DETECTION OF MULTIPATH RANDOM SIGNALS BY MULTIRESOLUTION SUBSPACE DESIGN

Chuang He and José M. F. Moura

Department of Electrical and Computer Engineering
Carnegie Mellon University
5000 Forbes Ave.
Pittsburgh, PA 15213–3890

ABSTRACT

In our earlier work [1, 2], we developed a robust detector for multipath constrained environments when the transmitted signal is known. In this paper, we extend these results to the case where the transmitted signal is a random process. The approach in [1, 2] is to replace the orthogonal projection on the multipath signal subspace $\mathcal{S}$ by the orthogonal projection on a representation subspace $\mathcal{G}$, such that $\mathcal{G}$ and $\mathcal{S}$ are close in the gap metric sense. When the signal is random, $\mathcal{S}$ is no longer a linear subspace but a set with a given structure. The gap metric applies only when $\mathcal{S}$ and $\mathcal{G}$ are subspaces. In this paper, we introduce the *modified deflection* as the appropriate measure to be used in the random signal case. We design the representation subspace $\mathcal{G}$ to match the multipath signal set $\mathcal{S}$ in the modified deflection sense. Wavelet multiresolution tools are used to facilitate the design.

1. INTRODUCTION

Multipath signal detection has been a problem of major concern in many areas, such as wireless communications, sonar, and radar. When the transmitted signal $s(t)$ is known, the traditional detector is the correlator receiver. The correlator receiver correlates the received signal with the transmitted signal and uses the peaks in the correlator output to detect the signal. It is a simple receiver, but its performance is poor when different delayed replicas of the transmitted signal overlap. On the other hand, the optimal receiver, the generalized likelihood ratio test (GLRT) receiver, is computationally very intensive. It requires maximum likelihood (ML) estimation of the channel parameters, which is a multidimensional nonlinear optimization problem. In our earlier work [1, 2], we developed a new receiver which is simple and can approximate the GLRT receiver. Realizing that the ML estimate of the multipath signal is the orthogonal projection of the received signal on the multipath signal subspace $\mathcal{S}$, we designed a second subspace $\mathcal{G}$, the representation subspace, that is close to $\mathcal{S}$, but whose orthogonal projection is easily computed. "Closeness" between the subspaces $\mathcal{S}$ and $\mathcal{G}$ is measured by the gap metric.

In this paper, we extend our approach by allowing the transmitted signal $s(t)$ to be random. When $s(t)$ is random, the detection problem becomes a composite hypothesis testing problem. The structure of the optimal receiver is complicated because the multipath signal set $\mathcal{S}$ is no longer a subspace. In this paper, we still use the energy of the orthogonal projection of the received signal on a representation subspace $\mathcal{G}$ as the test statistic and we design $\mathcal{G}$ to be close to $\mathcal{S}$ in some sense. However, since in this case, the multipath signal set $\mathcal{S}$ is a set of random processes, it is no longer a subspace and the gap metric used in our earlier work does not apply. A different measure has to be used to measure the similarity between $\mathcal{S}$ and $\mathcal{G}$. We introduce the modified deflection. The modified deflection is a generalization of one side of the gap metric. Being one-sided, it has difficulties. If the modified deflection between $\mathcal{S}$ and $\mathcal{G}$ is zero, then the modified deflection between $\mathcal{S}$ and any subspace that includes $\mathcal{G}$ is also zero. This means that the modified deflection by itself is not sufficient. What is needed is the "smallest" subspace that can minimize the modified deflection. This is where the role of multiresolution analysis [3] comes into play. We want to find the lowest resolution/scale subspace that still minimizes the modified deflection. Once we have designed the representation subspace, we use the energy of the orthogonal projection of the received signal on the representation subspace as the test statistic.

Simulation results using whale sounds show that our new receiver provides on average better performance over some alternative simple receivers.

This work was partially supported by DARPA through AFOSR grant # F49620-96-1-0436.

2. PROBLEM FORMULATION

In this section, we first briefly review the formulation of the multipath detection problem and its geometric interpretation. Then, we will introduce the definition of the modified deflection.

2.1. Multipath detection and its geometric interpretation

The detection problem is a standard binary hypothesis testing problem:

$$H_1 : r(t) = s_m(t) + n(t) \tag{1}$$
$$H_0 : r(t) = n(t) \tag{2}$$

where the multipath noise free signal $s_m(t)$ is

$$s_m(t) = \sum_{k=1}^{K} \alpha_k s(t - \tau_k) \tag{3}$$

We assume that the channel parameters, K, $\{\tau_k\}$, and $\{\alpha_k\}$ are all deterministic unknown. For simplicity, we assume that the noise $n(t)$ is white and Gaussian. In our prior work, we dealt with the case where the transmitted signal $s(t)$ is known. Here, we will assume that $s(t)$ is a zero mean random process and the autocorrelation function $K(t,u)$ of $s(t)$ is either known or can be estimated.

