CYCLOSTATIONARITY IN PARTIAL RESPONSE SIGNALING: A NOVEL FRAMEWORK FOR BLIND EQUALIZATION

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ABSTRACT

When fractional samples are available at the receiver, blind channel estimation methods can be developed exploiting the cyclostationary nature of the received signal. In this paper, we show that different solutions are possible if cyclostationarity is introduced at the transmitter instead of the receiver. We propose specific coding and interleaving strategies at the transmitter which induce cyclostationarity and facilitate the equalization task. Novel subspace equalization algorithms are derived which make no assumptions whatsoever on the channel zeros locations. Synchronization issues are briefly discussed and some simulation examples are presented.

1. INTRODUCTION

Recent advances in Fractionally Spaced Equalizers (FSEs) have made blind equalization possible for a large class of channels, without resorting to higher-order statistics [5], [3], [1]. In this way, blind equalizers have been significantly improved in terms of convergence speed.

Sampling at a fraction of the symbol rate at the receiver, introduces diversity and transforms the stationary, channel estimation problem to a cyclostationary one [1]. It is precisely the cyclostationarity of the received signal that facilitates the estimation of the channel from second order information only. A natural question raised by this observation, is whether other ways of inducing cyclostationarity at the received signal can be equally (or more) beneficial; for example, by manipulating the information signal prior to transmission (coding/interleaving).

In this paper we propose a novel transmission strategy, which incorporates elements of both partial response channels and FSEs. Coding and interleaving of the input symbols prior to their transmission is introduced to facilitate ISI removal from output data only. The proposed method overcomes limitations of FSE approaches because it guarantees identifiability of *all FIR channels* regardless of zero locations without resorting to high-order statistics. In contrast to FSEs, the new method is also robust to model order mismatch.

Traditionally, channel coding has been performed with the sole objective of error correction in mind and with little concern about channel dispersion problems. On the other hand, channel equalization methods typically assume i.i.d. inputs ignoring any possible coding at the transmitter. In this paper, we introduce a novel viewpoint where coding information can be exploited to facilitate the receiver's equalization task.

The price paid for these advantages is the introduction of a small decoding delay, equal to a few symbol periods, due to coding and interleaving in the transmitter. Also a moderate increase in the transmitter complexity is introduced, along with some controlled ISI.

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2. PROBLEM STATEMENT

Let us consider the interleaving procedure of Fig. 1, where the input signal is partitioned in successive blocks of length M, $\mathbf{w}_M(l) = [w(Ml), \ldots, w(Ml+M-1)]^T$ and each block is transmitted twice, i.e., $\mathbf{w}_b = [\mathbf{w}_M^T(0), \mathbf{w}_M^T(0), \mathbf{w}_M^T(1), \mathbf{w}_M^T(1), \ldots]^T$; more formally, the transmitted signal is

$$w_b(2Ml+k) = \begin{cases} w(Ml+k) & 0 \le k < M \\ w(Ml+k-M) & M \le k < 2M \end{cases} . \quad (1)$$

If we consider linear modulation, then the received continuous time signal is

$$y_c(t) = \sum_{k=-\infty}^{\infty} w_b(k) h_c(t - kT/2) + v_c(t) \ , \quad (2)$$

where T is the symbol period, $h_c(t)$ is the impulse response of the channel (and spectral shaping filters) and $v_c(t)$ is additive noise. After sampling at the receiver, the equivalent baseband signal is

$$y(n) = \sum_{k=0}^{q} h(k) w_b(n-k) + v(n) \ , \quad (3)$$

where $h(n) := h_c(nT/2)$ is an FIR impulse response of order q, and $v(n)$ is additive, white, zero mean noise.

A number of different interpretations can be given to this repetition framework. If the pulse bandwidth remains unchanged (equal to $1/T$) then some controlled ISI is introduced at the transmitter (there is more overlap between successive pulses due to the increased data rate). In this respect, the scheme is similar to partial response signaling (e.g., [4, pg. 548]), where controlled ISI is introduced to simplify the pulse design. The induced ISI is expected to have a negative effect in performance, but in partial response channels, this effect has been observed to be minimal [4]. One might be tempted to discard all repetition based techniques by arguing that it is preferable to insert training symbols in the place of repeated symbols and perform trained equalization. This argument ignores the fact that in this case half of the transmitter's power would be devoted to training (for $P = 2$), resulting in a 3dB penalty even under perfect ISI removal.

