BLIND WIENER FILTER ESTIMATION FOR MULTI-CHANNEL SYSTEMS BASED ON PARTIAL INFORMATION

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ABSTRACT

In a multi-user system where training is not available, blind channel identification and equalization become essential. In this paper, we present a new method that utilizes second order statistics for channel parameter estimation and optimum filtering. The identification algorithm is simple and relies on an outer-product decomposition and partial information of the desired signal channel. It allows the identification of individual user channels for which partial information is known under a specific condition. An optimum receiver structure can then be established for the desired signal channel.

1. INTRODUCTION

In many data communication systems such as the digital wireless systems, data signals are often transmitted through unknown channels which may introduce severe linear distortion. In order to improve the system performance, it is important for the receiver to identify and remove channel distortions through equalization or sequence estimation. Because input training signal may be too short or even nonexistent for channel identification, blind channel identification can play useful roles in these systems. The blind identification of single user system can rely on second order cyclic statistics as shown in [1].

In multi-user and multi-channel systems, the blind identification of multiple channel responses often produce a set of ambiguous results. It has been previously established [2] that blind channel identification can only identify channel responses subject to an instantaneous mixing matrix. Hence, signal separation becomes necessary [3]. Traditionally, signal separation is based on additional information of the source signals. When several users transmit similar signals, source separation becomes a challenging problem.

In this paper, a knowledge based method is presented to resolve this ambiguity. Given the identified ambiguous channel response matrix, a priori information on desired users can be used to further identify the unknown channels uniquely. Their Wiener filters can subsequently be estimated for signal recovery and separation. This prior knowledge may take the form of CDMA spreading codes or pulse-shaping filters of the desired signal channel.

2. PROBLEM FORMULATION

A multi-user QAM data communication system can be described using a baseband representation. Assume that the N user channels are all linear and causal with impulse response $\{h_u(t),\ \ u=1,2,\ldots N\ \}$, the received output signal can be written as

$$x(t)=\sum_{u=1}^{N}\sum_{k=-\infty}^{\infty} s_{k,u}h_u(t-kT)+w(t), \quad s_{k,u}\in\mathcal{A}_u, \quad (2..1)$$

where T is the symbol baud period and $\mathcal{A}_u$ is the input signal set of user u. The channel input sequences $\{s_{k,u}\}$ are typically independent for different users and are also i.i.d. The noise $w(t)$ is stationary, white, and independent of channel input sequences $s_{k,u}$, but not necessarily Gaussian.

Note that $h_u(t)$ is a "composite" channel impulse response that includes transmitter and receiver filters as well as the physical channel response. In a typical multi-user system, multiple channels of observations will be available from multiple sensors. If J sub-channels (sensors or antennas) exist, then $x(t)$, $h_u(t)$, and $w(t)$ are all $J\times 1$ vectors.

It has been shown [1] that more channel diversity may result from oversampling the channel outputs. Let the sampling interval be $\Delta=T/p$ where p is an integer. The oversampled discrete signals and responses are

$$x_i \triangleq x(i\Delta), \quad h_u[i]\triangleq h_u(i\Delta) \quad \text{and} \quad w_i\triangleq w(i\Delta). \quad (2..2)$$

The channel output samples are hence

$$x_n=\sum_{u=1}^{N}\sum_{k=-\infty}^{\infty} s_{k,u}h_u[n-kp]+w_n.$$

Suppose $\{h_u(t)\}$ has joint finite support $[0,T_h)$, which spans m_0+1 integer baud periods. By defining notations

$$\begin{aligned}
s_k &\triangleq [s_{k,1}\ \ s_{k,2}\ \ \ldots\ \ s_{k,N}\ ];\\
\mathbf{s}[k] &\triangleq [s_k\ \ s_{k-1}\ \ \ldots\ \ s_{k-m_0-M+1}]'\\
\mathbf{w}[k] &\triangleq [w_{kp}\ \ w_{kp+1}\ \ \ldots\ \ w_{kp-Mp+1}]'\\
\mathbf{h}_u[i] &\triangleq \begin{bmatrix} h_u[ip]\\ h_u[ip+1]\\ \vdots\\ h_u[ip+p-1]\end{bmatrix},\\
\mathbf{H}_i &\triangleq [\mathbf{h}_1[i]\ \mathbf{h}_2[i]\ \ldots\ \mathbf{h}_N[i]],
\end{aligned}$$

we form an $MpJ\times(m_0+M)N$ block Toeplitz matrix

$$\mathbf{H}=\begin{bmatrix}
\mathbf{H}_0 & \mathbf{H}_1 & \ldots & \mathbf{H}_{m_0} & \mathbf{0} & \ldots & \mathbf{0}\\
\mathbf{0} & \mathbf{H}_0 & \mathbf{H}_1 & \ldots & \mathbf{H}_{m_0} & \ddots & \vdots\\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \mathbf{0}\\
\mathbf{0} & \ldots & \mathbf{0} & \mathbf{H}_0 & \mathbf{H}_1 & \ldots & \mathbf{H}_{m_0}
\end{bmatrix}. \quad (2..3)$$