If $s(t)$ is known, the generalized likelihood ratio test (GLRT) statistic is the norm square of the orthogonal projection of the received signal on the multipath signal subspace, i.e.,

$$L = \| P_{\mathcal{S}}(r(t)) \|_2^2 \tag{4}$$

where

$$\mathcal{S} = \left\{ s_m(t) = \sum_{k=1}^{K} \alpha_k s(t-\tau_k), K \in \mathbb{Z}^+, \alpha_k, \tau_k \in \mathbb{R} \right\} \tag{5}$$

is the multipath signal subspace and $P_{\mathcal{S}}$ is the orthogonal projection operator on $\mathcal{S}$.

Evaluating the orthogonal projection $P_{\mathcal{S}}(r(t))$ is not computationally feasible. Instead, in [1, 2], we developed an algorithm to approximate the multipath signal subspace $\mathcal{S}$ by a representation subspace $\mathcal{G}$ whose orthogonal projection is easily computed. We used the gap metric to measure the similarity between $\mathcal{S}$ and $\mathcal{G}$. Once we have designed the representation subspace $\mathcal{G}$, we use

$$L' = \| P_{\mathcal{G}}(r(t)) \|_2^2 \tag{6}$$

as the test statistic.

In this paper, we extend our approach in [1, 2] to include the random signal case. We design a representation subspace $\mathcal{G}$ to match $\mathcal{S}$. However, since $\mathcal{S}$ is no longer a subspace if $s(t)$ is random, the gap metric does not apply. We have to use a different measure. We observe that, with $s(t)$ being a random process, the signal set $\mathcal{S}$ given by (5) is essentially an ensemble of subspaces in that, for a fixed realization of $s(t)$, $\mathcal{S}$ is a subspace. Therefore, our goal is to design a representation subspace $\mathcal{G}$ to approximate $\mathcal{S}$ in an ensemble average sense. To accomplish this goal, we introduce a different measure tailored to the special structure of the signal set $\mathcal{S}$. We propose to use the modified deflection as the measure. Once the representation subspace $\mathcal{G}$ has been designed, the test statistic is as simple as (6).

2.2. Deflection and modified deflection

Since the modified deflection is a modified version of the deflection [4], we first introduce the deflection measure.

Deflection. The definition of the gap metric applies only when both $\mathcal{S}$ and $\mathcal{G}$ are subspaces. If one of them, $\mathcal{S}$ or $\mathcal{G}$, is not a subspace but a general set, we have to use a different measure called deflection [4]. Assuming that $\mathcal{S}$ is a set in L^2 and $\mathcal{G}$ is a subspace in L^2, the deflection between $\mathcal{S}$ and $\mathcal{G}$ is given by

$$\hat{\delta}(\mathcal{S}, \mathcal{G}) = \sup_{u \in \mathcal{S}_S} \operatorname{dist}(u, \mathcal{G}) \tag{7}$$

where

$$\operatorname{dist}(u, \mathcal{G}) = \inf_{v \in \mathcal{G}} \| u - v \|_2 \tag{8}$$

$\mathcal{S}_S$ is the unit sphere of $\mathcal{S}$ and $\operatorname{dist}(u,\mathcal{G})$ is the distance from u to $\mathcal{G}$.

Modified deflection. When the set $\mathcal{S}$ has random elements, we need to modify the deflection measure. Assuming that $\mathcal{S}$ is a set with random elements and $\mathcal{G}$ is a subspace, the modified deflection between $\mathcal{S}$ and $\mathcal{G}$ is defined as

$$\hat{\delta}_{\text{mod}}(\mathcal{S}, \mathcal{G}) = \sup \left\{ E\{\operatorname{dist}^2(u, \mathcal{G})\} \right\}^{1/2} \tag{9}$$

where $E\{\cdot\}$ is the expectation, taken with respect to u, and the supremum is subject to

$$E\{\| u \|_2^2\} = 1 \tag{10}$$

Both the deflection and the modified deflection are one-side measures, i.e., they can only determine how $\mathcal{S}$ is "included" in $\mathcal{G}$, but not vice versa. Specifically, if the modified deflection between $\mathcal{S}$ and $\mathcal{G}$ is zero, then the modified deflection between $\mathcal{S}$ and any subspace that includes $\mathcal{G}$ is also zero. What we really need to find is the "smallest" subspace that can minimize the modified deflection between $\mathcal{S}$ and $\mathcal{G}$. The question is how to quantify the term "smallest". With the multiresolution subspaces, our problem is reduced to finding the lowest resolution/scale subspace that can minimize the modified deflection between $\mathcal{S}$ and $\mathcal{G}$.

