ROBUST SOURCE DETECTION IN SHALLOW WATER

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ABSTRACT

It is not possible, in practice, to precisely model a complex propagation channel, such as shallow water. This lack of accuracy causes a deterioration in the performance of the optimal detector and motivates the search for sub-optimal detectors which are insensitive to uncertainties in the propagation model. We present a novel, robust detector, which measures the degree of *spatial-stationarity* of the received field, exploiting the fact that a signal propagating in a *bounded* channel induces non-spatial-stationarity. The performance of the proposed detector is evaluated using both simulated data and experimental data collected in the Mediterranean Sea. This performance is compared to those of three other detectors, employing different extents of prior information. It is shown that when the propagation channel is not completely known, as is the case of the experimental data, the novel detector outperforms the others. That is, this detector couples good performance with robustness to propagation uncertainties.

1. INTRODUCTION

Detection of a signal in noise is a classic problem with applications in many fields. The optimal procedure, i. e., the likelihood ratio test, can only be applied when the joint probability density function (pdf) of the received signal and noise are known. This, in turn, requires perfect knowledge of the propagation channel. Shallow water, however, is an example for a propagation channel that is, in practice, too complex to be fully characterized. It is therefore necessary to revert to sub-optimal methods relying on lesser degrees of prior knowledge of the propagation channel. In this work, we compare the performance of four detectors that employ different extents of such information. These detectors range from the optimal two-sided maximum-likelihood detector to the simple and robust energy-detector. In particular, we study a novel detector first suggested in [2].

This detector is based on the following condition: if a field is spatially-stationary, then the generating sources must be uncorrelated and located in the far field zone. Propagation of a signal in a *bounded* channel, such as an underwater one, causes correlative echoes of the source (multipath) to reach the receiver, thereby violating this condition. These echoes increase the level of non-spatial-stationarity of mere underwater background noise. This, then, is the motivation for using the level of spatial-stationarity as a measure for the existence of a source. It is apparent that for such a detector the only prior information assumed is that the propagation channel is bounded, and that the additive noise is of a low degree of non-spatial-stationarity. That is, the *exact* nature of the propagation channel, is of no direct importance, thus, we expect the proposed detector to be robust to channel uncertainties.

In order to compare the performance of the stationarity-based detector, as well as the other detectors, we used both simulated data and experimental data. The simulations were carried out on one of the benchmarks ("genlmis") used in the May 1993 NRL Workshop on Acoustic Models in Signal Processing [7]. Experimental results were obtained by processing experimental data collected in the Mediterranean Sea by the NATO SACLANT Center [4].

We proceed next with a formulation of the detection problem. A brief[1] description is then given for the novel spatial-stationarity detector and the three other detectors. Simulation and experimental results are presented, followed by a discussion.

2. PROBLEM FORMULATION AND DEFINITIONS

Consider a stochastic, temporally-ergodic, temporally-stationary, field, $y(t, \boldsymbol{X})$. t denotes the time coordinate and $\boldsymbol{X}$ is a vector representing a point in a spatial coordinate system. The field is assumed to exist in a horizontally-bounded channel, and is spatially sampled using a uniform vertical array of P sensors located at known depths $\{\boldsymbol{X}_p\}_{p=1}^{P}$. The field is also assumed narrow-band, and is temporally sampled at L instants, $\{t_l\}_{l=0}^{L-1}$, so that the temporal samples at different l's are uncorrelated. Denote for brevity: $y(l,p) = y(t_l, \mathbf{X}_p)$ and $\boldsymbol{y}(l) \equiv [y(l,1), \ldots, y(l,P)]^T$ With these available tempo-spatial samples, we confront the classical detection problem in which a decision must be made between the following two hypotheses:

$$\begin{aligned} H_0 &: \boldsymbol{y}(l) = \text{noise only} \\ H_1 &: \boldsymbol{y}(l) = \text{signal + noise.} \end{aligned} \tag{1}$$

Under both hypotheses, $\boldsymbol{y}$ is assumed zero-mean with a spatial covariance matrix defined as:

$$[\boldsymbol{Cov_y}]_{p_1,p_2} = Cov_y(0;p_1,p_2) \equiv E\left\{y(l;p_1)y^*(l;p_2)\right\} \tag{2}$$

where the independence of l is due to the temporal-stationarity of $\boldsymbol{y}$.