On the other hand, if the transmitter's spectral pulse bandwidth increases by a factor of 2 to avoid inducing ISI, then the scheme resembles a repetition coding setup. However, due to the poor error correcting performance of repetition codes, we will not pursue this direction any further.

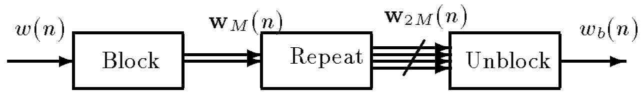

Figure 1. Repetition Coding with Interleaving

Note that our main goal in this paper is concerned with combating ISI rather than achieving coding gain. One exception where the increased bandwidth could be tolerated is in spread spectrum and CDMA applications (see [8] for related results in a multiuser setup and for a repetition factor greater than two).

The problem addressed in this paper is the estimation of $h(n)$ given the setup of (1), (3). It was shown in [6] that in this setup (and if M is chosen such that $q < M$), the impulse response $h(n)$ is a scalar multiple of some selected autocorrelation lags. A simple adaptive algorithm was developed based on this observation, assuming that the input is i.i.d. Here, we propose a subspace method which in the absence of noise guarantees perfect channel estimates. The inaccessible input must be persistently exciting (see also [3]), but apart from that it can be deterministic or random (white or colored).

3. VECTOR FORMULATION

It is easy to verify from (1), that the maximum rate change is $2M$ and that $w_b(n)$ (and hence $y(n)$) has periodically time-varying statistics with period $2M$. Therefore, in order to stationarize the (repetition induced) cyclostationary problem, we will consider a polyphase vector representation of order $2M$. The polyphase components of $h(n)$ are defined as $h_k(n) \triangleq h(2Mn + k)$, $k = 0, 1, \ldots, 2M-1$, and represent different decimated versions of the original impulse response $h(n)$. Using vector notation, we define $\mathbf{h}_{2M}(n) \triangleq [h_0(n), \ldots, h_{2M-1}(n)]^T$ and its z-transform $\mathbf{h}_{2M}(z) \triangleq [h_0(z), \ldots, h_{2M-1}(z)]^T$. Similar representations of $w_b(n)$, $v(n)$ and $y(n)$ are also possible.

The channel input/output relationship can be expressed in a polyphase form as [9, p. 431],

$$\mathbf{y}_{2M}(z) = \mathbf{H}_{2M}(z)\mathbf{w}_{b,2M}(z) + \mathbf{v}_{2M}(z) \ , \tag{4}$$

where $\mathbf{H}_{2M}(z)$ is a pseudo-circulant matrix of the polyphase vector $\mathbf{h}_{2M}(z)$,

$$\mathbf{H}_{2M}(z) = \begin{bmatrix} h_0(z) & z^{-1}h_{2M-1}(z) & \cdots & z^{-1}h_1(z) \\ h_1(z) & h_0(z) & \ddots & \vdots \\ \vdots & \vdots & \ddots & z^{-1}h_{2M-1}(z) \\ h_{2M-1}(z) & h_{2M-2}(z) & \cdots & h_0(z) \end{bmatrix} \tag{5}$$

With $q < M$, matrix $\mathbf{H}_{2M}(z)$ in (5) is a constant matrix. Hence, equation (4) can be easily written in the time domain as

$$\begin{aligned} \mathbf{y}_{2M}(n) &= \begin{bmatrix} \mathbf{H}_{11} \\ \mathbf{H}_{11} + \mathbf{H}_{21} \end{bmatrix} \mathbf{w}_M(n) \\ &+ \begin{bmatrix} \mathbf{H}_{21} \\ \mathbf{0} \end{bmatrix} \mathbf{w}_M(n-1) + \mathbf{v}_{2M}(n) \ , \end{aligned} \tag{6}$$

where[2] $\mathbf{H}_{11}$ is lower-triangular and $\mathbf{H}_{21}$ is upper-triangular with first all zero column,

$$\mathbf{H}_{11} = \begin{bmatrix} h(0) & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ h(M-1) & \cdots & h(0) \end{bmatrix} , \tag{7}$$

$$\mathbf{H}_{21} = \begin{bmatrix} 0 & h(M-1) & \cdots & h(1) \\ \vdots & \ddots & \ddots & \vdots \\ 0 & & 0 & h(M-1) \\ 0 & \cdots & & 0 \end{bmatrix} . \tag{8}$$

[2] If $q < M-1$, the samples $h(q+1), \ldots, h(M-1)$ equal to zero and simply denote the zero padded extention of $h(n)$ in what follows.

