadratic problem, and the solution is the minimal eigenvector of $B(h)$, this solution is used to reevaluate $B(h)$ and other iterations may be done.

For blind GML, the decomposition is of the form $B(h)h$, for semi-blind GML, it is of the form $B(h)h + D(h)$, where $B(h)$ is now positive definite. For the semi-blind GML, both $B(h)$ and $D(h)$ are supposed constant in the first step of the procedure, h is the solution of a linear system. The difficulty is to find the right decomposition.

For a channel of length 1, we derived the PQML method, the general case is still under study. In fig.1, we show the normalized errors on h , at each step of PQML for 20 known symbols and SNR=10dB. The channel considered is $[1 \ -1.5]^H$.

3.5.2. Approximate semi-blind DML

The following one-shot method can be used in its own right or as initialization step for PQGML. For conciseness, we shall explain the case of $m = 2$ channels. Several popular methods are based on the observation that for the noise-free signals, we have $H_2(z)y_1(k) - H_1(z)y_2(k) = 0$ or in matrix form $\mathcal{T}(H_2)Y_2^{(2)} - \mathcal{T}(H_1)Y_1^{(2)} = \mathcal{Y}^{(2)}h = 0$ where superscript (2) denotes the data used for the blind part. In the presence of noise, we can solve this equation in a least-squares sense for h . Let $\mathbf{Y}^{(1)}$ denote the data corresponding to the training symbols A_1° . If $\mathbf{Y}^{(1)}$ and $\mathbf{Y}^{(2)}$ do not overlap, then an optimally weighted semi-blind least-squares criterion is

$$\min_h \left\| \begin{bmatrix} \mathbf{Y}^{(1)} \\ 0 \end{bmatrix} - \begin{bmatrix} \mathcal{A}^\circ \\ \mathcal{Y}^{(2)} \end{bmatrix} h \right\|_{\begin{bmatrix} I & 0 \\ 0 & W^{-1} \end{bmatrix}}^2 \quad (16)$$

where in $W = \mathcal{T}(H_1)\mathcal{T}^H(H_1) + \mathcal{T}(H_2)\mathcal{T}^H(H_2)$ the two terms can be estimated consistently from $\frac{1}{\sigma_v^2} \left(\widehat{R}_{YY} - \lambda_{\min}(\widehat{R}_{YY})I \right)$.

4. PREDICTION BASED STOCHASTIC ML.

The GMAPML minimization criterion can be parametrized in terms of linear prediction quantities, which has for advantage to be robust to channel order overestimation. The extended noise subspace (corresponding to \mathcal{T}') can be linearly parametrized. Let $\mathcal{T}'^\perp(G)$, parameterized by G , be of full column rank equal to the noise subspace dimension such that $\mathcal{T}'^{\perp H} \mathcal{T}' = 0$. Then also $\left(R^{H/2} \mathcal{T}'^\perp \right)^H \left(R^{-1/2} \mathcal{T}' \right) = 0$.

After substituting A from (7) in (6), we get:

$$\begin{aligned} \min_{\mathbf{H}} \left\| P_{R^{-1/2} \mathcal{T}'}^\perp R^{-1/2} \mathbf{Y}' \right\|^2 &= \min_G \left\| P_{R^{H/2} \mathcal{T}'^\perp} R^{-1/2} \mathbf{Y}' \right\|^2 \\ &= \min_G \mathbf{Y}'^H \mathcal{T}'^\perp \left(\mathcal{T}'^{\perp H} R \mathcal{T}' \right)^{-1} \mathcal{T}'^{\perp H} \mathbf{Y}' \end{aligned} \quad (17)$$

The signal subspace is parameterized linearly by \mathbf{H} . Now let $\mathbf{P}(z) = \sum_{i=0}^L \mathbf{p}(i) z^{-i}$ with $\mathbf{p}(0) = I_m$ be the MMSE multivariate prediction error filter of order L for the noise-free received signal $\mathbf{y}(k)$. If $L \geq \underline{L} = \lceil \frac{N-1}{m-1} \rceil$, then it can be shown [9] that

$$\mathbf{P}(z)\mathbf{H}(z) = \mathbf{h}(0). \quad (18)$$

From (18) it is clear that $\mathbf{H}(z)$ and $\mathbf{P}(z), \mathbf{h}(0)$ are equivalent parameterizations.