[1] Detailed descriptions and analyses of these detectors are given in [3].

The hypotheses (1) are only vaguely defined. This enables us to pose variations of these hypotheses for each considered detector, underlining the particular principle on which it is based. The only common assumption for all detectors is that the signal, s, and noise, $\boldsymbol{n}$, are temporally-stationary, zero-mean, independent processes. The spatial covariance matrix of the noise is defined as:

$$[\boldsymbol{Cov_n}]_{p_1,p_2} = Cov_n(0;p_1,p_2) \equiv E\left\{n(l;p_1)n^*(l;p_2)\right\}. \quad (3)$$

3. THE PROPOSED DETECTOR

The proposed detector is based on a test for spatial-stationarity proposed in [2]. This test is in fact a *weighted* energy test, where by *weighted* refers to extracting the non-spatially-stationary part of the received field. The application of this test to the decision problem (1) is based on the fact that the existence of sources in a bounded propagation channel increases the *non-spatial-stationarity* level of the field. If the non-spatially-stationary part of the additive noise is sufficiently low, we expect that the level of non-spatial-stationarity would serve as a good measure for the existence of a source. We can therefore restate the hypotheses problem (1):

H_0 : The field is spatially-stationary to a low (or, to a *known* arbitrary) degree.

H_1 : The field is highly non-spatially-stationary.

Extracting the non-spatially-stationary part of the sampled signal is achieved via the second-order Spatial Cumulant-Spectrum (SCS):

$$SCS_y(\kappa_1,\kappa_2) \equiv \sum_{p_1,p_2=-\infty}^{\infty} Cov_y(0;p_1,p_2)e^{-j(p_1\kappa_1+p_2\kappa_2)} \quad (4)$$

where $Cov_y(0;p_1,p_2)$ has been defined in (2), and κ denotes a spatial angular frequency [5]. The spatial cumulant-spectrum is well defined for any field. Theoretically, however, when spatial-stationarity holds, $SCS_y(0;\kappa_1,\kappa_2)$ vanishes *identically* (e. g., [1]) for $\kappa_1+\kappa_2 \neq 0$. In the practical case, when only P spatial samples are available, we expect the spatial cumulant-spectrum matrix, $[\boldsymbol{SCS_y}]_{\kappa_1,\kappa_2} = SCS_y(\kappa_1,\kappa_2)$, to be nearly diagonal[2]. We therefore intuitively define the non-spatially-stationary part of the field's energy as the sum of the magnitude-square of all the non-diagonal elements. This measure is thus proposed as a basis for deciding H_0 or H_1.

The following portrays the algorithm for this detector:

1. Estimate the second-order spatial covariance matrix (y is temporally ergodic):

$$\widehat{\boldsymbol{Cov_y}} = \frac{1}{L}\sum_{l=0}^{L-1}\boldsymbol{y}(l)\boldsymbol{y}^H(l). \quad (5)$$

[2]In fact, the finite number of spatial samples introduces a certain *leakage* from the diagonal to the non-diagonal elements of the spatial cumulant-spectrum matrix. This leakage is assumed to be negligible in comparison to the diagonal elements.

2. Estimate the second-order spatial cumulant-spectrum matrix:

$$\left[\widehat{\boldsymbol{SCS_y}}\right]_{\kappa_1,\kappa_2} = \widehat{SCS}_y(\kappa_1,\kappa_2) = \sum_{p_1,p_2=1}^{P} e^{-j(\kappa_1 p_1+\kappa_2 p_2)}\left[\widehat{\boldsymbol{Cov_y}}\right]_{p_1,p_2}. \quad (6)$$

3. Form the test statistic as the sum of the magnitude-square of the off-diagonal elements of (6), and perform the test:

$$\zeta = \sum_{i\neq j}\left|\left[\widehat{\boldsymbol{SCS_y}}\right]_{\kappa_i,\kappa_j}\right|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \gamma. \quad (7)$$

where the threshold γ determines the probability of "false alarm", P_{fa}.