4. PROPOSED METHOD

Let us write the received data vector $\mathbf{y}_{2M}(n) \triangleq [y(2Mn), y(2Mn+1), \ldots, y(2Mn+2M-1)]^T$ as $\mathbf{y}_{2M}(n) = [\mathbf{y}_{M,1}^T(n),\ \mathbf{y}_{M,2}^T(n)]^T$, where $\mathbf{y}_{M,1}(n)$ [$\mathbf{y}_{M,2}(n)$] denotes the first [last] M components of $\mathbf{y}_{2M}(n)$. With similar notation for the noise vector $\mathbf{v}_{2M}(n) = [\mathbf{v}_{M,1}^T(n),\ \mathbf{v}_{M,2}^T(n)]^T$ and the transmitted data vector $\mathbf{w}(n)$, it follows from (6) that

$$\begin{aligned} \mathbf{y}_{M,2}(n) - \mathbf{y}_{M,1}(n) &= \mathbf{H}_{21}[\mathbf{w}_M(n) - \mathbf{w}_M(n-1)] \\ &+ \mathbf{v}_{M,2}(n) - \mathbf{v}_{M,1}(n) \ , \end{aligned} \tag{9}$$

and

$$\begin{aligned} \mathbf{y}_{M,1}(n) - \mathbf{y}_{M,2}(n-1) &= \mathbf{H}_{11}[\mathbf{w}_M(n) - \mathbf{w}_M(n-1)] \\ &+ \mathbf{v}_{M,1}(n) - \mathbf{v}_{M,2}(n) \ . \end{aligned} \tag{10}$$

Based on (9), (10), we will estimate the channel's impulse response. If we let $\tilde{\mathbf{w}}_M(n) = \mathbf{w}_M(n) - \mathbf{w}_M(n-1)$, $\tilde{\mathbf{y}}_{2M}(n) = [(\mathbf{y}_{M,2}(n) - \mathbf{y}_{M,1}(n))^T,\ (\mathbf{y}_{M,1}(n) - \mathbf{y}_{M,2}(n-1))^T]^T$ and $\tilde{\mathbf{v}}_{2M}(n) = [(\mathbf{v}_{M,2}(n) - \mathbf{v}_{M,1}(n))^T,\ (\mathbf{v}_{M,1}(n) - \mathbf{v}_{M,2}(n-1))^T]^T$, then (9) becomes

$$\tilde{\mathbf{y}}_{2M}(n) = \mathcal{T}(\mathbf{h})\tilde{\mathbf{w}}_M(n) + \tilde{\mathbf{v}}_{2M}(n) \ , \tag{11}$$

where the $2M \times M$ matrix $\mathcal{T}(\mathbf{h})$ is given by

$$\mathcal{T}(\mathbf{h}) = \begin{bmatrix} h(0) & & 0 \\ \vdots & \ddots & \\ h(M-1) & & h(0) \\ 0 & \ddots & \vdots \\ \vdots & \ddots & h(M-1) \\ 0 & \cdots & 0 \end{bmatrix} . \tag{12}$$

Based on (12), the correlation matrix of $\tilde{\mathbf{y}}_{2M}(n)$ is

$$\mathbf{R}_{\tilde{\mathbf{y}}}(0) = \mathcal{T}(\mathbf{h})\mathbf{R}_{\tilde{\mathbf{w}}}(0)\mathcal{T}^{*T}(\mathbf{h}) + \mathbf{R}_{\tilde{\mathbf{v}}}(0) \ , \tag{13}$$

and has full rank under a persistence of excitation assumption. Then, it was shown in [3] in a different context, that $h(k)$ can be uniquely identified (within a scaling ambiguity), from the equations

$$\mathbf{G}^{*T}\mathcal{T}(\mathbf{h}) = \mathbf{0} \tag{14}$$

where $\mathbf{G} \triangleq [\mathbf{g}_1, \ldots, \mathbf{g}_M]$ is a collection of the null subspace eigenvectors. Equation (14) was used in [3] to estimate a FS channel. Similarly, (14) can be used in the current framework, after an eigenvalue decomposition of $\mathbf{R}_{\tilde{\mathbf{y}}}(0)$ in (13). The Toeplitz matrix $\mathcal{T}(\mathbf{h})$ is not parametrized in exactly the same way as in [3]. However, the identifiability result of [3] holds here too (see [7]).