3. MULTIRESOLUTION SUBSPACE DESIGN

Our goal is to find the lowest resolution/scale subspace that minimizes the modified deflection between $\mathcal{S}$ and $\mathcal{G}$. In other words, we want to find the multiresolution subspace

$$\mathcal{G}_j = \left\{ \sum_{n=-\infty}^{+\infty} 2^{j/2}\alpha_n g(2^j t - n), \alpha_n \in \mathbb{R} \right\} \tag{11}$$

to minimize the modified deflection between $\mathcal{S}$ and $\mathcal{G}_j$ with the smallest index j. We call the function $g(t)$ the generating function. With the subspace structure given in (11), the subspace design problem is reduced to a functional design problem. We only need to design the generating function $g(t)$. In this paper, we restrict the generating function $g(t)$ to be a compactly supported orthonormal scaling function. In [5], the authors showed that all orthonormal scaling functions of support $2M-1$ are parameterizable by choosing just $M-1$ parameters, $(\zeta_1, \zeta_2, \cdots, \zeta_{M-1})$, over $[0, 2\pi]^{M-1}$. We will use this parameterization in our design algorithm to perform the optimization.

Solving the minimization problem directly is difficult. Instead, we solve the design problem in two major steps. In the first step, we design the generating function $g(t)$ to minimize the modified deflection between an integer shifted signal set $\mathcal{S}_{int}$ given by

$$\mathcal{S}_{int} = \left\{ \sum_{n=-\infty}^{+\infty} \alpha_n s(t-n), \alpha_n \in \mathbb{R} \right\} \tag{12}$$

and $\mathcal{G}_j$.

In the second step, we reshape the optimal generating function $g^*(t)$ obtained from the first step to make it nearly shiftable [6]. The following is the subspace design algorithm

1. Parameterize the generating function $g(t)$ using the parameterization given in [5];
2. Find the generating function $g^*(t)$ and the scale index j^* such that, for a given error threshold ϵ, $\hat{\delta}_{\text{mod}}(\mathcal{S}_{int}, \mathcal{G}_{j^*}) < \epsilon$ and $\hat{\delta}_{\text{mod}}(\mathcal{S}_{int}, \mathcal{G}_{j^*-1}) > \epsilon$;
3. Reshape $g^*(t)$ to make it nearly shiftable using Benno and Moura's algorithm given in [6].

Items 1 and 2 together form the first step. In step 1 of our design algorithm, we want to find a subspace $\mathcal{G}_j$ such that $\hat{\delta}_{\text{mod}}(\mathcal{S}_{int}, \mathcal{G}_j) < \epsilon$ and j is as small as possible. The following theorem provides a way to compute the modified deflection $\hat{\delta}_{\text{mod}}(\mathcal{S}_{int}, \mathcal{G}_j)$.

Theorem 1 *Let $\mathcal{S}_{int}$ be the set given by (12) and $\mathcal{G}_j = \{\sum_n 2^{j/2}\alpha_n g(2^j t - n)\}$ where $g(t)$ is a compactly supported orthonormal scaling function. Then the modified deflection between $\mathcal{S}_{int}$ and $\mathcal{G}_j$ is given by*

$$\hat{\delta}_{\text{mod}}(\mathcal{S}_{int}, \mathcal{G}_j) = \sqrt{1 - \inf_{f \in [0,1)} C(f)} \tag{13}$$

where $C(f)$ is

$$\frac{\sum_{m=1}^{\infty} \sigma_m^2 C_{hg}^m(f)}{\sum_{m=1}^{\infty} \sigma_m^2 \sum_{n=-\infty}^{+\infty} |\mathcal{F}_{h_m}(f+n)|^2} \tag{14}$$

The functions $\{C_{hg}^m(f), m = 1, \cdots, \infty\}$ are given by

$$\sum_{k=0}^{2^j-1} \left| \sum_{n=-\infty}^{+\infty} \mathcal{F}_{h_m}(f + 2^j n + k) \mathcal{F}_g^*(2^{-j} f + n + 2^{-j} k) \right|^2 \tag{15}$$

where $\{h_m(t), m = 1, \cdots, \infty\}$ and $\{\sigma_m^2, m = 1, \cdots, \infty\}$ are the eigenfunctions and eigenvalues of the autocorrelation function $K(t,u)$. $\mathcal{F}_{h_m}(f)$ and $\mathcal{F}_g(f)$ are the Fourier transforms of h_m and $g(t)$ respectively. $\mathcal{F}_g^*(f)$ is the complex conjugate of $\mathcal{F}_g(f)$. The infimum is taken over the regions where the function $C(f)$ is continuous.