The test procedure has been well defined without any information on the distribution of the received signal. However, to fix γ, the distribution of the noise (H_0) is required[3].

4. THE ALTERNATIVE TESTS

It is instructive to compare the proposed novel detector to others that combine different degrees of computational complexity, a-priori assumptions and performance. In particular, we selected an energy detector (ED), a one-sided likelihood ratio test (LRT1), and a two-sided generalized likelihood ratio test (GLRT2).

The energy detector (ED) uses a simple comparison of the measured energy of the received signal to a threshold:

$$\zeta_0 = trace\left(\widehat{\boldsymbol{Cov_y}}\right) \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_0. \quad (8)$$

where $\widehat{\boldsymbol{Cov_y}}$ is defined in (5). The test statistic is thus ignorant of any prior information. However, as in the case of the novel detector, setting the threshold, γ_0, requires knowledge of the distribution of the noise (H_0).

The next detector is a one-sided likelihood ratio test (LRT1) which accepts or reject the noise-only hypothesis. To avoid full specification of the distribution of the ocean noise (usually unknown), H_0 is only specified in terms of the spatial covariance matrix of the noise (can be estimated in practice). The null hypothesis is thus defined:

$$H_0: \quad \boldsymbol{Cov_n} = \boldsymbol{\Lambda_n}, \quad \boldsymbol{\Lambda_n} \text{ is known.}$$

To perform the test, $\widehat{\boldsymbol{Cov_y}}$ is obtained via (5), and its elements are strung in a vector, $\boldsymbol{v} = \left[\boldsymbol{v_1}^T;\boldsymbol{v_2}^T\right]$, where $\boldsymbol{v_1}$ contains the diagonal elements and $\boldsymbol{v_2}$ contains the off-diagonal elements. $\boldsymbol{v}$ is then tested to posses its predetermined mean and covariance under H_0.

[3]An example derivation for the distribution of $\zeta|_{H_0}$ is shown in [3]

For example, when the noise process is Gaussian, $\boldsymbol{\Lambda_n} = \sigma_n^2 \boldsymbol{I}$, the sample mean and covariance can be analytically found, and LRT1 assumes the form:

$$\zeta_1 = \frac{L}{\sigma_n^4} \left[\left\| \boldsymbol{v_1} - \sigma_n^2 \mathbf{1} \right\|^2 + \|\boldsymbol{v_2}\|^2 \right] \underset{\text{reject } H_0}{\overset{\text{accept } H_0}{\lessgtr}} \gamma_1 \qquad (9)$$

where $\mathbf{1}$ is an all-ones vector, and γ_1 is a threshold that can be fixed for a given P_{fa}.

The last test considered, (GLRT2), is a two-sided generalized likelihood ratio test for the hypotheses:

$$\begin{aligned} H_0 : \ \boldsymbol{y}(l) &= \boldsymbol{n}(l) \\ H_1 : \ \boldsymbol{y}(l) &= \sigma_s \boldsymbol{g}(r_s, z_s, \boldsymbol{\theta}) s(l) + \boldsymbol{n}(l). \end{aligned} \qquad (10)$$

$\boldsymbol{g}(\cdot)$ is a known spatial transfer function from the source to the array, and is dependent on unknown parameters: r_s, the distance of the source from the array; z_s, the depth of the source; and $\boldsymbol{\theta}$, parameters characterizing the propagation channel. $\boldsymbol{g}(\cdot)$ is normalized so that σ_s^2 is the unknown total signal power at the array. We assume Gaussian distributions for the signal and for the noise (zero mean, $\boldsymbol{Cov_n} = \sigma_n^2 \boldsymbol{I}$). Under these assumptions, the detector, which involves a multi-dimensional search (over $r_s, z_s, \boldsymbol{\theta}$), becomes:

$$\zeta_2 = \min_{r_s, z_s, \boldsymbol{\theta}} [ln\phi(\boldsymbol{y}, r_s, z_s, \boldsymbol{\theta}) - \phi(\boldsymbol{y}, r_s, z_s, \boldsymbol{\theta})] \underset{H_1}{\overset{H_o}{\gtrless}} \gamma_2. \qquad (11)$$

$\phi(\boldsymbol{y}, r_s, z_s, \boldsymbol{\theta}) \equiv \frac{\boldsymbol{g}^H \widehat{\boldsymbol{Cov_y}} \boldsymbol{g}}{\sigma_n^2 \boldsymbol{g}^H \boldsymbol{g}}$ and $\widehat{\boldsymbol{Cov_y}}$ is defined in (5).