Notice that in the absence of noise, (13) holds true even when $\mathbf{R}_{\tilde{\mathbf{y}}}(0)$, $\mathbf{R}_{\tilde{\mathbf{w}}}(0)$ are replaced by $\hat{\mathbf{R}}_{\tilde{\mathbf{y}}}(0)$, $\hat{\mathbf{R}}_{\tilde{\mathbf{w}}}(0)$, and

(14) is still an exact solution, as long as $\hat{\mathbf{R}}_{\tilde{\mathbf{w}}}(0)$ has full rank. Hence, no independence assumption on the input is required, and exact solutions can be found from finite data lengths, so long as the input is persistently exciting (see also [3]).

If noise is present however, the structure of $\mathbf{R}_{\tilde{\mathbf{v}}}(0)$ needs to be considered. If we assume $v(n)$ to be additive white noise of variance σ_v^2, then from the definition of $\tilde{\mathbf{v}}_{2M}(n)$, we can verify that

$$\mathbf{R}_{\tilde{\mathbf{v}}}(0) = \sigma_v^2 \mathcal{I}_{2M} = \sigma_v^2 \begin{bmatrix} 2\mathbf{I} & -\mathbf{I} \\ -\mathbf{I} & 2\mathbf{I} \end{bmatrix} . \tag{15}$$

In this case the matrix pencil of the matrices $(\mathbf{R}_{\tilde{\mathbf{v}}}(0), \mathcal{I}_{2M})$ needs to be used. It can be shown that the last M generalized eigenvalues of the two matrices equal σ_v^2, while the corresponding generalized eigenvectors span the null subspace of $\mathcal{T}(h)\mathbf{R}_{\tilde{\mathbf{w}}}(0)\mathcal{T}^{*T}(h)$. Therefore, a QZ decomposition algorithm should be used [2], to obtain the eigenvector matrix $\mathbf{G}$ needed in (14).

The resulting subspace algorithm can provide an exact solution from a finite number of data points, in the absence of noise. Contrary to FSE subspace solutions [3], the proposed method is not sensitive to channel order overestimation, or to the zeros' location (provided that $q < M$). It therefore obviates the need for statistical tests on the eigenvalues to estimate the correct channel order.

5. SYNCHRONIZATION

The subspace channel estimation method developed in the previous section illustrate the versatility of the proposed transmission framework. However, it relies on the tacit assumption that the receiver has knowledge of the timing instant at which each block of $2M$ symbols begins. While carrier and symbol timing information is crucial in many equalization methods, the current approach requires *block* timing on top of symbol timing information. Since the availability of such timing information in a blind scenario is not obvious, this matter deserves further discussion.

In the sequel, we will show that block timing information can be retrieved from the statistics of the received signal. We focus on suboptimal but simpler solutions, as opposed to more involved maximum likelihood formulations.

Let us assume that the observed signal is

$$\tilde{y}(n) = y(n-d) \quad , \quad 0 \le d \le 2M-1 \ \ mod \ \ 2M \ , \tag{16}$$

received with a delay of d symbol periods (symbol synchronization is assumed). Then, the correlation of two data points that are M samples apart is

$$\begin{aligned} r(\tau) &= E\{\tilde{y}^*(2Mn+\tau)\tilde{y}(2Mn+M+\tau)\} \qquad (17)\\ &= E\{y^*(2Mn+\tau-d) - y(2Mn+M+\tau-d)\} \end{aligned}$$

It can be shown that for $\tau - d = 0$, $r(0) = \sigma_w^2 |h(0)|^2$. This is so because the data points $y^*(2Mn)$, $y(2Mn+M)$ share only one common input point $w(Mn)$ multiplying the factors $h^*(0)$, $h(0)$ respectively. If $0 < \tau - d < M$ $(mod\ 2M)$, it follows from (1), (3) that the two data points $y^*(2Mn+\tau-d)$, $y(2Mn+M+\tau-d)$ depend on the common inputs $w(Mn), \ldots, w(Mn+\tau-d)$, while if $M \le \tau - d < 2M$, they depend on $w(\tau-d-M+1), \ldots, w(M-1)$. Hence, $r(\tau)$ may expressed in terms of the channel parameters as

$$r(\tau) = \begin{cases} \sigma_w^2 \sum_{k=0}^{\tau-d} |h(k)|^2 & 0 \le \tau - d < M \\ \sigma_w^2 \sum_{k=\tau-d-M+1}^{M-1} |h(k)|^2 & M \le \tau - d < 2M \end{cases} \tag{18}$$