For the noise subspace parameterization, note that from (18) we get

$$\begin{aligned} \mathcal{T}_{M-L}(\mathbf{P})\mathcal{T}_M(\mathbf{H}) &= [0_{(M-L) \times (L+N-1)} \ I_{M-L}] \otimes \mathbf{h}(0) = \\ \tilde{I} \otimes \mathbf{h}(0) &\Rightarrow \left[\mathcal{T}_{M-L}(\mathbf{P}) \mid -\tilde{I} \otimes \mathbf{h}(0) \right] \mathcal{T}'(\mathbf{H}) = 0 \end{aligned} \quad (19)$$

However

$$\text{Range} \left(\left[\mathcal{T}_{M-L}(\mathbf{P}) \mid -\tilde{I} \otimes \mathbf{h}(0) \right]^H \right) \subset \left(\text{Range} \left(\mathcal{T}' \right) \right)^\perp \quad (20)$$

(the noise subspace is not completely spanned) so that replacing $\mathcal{T}'^{\perp H}$ by $\left[\mathcal{T}_{M-L}(\mathbf{P}) \mid -\tilde{I} \otimes \mathbf{h}(0) \right]$ in (17) (with the role of G played by $\mathbf{p}(1:L), \mathbf{h}(0)$) leads to estimates that are only asymptotically (in M) equal to the ML estimates.

We shall specialize now this general formulation to the case of semi-blind equalization. (17) now becomes:

$$\min_{\mathbf{p}(1:L), \mathbf{h}(0)} \left\| \mathcal{T}(\mathbf{P})\mathbf{Y} - (\tilde{I}\mathcal{P} \begin{bmatrix} A_1^\circ \\ 0 \end{bmatrix}) \otimes \mathbf{h}(0) \right\|_{\mathcal{R}^{-1}} \quad (21)$$

$\mathcal{R} = \sigma_v^2 \mathcal{T}(\mathbf{P})\mathcal{T}^H(\mathbf{P}) + (\tilde{I}R_{AA}\tilde{I}^H) \otimes (\mathbf{h}(0)\mathbf{h}^H(0))$ This equation can be seen as the (maximized w.r.t. A) GMAPML log likelihood of $\mathcal{T}(\mathbf{P})\mathbf{Y} \sim \mathcal{N}(\left(\tilde{I}\mathcal{P} \begin{bmatrix} A_1^\circ \\ 0 \end{bmatrix}\right) \otimes \mathbf{h}(0), \mathcal{R})$. $\mathcal{T}(\mathbf{P})\mathbf{Y}$ represents a reduction of data w.r.t. \mathbf{Y} which is however asymptotically negligible.

We can then define 2 approaches to ML based on \mathbf{P} and $\mathbf{h}(0)$: the GMAPML approach described in (21) and the GML approach which minimizes:

$$\ln(\det \mathcal{R}) + \left\| \mathcal{T}(\mathbf{P})\mathbf{Y} - (\tilde{I}\mathcal{P} \begin{bmatrix} A_1^\circ \\ 0 \end{bmatrix}) \otimes \mathbf{h}(0) \right\|_{\mathcal{R}^{-1}}^2 \quad (22)$$

Like for the channel based approaches, the GML method appears better than the GMAPML.