Despite the formidable appearance of equation (14), it is very easy to compute. The terms on the numerator $\{C_{hg}^m(f), m = 1, \cdots, \infty\}$ are essentially the discrete time Fourier transform (DTFT) of the downsampled autocorrelation sequences

$$R_m^j[l] = \sum_{k=-\infty}^{+\infty} c_m^j[k] c_m^j[2^j l + k] \qquad m = 1, \cdots, \infty \tag{16}$$

The sequences $\{c_m^j[k],\ m = 1, \cdots, \infty\}$ are given by

$$< h_m(\cdot)\ ,\ 2^{j/2} g(2^j \cdot - k) > \qquad m = 1, \cdots, \infty \tag{17}$$

which consist of the orthogonal projection coefficients of $\{h_m(t),\ m = 1, \cdots, \infty\}$ on the subspace $\mathcal{G}_j$.

4. EXPERIMENTAL RESULTS

We test the performance of our new receiver using a database of whale sounds and compare it with some other alternative receivers. The database consists of 50 realizations of whale sounds. We separate the database into two sets: a training set and a testing set. The training set is used to estimate the autocorrelation function of the signal $s(t)$ and the testing set is used to test the performance. The whale sounds in the database are not multipath signals. We simulate the multipath effect by generating the channel parameters using a random number generator.

We use the algorithm described in section 3 to design the generating function of the representation subspace. Two parameters: $[\zeta_1, \zeta_2] \in [0, 2\pi]$, are used. We choose two parameters because in our simulation, it provides a good tradeoff between the computational complexity and the performance. The optimization is done by computing the modified deflection for $\zeta_k = 2\pi l/50, \quad k = 1, 2, \; l = 0, \cdots, 49$ and finding the $\zeta^* = [\zeta_1^*, \zeta_2^*]$ that leads to the minimum value of the modified deflection. The optimal scaling function $g^*(t)$ is reconstructed using ζ^*. Then we reshape it to make it nearly shiftable.

Then, we test the performance of our receiver with the reshaped optimal generating function. The number of paths K is set to 8. For simplicity, we set all attenuation factors $\{\alpha_k\}$ to be equal to 1. The set of delays $\{\tau_k, k = 1, \cdots, 8\}$ are generated by a random number generator. A total of 100 delay patterns are generated.

Fig. 1 shows the average detection probability P_D as a function of the signal-to-noise ratio (SNR). The average is taken over all the testing samples and all the delay patterns. The false alarm probability P_F is fixed at 0.01. There are 5 curves in the figure. The solid line is obtained by assuming perfect knowledge about the transmitted signal $s(t)$, the number of paths K, and the delays $\{\tau_k\}$. It is an over optimistic performance bound. The dashed line is our new receiver. The dashdotted and the dotted line represent the correlator receiver and the "Matched Filter with Integer Shifts" MFIS receiver [1] respectively. The "+" curve represents the energy detector.

Since the transmitted signal is random, the correlator receiver correlates the received signal with the most significant eigenfunction of $K(t, u)$. The MFIS receiver matches the received signal with integer shifts of the most significant eigenfunction of $K(t, u)$ and uses the sum of the magnitude square of the matching coefficients as the test statistic.

Fig. 1 shows that our receiver provides an average gain of about 3.0dB over the correlator receiver, a gain of about 1.4dB over the MFIS receiver and a gain about 2.5dB over the energy detector. Since delay patterns are chosen so that the delayed replicas of the transmitted signal have considerable overlap, the correlator receiver performs very poorly.

5. SUMMARY

In this paper, we develop a robust signal detector for multipath channels when the transmitted signal is random. Our approach is based on a geometric interpretation of the multipath detection problem. The new test statistic is the energy of the orthogonal projection of the received signal on a multiresolution representation subspace. The representation subspace is designed to match the multipath signal set. A new measure, the modified deflection, is proposed to compare the signal set with the representation subspace. Simulation results show the improvement in performance over the traditional correlator receiver, the MFIS receiver and the energy detector.

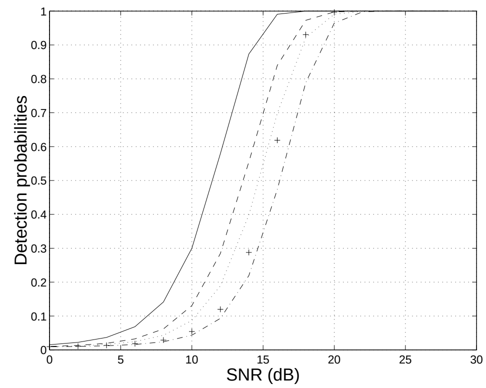

Figure 1: The average detection probabilities, $K = 8$, $P_F = 0.01$. The solid line is a performance bound, "$--$" is the new receiver, "$-.$" is the correlator receiver, "$\cdots$" is the MFIS receiver, and "+" is the energy detector.

6. REFERENCES

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