Clearly, m_0 is the order of the N dynamic FIR channels. With these notations, a sampled channel output signal vector of length Mp can be written as

$$\mathbf{x}[k] \triangleq \begin{bmatrix} x_{kp} \\ x_{kp+1} \\ \vdots \\ x_{kp+p-1} \\ x_{(k-1)p} \\ x_{(k-1)p+1} \\ \vdots \\ x_{(k-M+1)p+p-1} \end{bmatrix} = \mathbf{H}\mathbf{s}[k] + \mathbf{w}[k]. \qquad (2..4)$$

A rational fractional sampling generates an equivalent multi-user system. If $\Delta = qT/p$, $\mathbf{H}$ is simply an $MpJ \times (M + m_0)Nq$ block Toeplitz matrix with qN equivalent users [2].

It has been established [1] that the sufficient and necessary identification condition for $\mathbf{H}$ to be identifiable from second order statistics is that $\mathbf{H}$ must be full rank. A necessary dimensional condition requires that $Jp \geq Nq$.

3. CHANNEL IDENTIFICATION AND SIGNAL SEPARATION

3.1. Knowledge-Based Channel Identification

Assume that both the channel input signal and channel noise are white with zero mean. Let their respective covariance matrix be $R_s = E\{\mathbf{s}[k]\mathbf{s}[k]^H\} = \sigma_s^2 I$ and $R_w = E\{\mathbf{w}[k]\mathbf{w}[k]^H\} = \sigma_w^2 I$. Based on (2..4), the channel output covariance matrix becomes

$$R_{m_0} = E\{\mathbf{x}[k]\mathbf{x}[k]^H\} = \sigma_s^2 \mathbf{H}\mathbf{H}^H + \sigma_w^2 I \qquad (3..1)$$

We will form the channel parameter matrix

$$\mathcal{H} \triangleq \begin{bmatrix} \mathbf{H}_0 \\ \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_{m_0} \end{bmatrix}. \qquad (3..2)$$

In [2] we derived a method to estimate the channel parameter matrix

$$\mathcal{H}Q^H$$

where Q is an $N \times N$ unitary matrix. Recall from [3][4] that this memoryless mixing matrix is intrinsic to the multi-user blind identification problem and cannot be resolved unless additional information is available.

To identify a specific channel, note that in many communication systems, part of the composite signal channel, such as the pulse-shaping filter or the CDMA spreading signal, are known to the receiver. We would like to identify the unknown channel $c(t)$ based on the known filter response $f(t)$ for signal separation and channel identification. The development of this method is as follows. Let

$$c[i] \triangleq c(i\Delta), \quad i = 0,\ 1,\ \ldots,\ m_1 p - 1 \qquad (3..3)$$

$$f[i] \triangleq f(i\Delta), \quad i = 0,\ 1,\ \ldots,\ n_2. \qquad (3..4)$$

The sampled channel impulse response becomes

$$h[i] = \sum_{k=0}^{i} c[i-k] f[k]. \qquad (3..5)$$

Now define

$$C_i \triangleq [\ c[ip] \quad c[ip+1] \quad \ldots \quad c[ip+p-1]\ ]', \qquad (3..6)$$

$$\mathcal{F} \triangleq \begin{bmatrix} f_0 & 0 & \ldots & 0 \\ f_1 & f_0 & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ f_{n_2} & \ddots & \ddots & f_0 \\ 0 & f_{n_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ldots & f_{n_2} \end{bmatrix} \qquad (3..7)$$

and

$$\mathbf{c} \triangleq \begin{bmatrix} C_0 \\ C_1 \\ \vdots \\ C_{m_1-1} \end{bmatrix}. \qquad (3..8)$$

The overall channel response vector for i-th user can be written as

$$\mathcal{H} = [\ \mathcal{F}_1\mathbf{c}_1 \quad \mathcal{F}_2\mathbf{c}_2 \quad \ldots \quad \mathcal{F}_N\mathbf{c}_N\ ]. \qquad (3..9)$$

Given the ambiguous multi-channel matrix

$$B \triangleq \mathcal{H}Q^H \qquad (3..10)$$

we would like to identify each individual channel based on a priori knowledge. Without loss of generality, assume that a priori knowledge on $f_1(t)$ is available for the first channel.