5. SIMULATION RESULTS

The four detectors were tested using simulated data. The propagation model used for the underwater channel is one of the benchmarks ("genlmis") defined in the May 1993 NRL Workshop on Acoustic Models in Signal Processing [7]. This model is specified using seven unknown channel parameters and two unknown source-location parameters. The field in this channel was simulated[4] with a narrow band point source and additive temporally and spatially white Gaussian noise. 1000 frames, each of 100 temporal snapshots of the received field, were obtained by a uniform, vertical array of 13 sensors whose aperture is the depth of the water layer.

The detectors were implemented as described in (7), (8), (9) and (11). In (11), however, due to the computational complexity of the required minimization, the parameter search was carried over r_s and z_s (and the used $\boldsymbol{\theta}$ were the correct ones). Fig. 1 depicts the probability of detection (P_D) for each detector for different signal-to-noise ratios. For each detector the threshold was fixed to correspond to $P_{fa} = 10^{-3}$.

6. EXPERIMENTAL RESULTS

In addition to the simulations, we applied experimental data to the four detectors. The data were collected by the SACLANT Center in a shallow water area off the Italian coast in Oct. 1993. A detailed description of the experimental setup and data may be found in [4]. We were particularly interested in the data collected on Oct. 27, where a support ship towed a source away from an array of 48 hydrophones. The source transmitted in the 160–180Hz frequency band only 30 seconds out of every 60 seconds. These data were filtered, decimated, and grouped into frames, each of 100 complex snapshots (corresponding to 6sec approximately[5]).

From the available data we obtained 50 frames of noise only (n_k), and 50 frames of signal+noise (s_k). The LRT1 and GLRT2 detectors assume that $\boldsymbol{Cov_n}$ is known. It was therefore necessary to estimate the noise covariance matrix so that the data could be pre-whitened for these two detectors. This was done using 5 of the noise-only frames. Comparing the performances of the detectors at varying signal-to-noise ratios (snr) was not directly possible because the s_k frames are of a fixed and unknown snr. We therefore constructed compound frames, y_k, of varying *relative* snr's by adding together the remaining 45 signal+noise and noise-only frames:

$$y_k = s_k + (\alpha - 1) n_k, \qquad k = 1, \ldots, 45.$$

That is, α controls the relative snr of the compound frames y_k, and $20log(\alpha)$ measures this relative signal-to-noise ratio.

The 45 compound frames were processed using the four described detectors. As in the simulated data, the enormous computational burden involved with the multi-dimensional minimization in (11) forced us to a concession. Instead of searching over all unknown parameters, we searched only over the source's depth and range, confining our search to the approximate locations given in [4]. The rest of the parameters, however, were taken from [4]: some (depth, sound-speed profile) were measured on the day of the experiment, others, taken from measurements done on previous occasions in the nearby area. Another limitation we face is the lack of knowledge of the distribution of the ocean noise. The detectors' thresholds were therefore obtained by processing the noise-only frames, $\{n_k\}_{k=1}^{45}$ and selecting values matching a false-alarm probability of 1/45. Fig. 2 shows the total number of detections (of 45 frames) for different relative signal-to-noise ratios.

7. DISCUSSION

It is instructive to study the simulation results together with the experimental results. GLRT2, employing information on both H_0 and H_1 is clearly the best detector in the simulations, where that information is indeed correct. However, in practice, the propagation model under H_1 is only known to a certain degree, if at all. This fact is demonstrated in the experimental data where the channel model is only an approximation (based on measurements and historic data) to the true one. In this case, GLRT2 is seen in Fig. 2 to have an erratic behavior and its performance is drastically degraded. In theory, GLRT2 can be improved by carrying out the multi-dimensional search referred to in (11). However, this procedure is so computationally demanding that we consider it as impractical. We therefore regard GLRT2's performance in Fig. 1 (where the channel

[4]Using a normal mode propagation program, KRAKEN [6].