Notice that $r(\tau)$ is non-decreasing for $\tau \in [0, M-1]$ (more terms are added to (18)), while it is non-increasing for $\tau \in [M, 2M-1]$. Fig. 2 shows the values of $r(\tau)$ for $M = 5$ and for a particular channel of order $q = 3$, described in the simulations section. The offset here is $d = 5$ as can be deduced from the monotonicity of the graph in each block of length M.

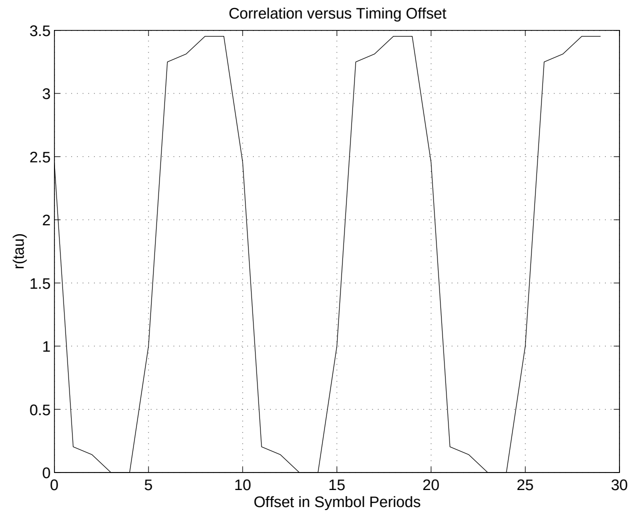

Figure 2. Correlation based synchronization

If an automated procedure for estimating d is desired based on (18), then a statistical test is needed to check the monotonicity of $r(\tau)$. Let $\hat{r}(\tau)$ be the sample estimate

$$\hat{r}(\tau) = \frac{1}{N_b} \sum_{n=0}^{N_b-1} \tilde{y}^*(2Mn+\tau)\tilde{y}(2Mn+M+\tau) \ , \tag{19}$$

and define the differences

$$\hat{e}(\tau) = \hat{r}(\tau) - \hat{r}(\tau-1) \ . \tag{20}$$

Then, d can be estimated by maximizing the following cost function

$$\hat{d} = \arg \max_{0 \le d' \le 2M-1} \left[\sum_{\tau=d'}^{d'+M-1} \hat{e}(\tau) - \sum_{\tau=d'+M}^{d'+2M-1} \hat{e}(\tau) \right] . \tag{21}$$

The test of (21) exploits the fact that $e(\tau) \ge 0$ for $\tau \in [d, d+M-1]$ and $e(\tau) \le 0$ for $\tau \in [d, d+M-1]$.

Statistical performance analysis of (21) (e.g., evaluation of the probability of incorrect decision) follows standard steps and exploits the asymptotic normality of $\hat{r}(\tau)$ (and hence of $\hat{e}(\tau)$). However, we will not pursue it any further here, due to lack of space.

6. SIMULATIONS

Some simulation results are presented in this section to illustrate the advantages of the proposed method when compared with alternative fractionally spaced schemes. In all the simulations that follow, 4-QAM i.i.d. symbols were generated and after being interleaved according to (1) (with $M = 5$), they were transmitted through the channel $h(z) = 1 - 1.5z^{-1} + 0.25z^{-2} - 0.375z^{-3}$, with zeros at $\pm 0.5j$, and 1.5. The channel was specifically chosen so that it is not identifiable using FSE methods (with an oversampling of 2), since the zeros at $\pm 0.5j$ are not resolvable. Moreover, we have overestimated the channel order (assuming $q = 4$ while the actual $q = 3$), to show the insensitivity of the proposed methods to model order mismatch.