We can write $Q = [\ \mathbf{q}_1 \quad \mathbf{q}_2 \quad \ldots \quad \mathbf{q}_N\ ]$. It is therefore easy to see that

$$\mathcal{F}_1\mathbf{c}_1 = B\mathbf{q}_1 \qquad (3..11)$$

Now we need to show that the non-zero solution to the equation

$$[\mathcal{F}_1 \quad B] \begin{bmatrix} \widehat{\mathbf{c}}_1 \\ -\widehat{\mathbf{q}}_1 \end{bmatrix} = 0, \qquad (3..12)$$

admits a unique channel estimate $\widehat{\mathbf{c}}_1 = \mathbf{c}_1$.

3.2. Conditions for Unique Solution

From

$$\mathcal{F}_1\widehat{\mathbf{c}}_1 = \sum_{i=1}^{N} \mathcal{F}_i\mathbf{c}_i\mathbf{q}_i^H\widehat{\mathbf{q}}_1 \qquad (3..13)$$

we have,

$$\mathcal{F}_1(\mathbf{c}_1\mathbf{q}_1^H\widehat{\mathbf{q}}_1 - \widehat{\mathbf{c}}_1) + \sum_{i=2}^{N} \mathbf{H}_i\mathbf{q}_i^H\widehat{\mathbf{q}}_1 = 0. \qquad (3..14)$$

Denote $\Delta\mathbf{c}_1 \triangleq \mathbf{c}_1\mathbf{q}_1^H\widehat{\mathbf{q}}_1 - \widehat{\mathbf{c}}_1$. We find

$$\mathcal{F}_1\Delta\mathbf{c}_1 = -\sum_{i=2}^{N} \mathbf{H}_i(\mathbf{q}_i^H\widehat{\mathbf{q}}_1).$$

To understand this condition, we can define the following $z-$transforms

$$F_i(z) = \sum_{k=0}^{n_2} f_i[k]z^{-k}; \quad C_i(z) = \sum_{k=0}^{m_1-1} c_i[k]z^{-k};$$

$$\Delta C_i(z) = \sum_{k=0}^{m_1-1} \Delta\mathbf{c}_i[k]z^{-k}.$$

Since $\mathcal{F}_1$ is Toeplitz, $z-$transform of equation (3..12) results in

$$F_1(z)\Delta C_1(z) + \sum_{i=2}^{N}(\mathbf{q}_i^H \widehat{\mathbf{q}}_1) H_i(z) = 0. \qquad (3..15)$$

If $\{H_i(z), \quad i = 2,\ 3,\ \ldots\ N\}$ are linearly independent of $F_1(z)L(z)$ for all $L(z)$, then we have

$$\Delta C_1(z) = 0 \quad \text{and} \quad \mathbf{q}_i^H \widehat{\mathbf{q}}_1 H_i(z) = 0, \quad i = 2,\ 3,\ \ldots,\ N. \qquad (3..16)$$

Because of the assumption

$$H_i(z) \neq 0 \quad \text{for} \quad i = 2,\ 3,\ \ldots, N,$$

the identified eigenvector $\widehat{\mathbf{q}}_1$ is orthogonal to $\mathbf{q}_i$

$$\mathbf{q}_i^H \widehat{\mathbf{q}}_1 = 0. \qquad (3..17)$$

Therefore, so long as $\widehat{\mathbf{q}}_1 \neq 0$,

$$\widehat{\mathbf{q}}_1 = ||\widehat{\mathbf{q}}_1|| \mathbf{q}_1. \qquad (3..18)$$

Finally, the desired channel response is identified from (3..12) since

$$\widehat{\mathbf{c}}_1 = (\mathbf{q}_1^H \widehat{\mathbf{q}}_1)\mathbf{c}_1 = ||\widehat{\mathbf{q}}_1||\mathbf{c}_1. \qquad (3..19)$$

To summarize, the nonzero solution to (3..12) allows the unique identification of $\widehat{\mathbf{c}}_1 = ||\widehat{\mathbf{q}}_1||\mathbf{c}_1$ *if* $\{H_i(z), \quad i = 2,\ 3,\ \ldots\ N\}$ *are linearly independent of* $F_1(z)L(z)$ *for all* $L(z)$.