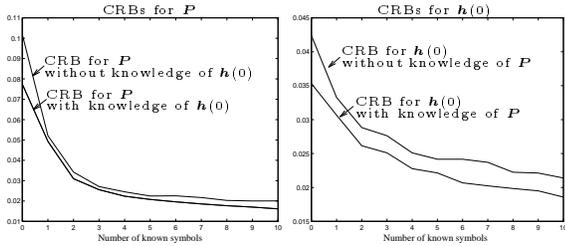


Figure 3. CRBs for \mathbf{P} and $\mathbf{h}(0)$

4.1. CRBs.

We study here the GML CRB for $\begin{bmatrix} \mathbf{p}(1:L) & \mathbf{h}(0) & \sigma_v^2 \end{bmatrix}$, which is asymptotically reached. The SNR is of 10dB, $M=50$. The channel is:

$$\begin{bmatrix} 1.0000 & 0.8000 \\ -1.5000 & 1.4000 \end{bmatrix}$$

In fig.4.1., again, a significant improvement has to be noticed as more and more symbols are known. The quality of estimation of $\mathbf{h}(0)$ has a certain influence on that of \mathbf{P} when very few symbols are known. The quality of estimation of $\mathbf{h}(0)$ is however dependent on that of \mathbf{P} even with a large number of known symbols.

4.2. Methods

4.2.1. Iterative Quadratic ML (IQML)

This method is based on GMAPML. At iteration (i), we get to solve:

$$\min_{\mathbf{p}^{(i)}(1:L), \mathbf{h}^{(i)}(0)} \left\| \mathcal{T}(\mathbf{P}^{(i)})\mathbf{Y} - (\tilde{\mathcal{I}}\mathcal{P} \begin{bmatrix} A_1^o \\ 0 \end{bmatrix}) \otimes \mathbf{h}^{(i)}(0) \right\|_{(\mathcal{R}^{(i-1)})^{-1}} \quad (23)$$

$$\mathcal{R}^{(i-1)} = \sigma_v^2 \mathcal{T}(\mathbf{P}^{(i-1)})\mathcal{T}^H(\mathbf{P}^{(i-1)}) + (\tilde{\mathcal{I}}R_{AA}\tilde{\mathcal{I}}^H) \otimes (\mathbf{h}^{(i-1)}(0)\mathbf{h}^{(i-1)H}(0)) \quad (24)$$

IQML is not guaranteed to converge. However, if it is initialized by a consistent estimate for \mathbf{P} and $\mathbf{h}(0)$, it converges sufficiently at the first iteration.

We simulated this algorithm for $M=100$, SNR=7dB and 10 dB, and the following channel:

$$\begin{bmatrix} 1.0000 & 0.8000 & 0.5000 & 0.6000 & 0.1000 \\ -1.5000 & 1.4000 & -0.9000 & 1.1000 & -0.0300 \end{bmatrix}$$

The number of known symbols is successively: 20, 10, 5 and 0. Two initializations were used: initialization by training sequence when 20 symbols are known and initialization by a blind algorithm based on linear prediction [6] tested for all cases. For 10 and 5 known symbols, we either kept the $\mathbf{h}(0)$ of the initialization or reevaluated it at each iteration. When no symbols are known, we kept the $\mathbf{h}(0)$ of the initialization. We get back the channel from \mathbf{P} and $\mathbf{h}(0)$ by solving equation (18) in the least squares sense.

We notice that even with 5 known symbols, the estimation of $\mathbf{h}(0)$ gets improved by the algorithm. It appears to be preferable to do only one iteration of the IQML. Furthermore, the performance is good when 20 symbols are known.

4.2.2. PQML

The estimate of $\mathbf{h}(0)$ can be refined by a PQML method applied to the GML cost function.

After the submission of this paper, we became aware of [10] in which a semiblind GML method and corresponding Cramer-Rao Bound have also been pursued. The modeling of the training-sequence information in [10] is incorrect

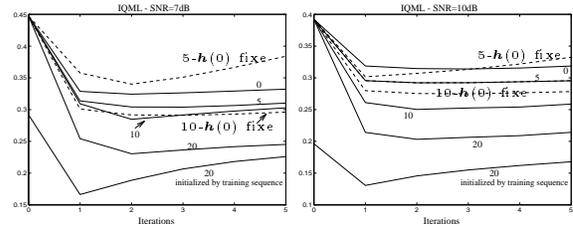


Figure 4. Normalized estimation errors for \mathbf{h} (averaged over 100 noise realizations)

though (instead of the training sequence, the information considered in [10] is the training sequence times an unknown zero-mean unit-variance normal variable).

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