[5]Cruising at 3.5kn, the source progressed appx. 10m in 6sec.

parameters are known exactly) as an unreachable *bound* in practice, rather than as achievable.

Ranking next in performance in Fig. 1 is LRT1. This detector assumes knowledge of the noise spatial-covariance matrix under H_0. In the simulations, this information is indeed accurate. However, in the experimental results, this covariance is unknown and must be *estimated*. Using this estimate the sampled field are pre-whitened before applying LRT1. This procedure introduces mismatches between the expected noise covariance under H_0 and the measured one. Fig. 2 demonstrates the drop in performance of LRT1 that occurs in practice due to this mismatch. Indeed, it performs the worst.

On the other hand, the energy detector that performed the worst in the simulations (Fig. 1), is not a bad option in the practical case. This detector assumes no prior information at all, hence its poor performance in the simulations. However, this also makes the ED robust in cases where the a-priori information is inaccurate. This is illustrated in Fig. 2, where the ED exhibits a performance corresponding to that of GLRT2.

The proposed, novel detector, is seen in Fig. 1 to have a comparable performance to those of the energy detector and LRT1. However, in the practical case of the experimental data, it performs the best (Fig. 2). That is, the qualitative nature of the assumptions in the novel detector, i. e., the difference in degree of spatial-stationarity, is enough to grant it both performance and robustness. Note that in both simulation and experimental results, the novel detector outperforms the ED. We thus deduce that the energy of the *non-spatially-stationary* part of the field is a better indicator for a source in a bounded channel than is the field's total energy.

8. SUMMARY

In selecting a detector one searches for good performance, robustness, and a low computational complexity. In this paper, we compared four detectors for a source in shallow water, in terms of these specifications using both simulated and experimental data. It was demonstrated that GLRT2, optimal in theory, is not robust to channel uncertainties and has a heavy computational demand. LRT1, which is easier to implement is also not robust and its performance decreases in practice. An easily implemented, novel detector, based on a test for spatial-stationarity was shown to have the best overall performance↔robustness tradeoff.

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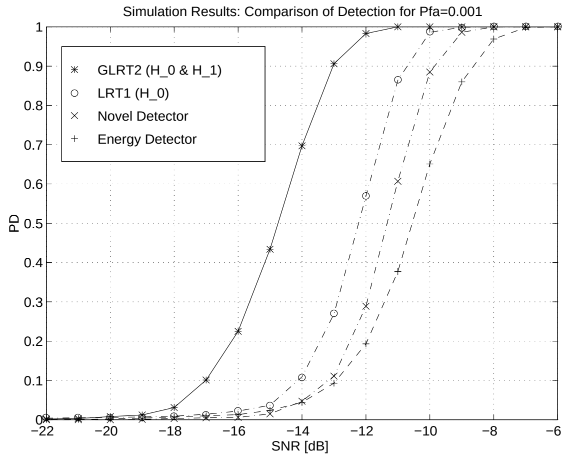

Figure 1: Simulation Results: Probability of detection as a function of SNR.

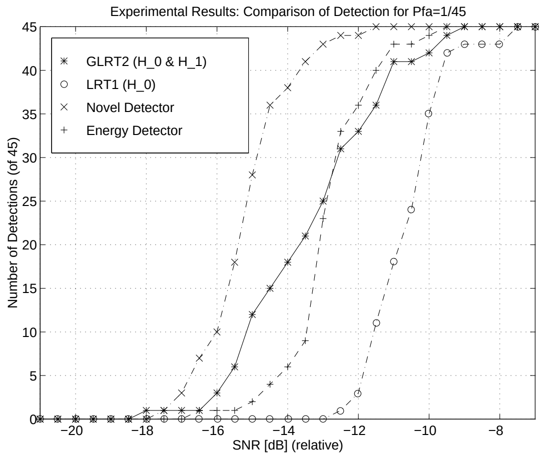

Figure 2: Experimental Results: Power of detection as a function of relative SNR.

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