When the number of users N is unknown, (3..16) shows that channel identification can still be identified. However, the first eigenvector $\mathbf{q}_1$ can only be identified if N is estimated accurately.

4. ESTIMATED WIENER FILTER AND OPTIMUM DELAY

Given the estimate of $\mathbf{h}_1[i]$, optimum equalizer (Wiener filter) can be derived for the first user to yield

$$\widehat{s}_{k-d,1} = \mathbf{g}^H \mathbf{x}[k]. \qquad (4..1)$$

The estimated Wiener filter can be found as

$$\begin{aligned} \mathbf{g} &= \left(E\{\mathbf{x}[k]\mathbf{x}[k]^H\}\right)^{-1} E\{\mathbf{x}[k]s_{k-d,1}^*\} \\ &= \sigma_s^2 R_{m_0}^{-1} \check{\mathbf{h}}_d, \end{aligned} \qquad (4..2)$$

where we have defined

$$\check{\mathbf{h}}_d \triangleq \begin{bmatrix} \mathbf{h}_1[d] \\ \mathbf{h}_1[d-1] \\ \vdots \\ \mathbf{h}_1[0] \\ \mathbf{0} \end{bmatrix}.$$

It is apparent that the estimated Wiener filter based on estimated channel impulse response differs for different system delay d. Since the estimated Wiener filter will result in different performances for different delays, we need to optimize the estimated Wiener filter by minimizing the MSE

$$MSE(d) \triangleq E\{|s_{k-d,1} - \widehat{s}_{k-d,1}|^2\}. \qquad (4..3)$$

The MSE can be found to be

$$MSE(d) = \sigma_s^2 - \sigma_s^4 \check{\mathbf{h}}_d^H R_{m_0}^{-1} \check{\mathbf{h}}_d.$$

Hence, optimum delay can be found by finding d that maximizes

$$\check{\mathbf{h}}_d^H R_{m_0}^{-1} \check{\mathbf{h}}_d.$$

5. SIMULATION

Consider a two user system in which both transmitted signals are i.i.d. QPSK. We select $p = 3$ in all simulations. The two ray multipath channel of user 1 and user 2 are given by

$$h_1(t) = p_1(t) - 0.7p_1(t - T/3),$$
$$h_2(t) = p_2(t) - 0.4p_2(t - 2T/3).$$

The two sampled pulses $p_1(t)$ and $p_2(t)$ are shown in Fig. 5., whereas the overall channel responses are given in Fig. 5..

400 data samples are used to estimate the channel. The two identified channels from 50 trials are shown in Fig. 3. Normalized channel estimation MSE averaged over 50 trials are given in Fig. 4 under various input SNR. Fig. 5 also shows the average MSE for equalized user 1 output under different SNR levels. Finally, the channel output, the Wiener filter output and the estimated Wiener filter output of the QPSK signal of user 1 are shown in Fig. 7..

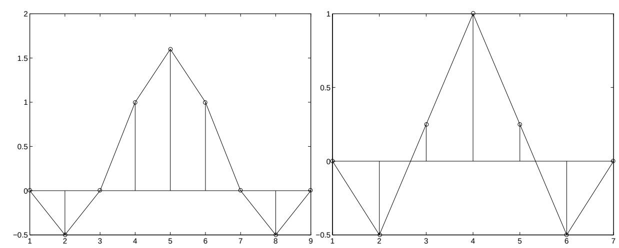

Figure 1. Two pulses $p_1(t)$ and $p_2(t)$.

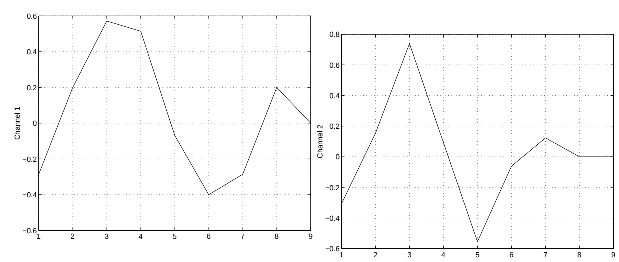

Figure 2. Overall sampled channel $h_1(t)$ and channel $h_2(t)$.

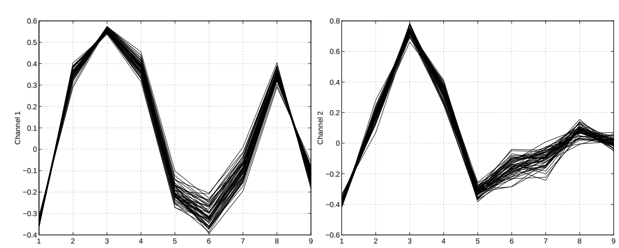

Figure 3. Identified channels under 0dB SNR.