Figure 3 shows the true channel coefficients as well as the estimated ones from 100 Monte Carlo realizations (mean $\pm$ standard deviation). In Fig. 3a the results of the FSE method of [3] are shown, which are not satisfactory, while in Figures 3b to 3c the performance of the method of [6] and the proposed method are depicted respectively. The SNR was 30 dB (relatively high), which explains the superiority of the exact subspace method depicted in Fig. 3c. The data length was 100 symbols for the exact methods of Fig. 3a and 3c, and 1,000 symbols for the statistical method of Fig. 3b. Block timing information was obtained from Fig. 2.

The difference in performance between the FSE and the proposed methods can also be seen in Fig. 4. In Fig. 4a, the received symbols are plotted on a constellation graph, while in Fig. 4b, the equalized symbols are plotted, using the MMSE equalizer based on the channel estimates. Similarly, Figures 4c and 4d show the same scenario when the equalizer is designed using the channel estimates provided by the FSE algorithm of [3]. Clearly, the equalizer in the latter case does not succeed in removing the ISI.

To obtain a more quantitative description of the above performance comparisons, we tested the FSE and the proposed subspace based equalizers with respect to the probability of symbol errors for different SNR levels. The resulting curves are shown in Fig. 5, where the FSE method clearly fails to decode the received symbols at any SNR level. In this experiment, 10,000 Monte Carlo realizations were averaged per SNR point.

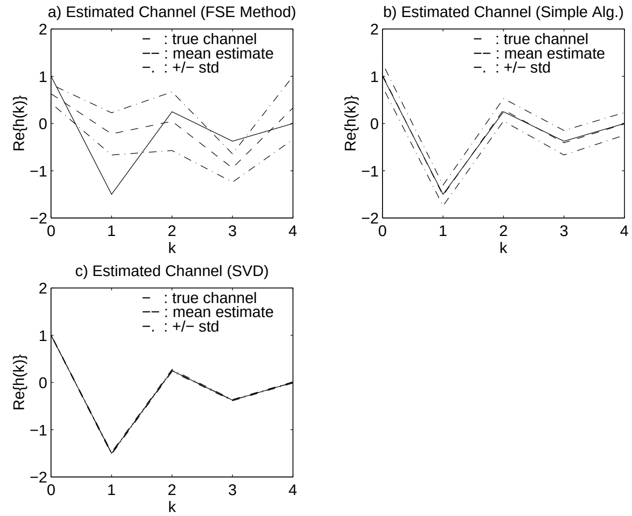

Figure 3. True and estimated channel tap coefficients

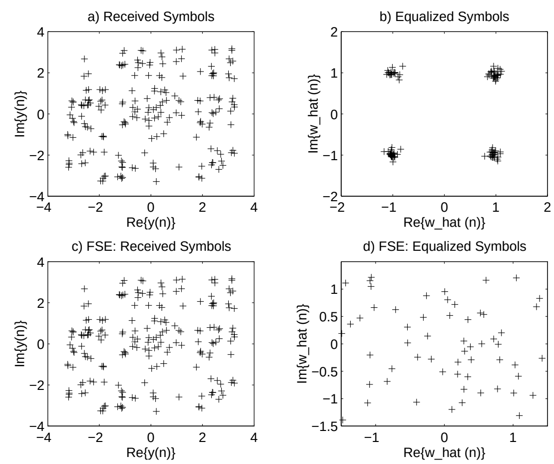

Figure 4. Received symbols: Before and after equalization

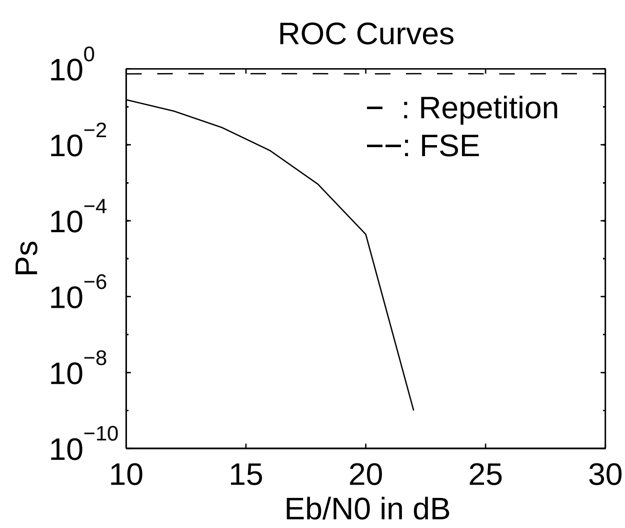

Figure 5. Probability of Symbol Error vs SNR

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