6. CONCLUSION

In this paper, we consider the problem of multi-channel identification and signal separation. We show that specific channel identification and signal separation can be uniquely determined if partial channel and signal information become available. A robust algorithm is presented that will identify the unknown part of the desired signal channel and generate a minimum mean square error Wiener filter. This algorithm is very useful in many communication systems including the IS-95 CDMA systems where a Walsh spreading function is used for each user.

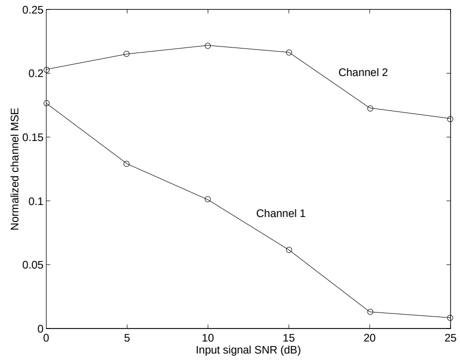

Figure 4. Normalized channel estimation MSE for 20dB SNR.

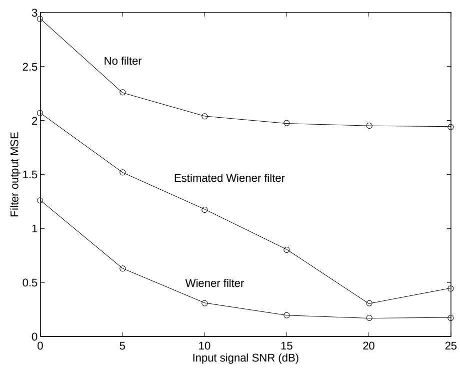

Figure 5. User 1 output MSE for SNR=20dB.

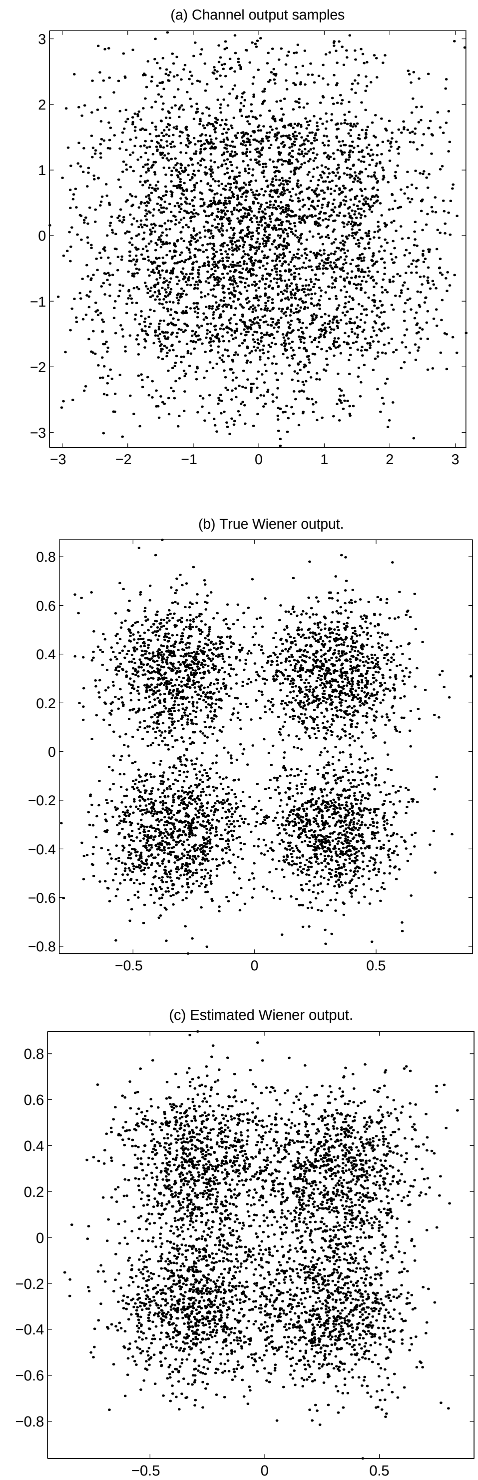

Figure 6. Sampled channel output and Wiener filter outputs at SNR=20dB.

7. ACKNOWLEDGMENT

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